About Use of Methods of Convex Programming for Synthesis of Conformal Arrays with Matched Dual-polarized Patterns

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Abstract

The solution of a task of the analysis and collecting polarizing information can improve considerably possibilities of radars in various appendices, such as: detection, assessment and tracking of radar targets. This task for a cage antenna lattice with the standard dual-polarized patterns is formulated in terms of convex optimization. The possibilities of the solution of an objective by means of a special Matlab CVX toolbox and various classical algorithms of convex optimization are considered. Also comparison of the results of the solution of an optimizing task received in the different ways is presented.

1 Introduction

At present, the directions of using mathematical optimization methods in radar problems are actively developing. Obtaining and analyzing polarization information in backscattered scattered waves can greatly improve the radar capabilities in various applications such as: detection, evaluation and tracking of various objects. To capture polarization information, the radar must measure two components of the orthogonal polarization of the target, which is the so-called polarization diversity. Taking into account the diversity of polarizations, the synthesis of a beam with the desired level of polarization and low side lobes is a new topic in studies for polarimetric radars that has attracted much attention in recent years. In the solution of this problem, various approaches have been applied in various works such as the iterative method and the method of least squares.

The idea of the above methods is to synthesize a beam with a high degree of polarization purity and low lateral lobes, but in some situations, for example high-frequency polarimetric radar measurements, alignment of beams between the double polarization directivity patterns is required. The authors of [Wanqiu Hu et al., 2015] attempted to transform the task to the problem of convex programming for the synthesis of conformal arrays in antenna arrays of arbitrary configuration with matched dual-polarization beams. This approach proved to be more effective for solving the problem of the synthesis of conformal arrays, in comparison with those considered in earlier works. In connection with these, the task arose, in more detail to open the topic of developing and projecting software for the synthesis of conformal arrays for cylindrical type antennas.

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2 Formulation

2.1 General Case

The formulation of the optimization problem is described in [Wanqiu Hu et al., 2015]. The object of research in the problem of synthesis of conformal arrays is the functions of co- and crosspolarization:

$$E^{\phi}(\theta,\phi) = \sum_{m=1}^{M} \omega_m^{\phi} exp(jka_r \cdot R_m) \begin{bmatrix} E_{m\phi}^{\phi}(\theta,\phi) \\ E_{m\theta}^{\phi}(\theta,\phi) \end{bmatrix}$$

$$E^{\theta}(\theta,\phi) = \sum_{m=1}^{M} \omega_m^{\theta} exp(jka_r \cdot R_m) \begin{bmatrix} E_{m\phi}^{\theta}(\theta,\phi) \\ E_{m\theta}^{\theta}(\theta,\phi) \end{bmatrix}$$

Here ϕ and θ indicate the direction of the angle in the spherical coordinate system with respect to which the polarization is measured. M is the total number of elements of the antenna array. ω^{ϕ} and ω^{θ} are complex vectors whose elements denote the excitation of the corresponding elements of the antenna array. Both these quantities are the desired characteristics in the original problem. $E_{m\phi}^{\phi}$, $E_{m\theta}^{\phi}$, $E_{m\phi}^{\theta}$ and $E_{m\theta}^{\theta}$ is components the polarization functions of the element with the number m. For different types of antenna arrays, these functions will be slightly different. Similarly, the polarization functions of a single element depend on the type of antenna.

2.2 Cylindrical Antenna Array

The authors of the article set and solved the task of implementing software for the synthesis of conformal arrays in the case of a cylindrical antenna array. By changing the parameters of the antenna array, we can obtain solutions for all possible configurations. In the case of a cylindrical antenna array [Voskresensky, 2012], the formulas take the following form:

$$E^{\phi}(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} \omega_{m,n}^{\phi} exp(j \cdot 2\pi(a \cdot sin(\theta) \cdot cos(\phi - \phi_m) - cos(\theta) \cdot z_n)) \cdot \overline{E_m^{\phi}(\theta,\phi)}$$

$$E^{\theta}(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} \omega_{m,n}^{\theta} exp(j \cdot 2\pi(a \cdot sin(\theta) \cdot cos(\phi - \phi_m) - cos(\theta) \cdot z_n)) \cdot \overline{E_m^{\theta}(\theta,\phi)}$$

ω^{ϕ} and ω^{θ} we represent in the form of a matrix since the configuration of a rectangular grid is natural for the arrangement of elements on a cylindrical antenna array. a is the diameter of the base of the antenna cylinder.

$$\overline{E_m^{\phi,\theta}(\theta,\phi)} = \overline{E_0^{\phi,\theta}(\theta,\phi - \phi_m)}$$

$$\begin{cases} \overline{E_0^{\phi}(\theta,\phi)} = \begin{bmatrix} cos(\phi)sin(\theta) \\ sin(\phi)cos(\theta) \end{bmatrix}, |\phi| < \pi/2 \\ \overline{E_0^{\phi}(\theta,\phi)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, |\phi| \geq \pi/2 \end{cases}$$

$$\begin{cases} \overline{E_0^{\theta}(\theta,\phi)} = \begin{bmatrix} cos(\phi)sin(\theta) \\ 0 \end{bmatrix}, |\phi| < \pi/2 \\ \overline{E_0^{\theta}(\theta,\phi)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, |\phi| \geq \pi/2 \end{cases}$$

It is seen that the polarization of a single element vanishes at an angle of $|\phi| \geq \pi/2$, this is due to the fact that a quenching winding is located in this antenna region.

ϕ_m we define in such a way that to center the main beam on $\phi = 0$:

$$\phi_m = \frac{(m - \frac{M+1}{2})}{2a}$$

$$z_n = \frac{n}{2}$$

When solving the problem in our case, we introduce some additional conditions:

$$ME = \left| E_{\phi}^{\phi}(\theta_d, \phi_d) - E_{\theta}^{\theta}(\theta_d, \phi_d) \right|$$

Thus, since there is no need to consider more than one point in the region of the fundamental beam, we take L = 1. (θ_d, ϕ_d) is the direction angle of the main beam. We also skip the step of calculating the parameter ς^{θ}, because the polarization component $E_{\phi}^{\theta} \equiv 0$. The parameter τ is defined as $\tau = max(\tau^{\phi}, \tau^{\theta})$, the parameter ς as ς^{ϕ}. In the rest, the implementation of the method remains pre-empted. Areas of limiting the side lobes and the level of cross-polarization are assumed to be equivalent:

$$\Omega_P = \Omega_S = ((\theta, \phi) : \theta = \theta_d, \phi \in \left[-\pi; -\Delta_w\right] \cup \left[\Delta_w; \pi\right])$$

Where Δ_w user-defined parameter.

3 Realization

When solving the problem, we had to work with complex-number vectors. The most convenient means for this was the CVX toolbox system [CVX Users Guide, 2012]. CVX is a modeling system based on Matlab for convex optimization. CVX turns Matlab into a modeling language, allowing you to define constraints and objective functions using the standard syntax of Matlab expressions. In its mode, by default it supports a special approach to convex optimization, called disciplined convex programming. In this approach, convex sets are constructed from a small set of rules of convex analysis starting from the base library of convex functions. Constraints and objective functions expressed through these rules are automatically converted into a canonical form and resolved. Disciplined convex programming is a methodology for constructing convex optimization problems proposed by Michael Grant, Stephen Boyd, and Yinyu Ye. It is meant to support the formulation and construction of optimization problems that the user intends from the outset to be convex.

Disciplined convex programming imposes a set of conventions or rules, which we call the DCP ruleset. Problems which adhere to the ruleset can be rapidly and automatically verified as convex and converted to solvable form. Problems that violate the ruleset are rejected even when the problem is convex. That is not to say that such problems cannot be solved using DCP; they just need to be rewritten in a way that conforms to the DCP ruleset. A detailed description of the DCP ruleset is given in The DCP ruleset. It is extremely important for anyone who intends to actively use CVX to understand it. The ruleset is simple to learn, and is drawn from basic principles of convex analysis. In return for accepting the restrictions imposed by the ruleset, we obtain considerable benefits, such as automatic conversion of problems to solvable form, and full support for nondifferentiable functions. In practice, we have found that disciplined convex programs closely resemble their natural mathematical forms. CVX solves the problems of convex programming using iterative methods. This system is flexible and has the ability to work with precision. To solve the task in the Matlab environment, it is necessary to implement the following functional modules:

- Co-polarization and cross-polarization functions;
- Functions for determining and selecting optimal parameters τ and ς;
- Basic computational function;
- Module for output and registration of graphs of the results.

The procedures for calculating the constraint parameters are represented by four functions that accept the configuration data for the antenna array and return the optimal parameters found. In the main calculation module, the parameters are chosen as a maximum among the values of the performance results of both computational procedures relating to each of the parameters.

The main computational module is a fully automated procedure that extracts input data from a file and writes the result of the work to another output file.

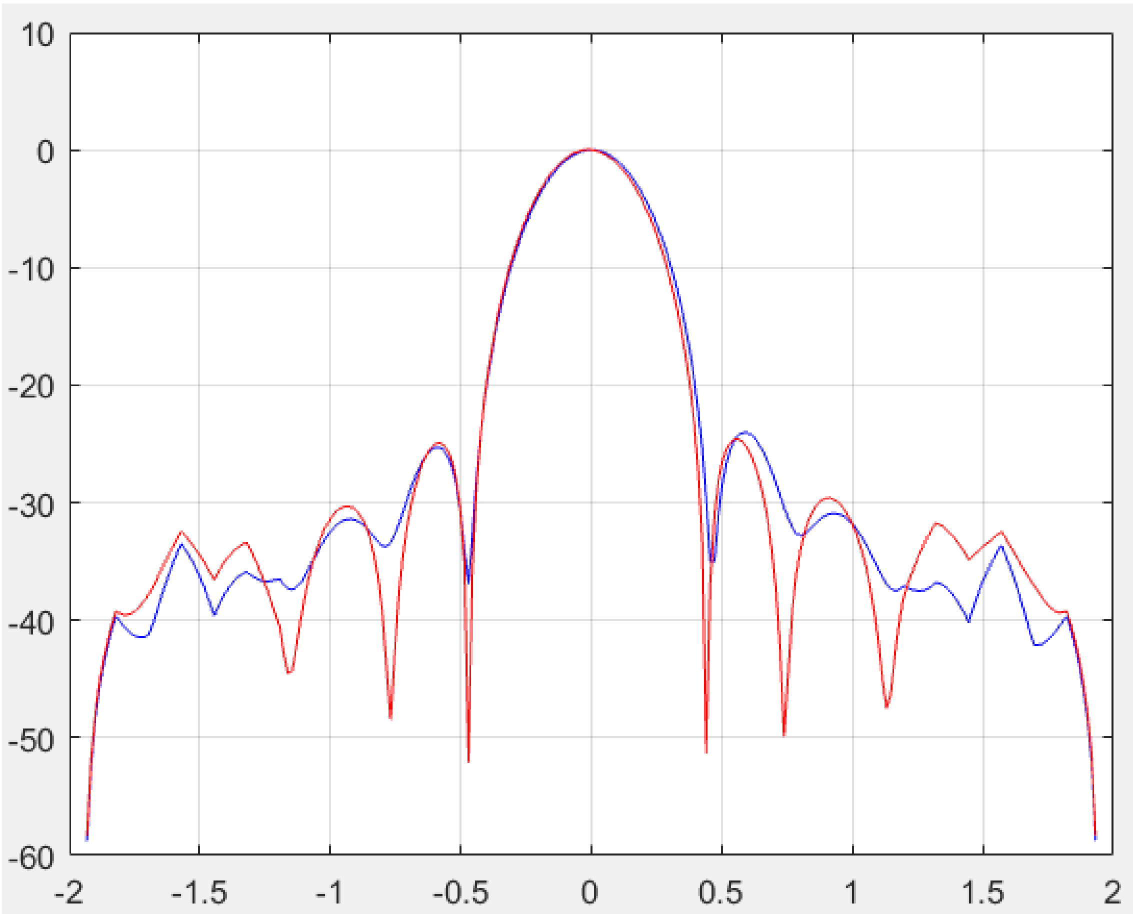

Figure 1: As a first example, let us consider the results of the program for a 7x7 antenna array with a base diameter of 4.0 and the direction of the main ray $(\theta_d, \phi_d) = (\pi\ 3.0)$, $\Delta_w = \pi/9$.

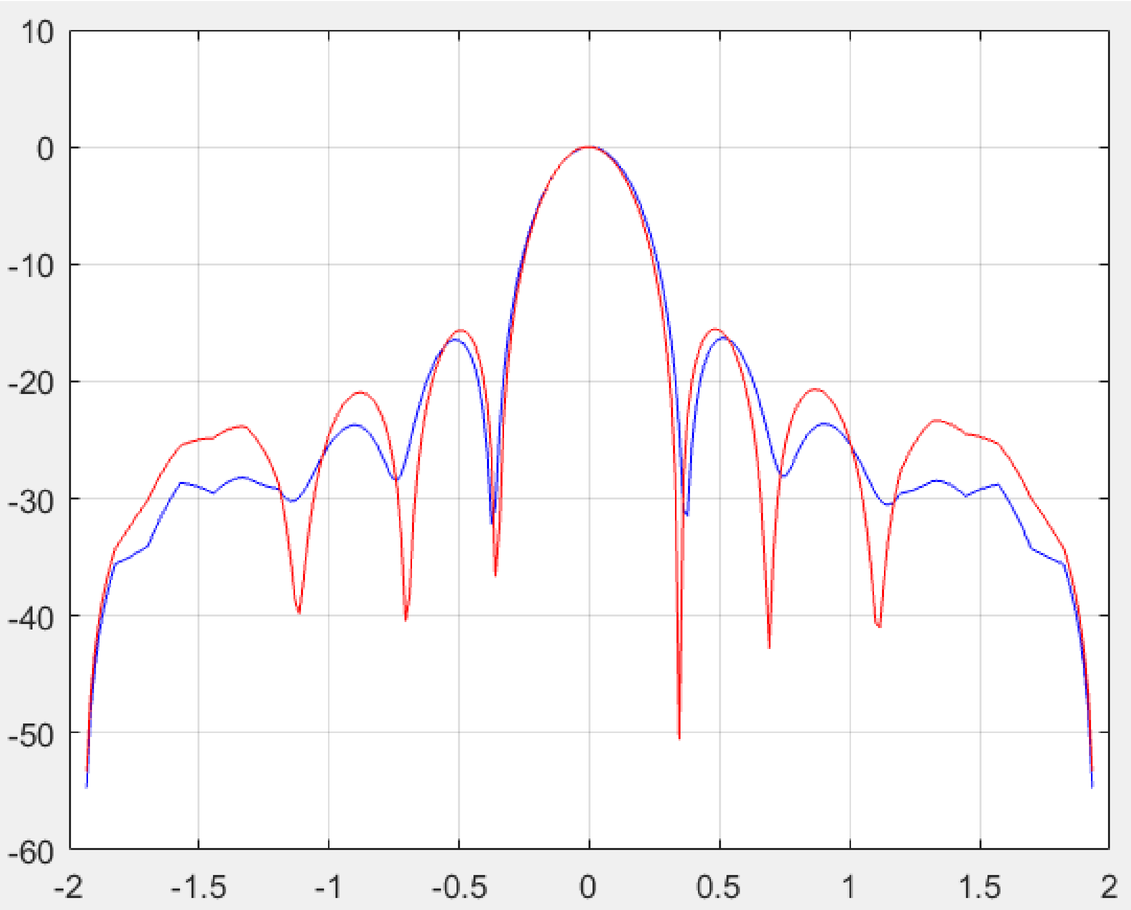

Figure 2: On the graph the squares of the modules are shown, the function E_ϕ^ϕ red line, E_θ^θ blue line for $\theta = \pi/3$. On the abscissa axis, the angle ϕ in radians. In the following example, the antenna array configuration remains predefined, but $\Delta_w = \pi/18$.

With decreasing deltas, we see a narrowing of the main beam and an increase in the level of the side lobes.

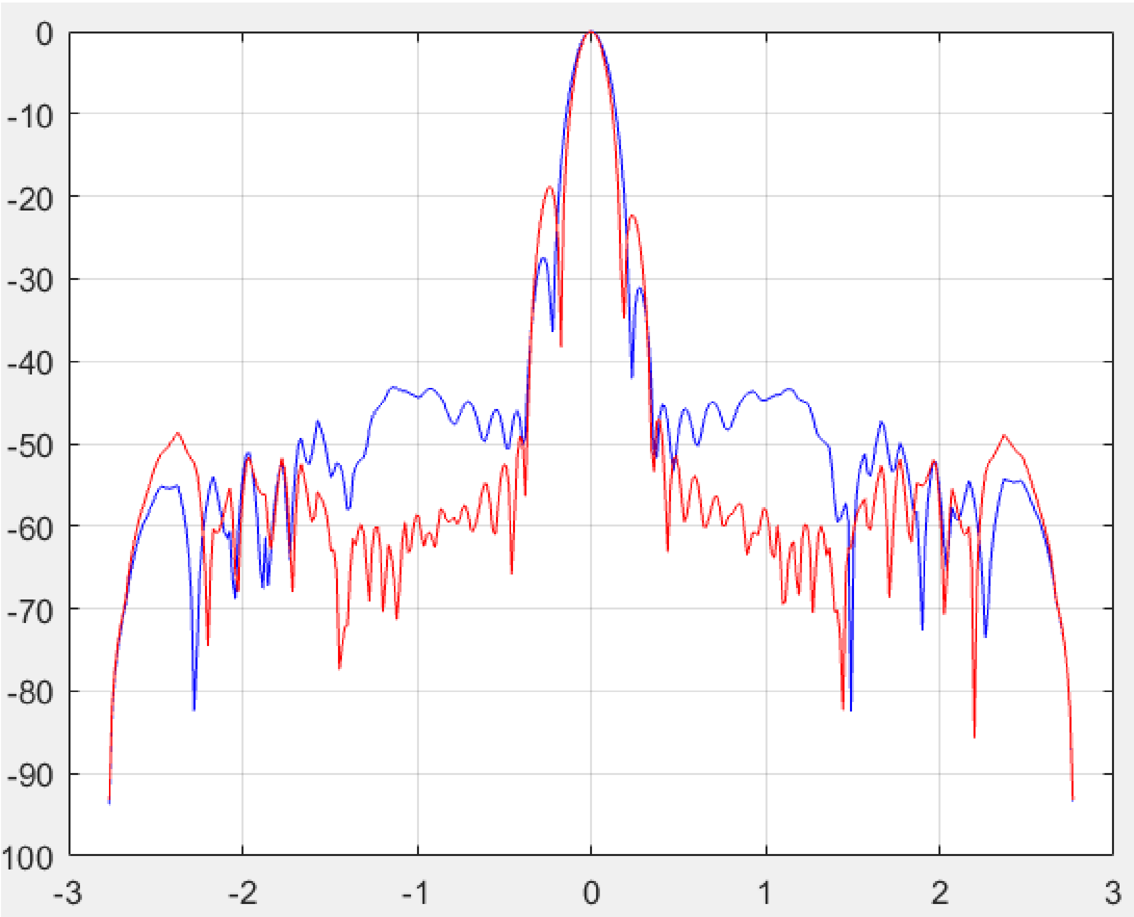

Figure 3: The running time of the algorithm in both situations is the same and is about 4 minutes. The next graph shows the result of the program with a large number of elements. Antenna grid consists of 625 (25x25) elements a base diameter of 5.0 and the direction of the main ray $(\theta_d, \phi_d) = (\pi\ 3.0)$, $\Delta_w = \pi/9$.

4 Results

The application of methods of mathematical optimization to problems of synthesizing the polarization flux is becoming increasingly popular, in view of its effectiveness and the quality of the results obtained. New approaches to the formulation of classical radar problems in the format of problems of convex programming, show their advantage in comparison with other methods.

In this paper, a software package for the synthesis of conformal arrays with two-polarized circuits in a cylindrical array is designed and implemented. The results of testing the program with a large number of elements showed an acceptable speed of operation.

We demonstrated the operation of the system with various configurations of antenna arrays. On the graphs of the results obtained, we can see that the radiation of the main polarization flux reaches a maximum at the angles given in the input data. The side lobes of polarization, entering the area of minimization, do not exceed the established limits. It is also possible to observe good consistency of beam diagrams in the region of the main ray, which was required in the initial formulation of the problem.

5 Conclusion

The software project was created based on the requirements of simplicity of modification of the source code. It should be noted that the software has been developed by the order of the All-Russian Scientific Research Institute of Radio Engineering, where it is now actively used to solve the corresponding problems. A user with a minimal knowledge of the use of the MATLAB system is able to write their own functions to work with different versions of antenna arrays. Also, the environment provides the ability to form dynamically linked libraries (dll) from individual functional modules, for their further use in high-level languages.

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