Numerical Damping of Vibrations of a Moving String

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Abstract

The basis of this research is the mechanical processes that take place in the production of paper. All papermaking machines contain open sections of the web between the support rollers. When the paper web moves, it may lose stability, break, or begin to make transverse vibrations. It is assumed that the amplitude of these vibrations is the same in the width direction of the web, so that the movement of the string in the axial direction with constant speed is considered as a model. It should be noted that because of the axial motion, the mechanics of moving materials are different from classical mechanics.

In the paper, the decrease of vibrations is realized with the help of point-type actuators. The motion of the string is modeled by the initial-boundary value problem for a partial differential equation of hyperbolic type, which is solved numerically by the method of characteristics. The vibration damping controlling actuators are modeled by the function on the right side of this equation. The problem is to damp the forced transverse vibrations of a moving string in a minimum time. Examples of calculations are given.

1 Formulation of the Problem

The small vibrations of a moving string $u(t, x)$, is governed by the hyperbolic partial differential equation [Archibald & Emslie, 1958], [Jeronen, 2011]

$$u_{tt} + 2V_0 u_{xt} + (V_0^2 - c^2)u_{xx} = f(t, x), \quad 0 \leqslant x \leqslant l, 0 \leqslant t \leqslant T, \tag{1}$$

where $V_0 > 0$ is the constant speed of the string, $c > V_0$ is the velocity of propagation of a disturbance wave along a string, l is the length of the string and T is the considered time.

Boundary conditions

$$u(t, 0) = 0, \quad u(t, l) = 0. \tag{2}$$

The initial disturbance

$$u(0, x) = \varphi(x), \quad u_t(0, x) = \psi(x) \tag{3}$$

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are known.

Let us find a function $f(t,x)$ from (1), that allows us to translate the string in a minimal time T from the initial perturbed state (3) to the final state

$$u(T,x)=0; \quad u_t(T,x)=0. \tag{4}$$

That is, to extinguish the initial perturbations in a time T. Conditions (3) are equivalent to equating of the next functional to zero.

$$J(T)=\int_0^l \left[u^2(T,x)+u_t^2(T,x)\right]dx=0. \tag{5}$$

To cancel the vibrations, we use a stationary pointwise actuator placed at the point x_0.Then the function $f(t,x)$ will be sought in the form:

$$f(t,x)=W(t)\delta(x-x_0), \tag{6}$$

where x_0 is the point of application of the actuator, δ is the Dirac delta function, $W(t)$ is the control function.

2 Method of Solution

2.1 Method of Characteristics

Consider the equation (1). We introduce auxiliary function $\upsilon(t,x)$ such that the equation (1) can be represented as two first-order equations

$$\begin{cases} u_t=-\upsilon_x-2V_0u_x \\ \upsilon_t=(V_0^2-c^2)u_x+g(t,x). \end{cases} \tag{7}$$

where

$$g(t,x)=-\int_0^x f(t,x)\,dx.$$

The initial conditions will take the form,

$$u(0,x)=\varphi(x),$$
$$\upsilon(0,x)=-2V_0\varphi(x)-\int_0^x \psi(x)dx$$

The system (7) can be written in the form of a characteristic system of ordinary differential equations

$$\frac{d}{dt}\big((V_0-c)u+\upsilon\big)=g \tag{8}$$

along the characteristics

$$x-(V_0-c)t=c_1=const, \tag{9}$$

$$\frac{d}{dt}\big((V_0+c)u+\upsilon\big)=g \tag{10}$$

along the characteristics

$$x-(V_0+c)t=c_2=const. \tag{11}$$

We introduce in the calculated domain a grid formed by the lines $x_m=mh$, $h=\dfrac{l}{M}$; $m=0,1,...,M$, $t_n=n\tau$; $n=0,1,...,N$, $T=\tau N$, where h and τ respectively the grid steps. We will assume that c, V_0 are integers and $\tau=h$ (then the characteristics of different families will pass through the nodal points of the grid). In grid nodes we introduce grid functions

$$\{u_m^n\}_{\substack{m\,\in\,0,\,M\\ n\,\in\,0,\,N}}, \quad \{\upsilon_m^n\}_{\substack{m\,\in\,0,\,M\\ n\,\in\,0,\,N}}.$$

Let $A \leqslant m \leqslant M + B$, where $A = V_0 + c$, $B = V_0 - c$. Consider the three nodes in Figure 1, that lie within the calculated area,

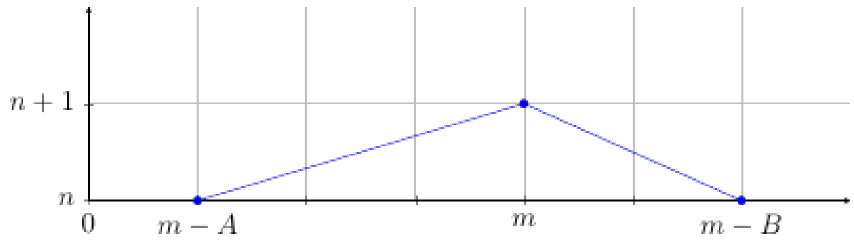

Figure 1:

We approximate the equation (1) with the second order along the characteristic, connected the points $\binom{n}{m-A}$ and $\binom{n+1}{m}$, by the finite-difference scheme

$$(Au+v)_m^{n+1} - (Au+v)_{m-A}^n = \tau \frac{g_m^{n+1} + g_{m-A}^n}{2},$$

And along the characteristic, connected the points $\binom{n+1}{m}$ and $\binom{n}{m-B}$, by a finite-difference scheme

$$(Bu+v)_m^{n+1} - (Bu+v)_{m-B}^n = \tau \frac{g_m^{n+1} + g_{m-B}^n}{2}.$$

From these equations, the values u_m^{n+1} and v_m^{n+1} are easily found.

Consider the case $m < A$.

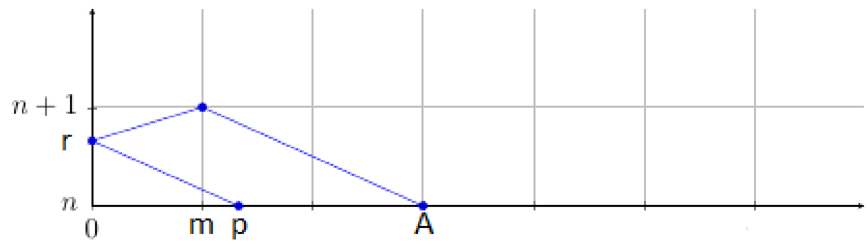

Figure 2:

In this case, for calculations, we find the values of u_p^n v_p^n using quadratic interpolation at the three nearest points. Taking into account, that from (2) $u_0^{n+r} = 0$, we find

$$v_0^{n+r} = (Bu+v)_p^n + \frac{\tau}{r}\frac{g_0^r + g_p^n}{2}$$

Now, finding v_0^{n+r}, we can compose and solve the following system of equations

$$\begin{cases} (Au+v)_m^{n+1} - (Au+v)_0^{n+r} = \dfrac{\tau}{1-r}\dfrac{g_m^{n+1} + g_0^{n+r}}{2} \\ (Bu+v)_m^{n+1} - (Bu+v)_{m-B}^n = \tau \dfrac{g_m^{n+1} + g_{m-B}^n}{2} \end{cases}$$

Similarly solved case $m > M + B$.

2.2 Method of Minimization

To solve the problem of damping the string, we seek a control function W(t) for minimizing the integral (5) using the Hook-Jeeves method.

To do this, we approximate the function $W(t)$ with a piecewise constant function: $\forall t \in [t_i, t_{i+1}]$, $W(t) = w_i$, $w_i = const, i = 0, 1, ..., N-1$.

Then the integral (5) will be a function of the variables $w_0, w_1, ..., w_{N-1}$

$$J(T) = J(w_0, w_1, ..., w_{N-1})$$

The optimal value $w_0, w_1, ..., w_{N-1}$ minimizing the integral with a given accuracy ε and being the desired solution of the problem, will be found by the Hook-Jeeves method. To calculate the integral numerically, we use the following approximation

$$J(T) = \sum_{m=1}^{M-1} \left[(u_m^N)^2 + (\frac{u_m^N - u_m^{N-1}}{\tau})^2 \right] \tag{12}$$

The condition for cancellating in the examples below is $J(T) \leqslant \varepsilon$.

3 Examples of Calculations

3.1 Example 1

As an example, let us consider the problem with the following parameter values: $V_0 = 1, c = 2, l = 1, M = 10$. The initial perturbations are given by the relations $\varphi(x) = \sin(\pi x)$, $\psi(x) = 0$.

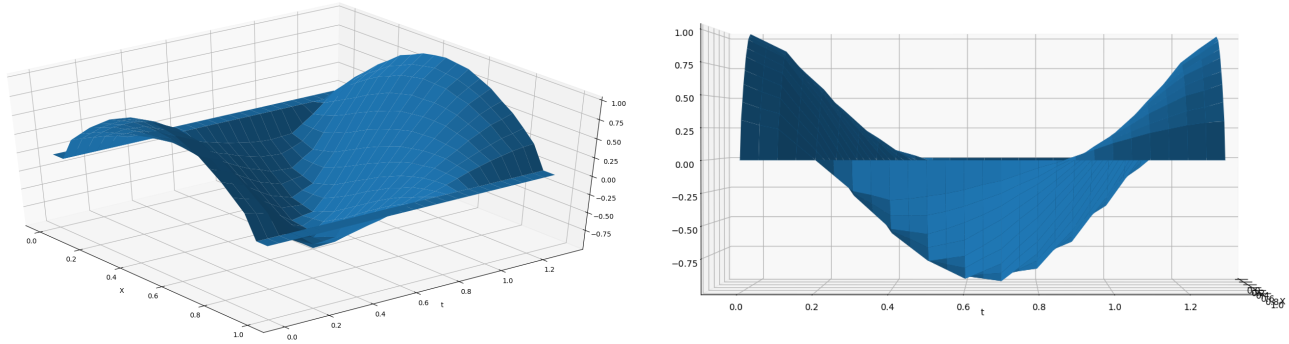

Figure 3: The function $u(t, x)$, free oscillations

Figure 3 shows the graph of the function $u(t, x)$ without a control action, when $f(t, x) = 0$. It can be seen that the string makes infinite fluctuations.

Let us see problem of demping this oscilation. The initial parameters is $x_0 = 0.5$,$\varepsilon = 0.005$, $T = 1.3$ and $N = 13$.

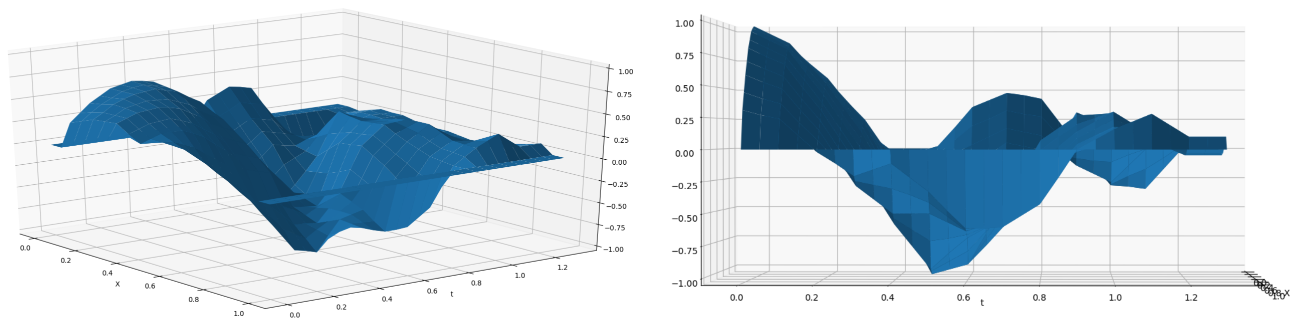

Figure 4: The function $u(t, x)$, with control

Figures 4 and 5 show process of damping for $u(t, x)$ and the control function $W(t)$ correspondingly.

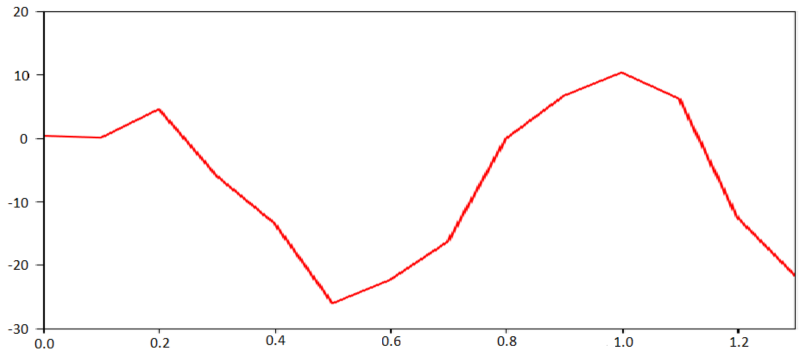

Figure 5: Control function W(t)

3.2 Example 2

As another example of calculations, consider the problem with the following parameter values: $V_0 = 1, c = 2, l = 1, M = 10$. The initial perturbations are given by the relations $\varphi(x) = -x(1-x)$, $\psi(x) = 0$.

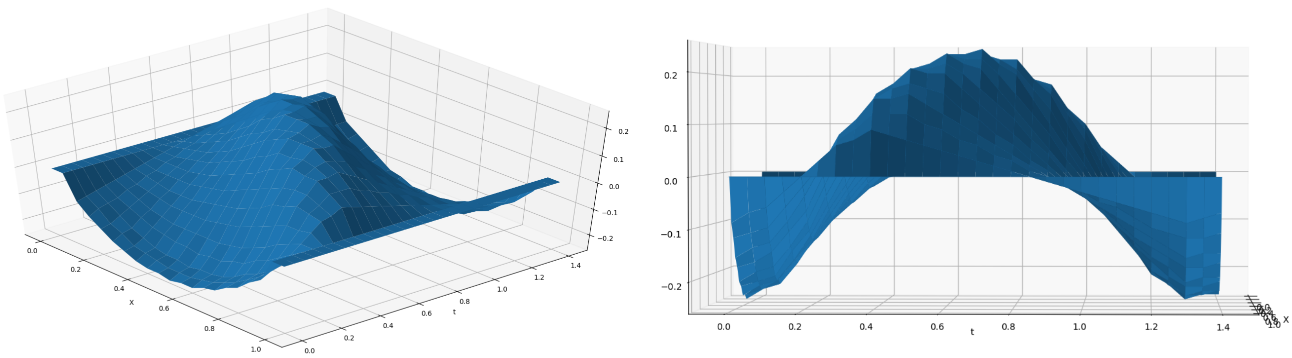

Figure 6: The function $u(t, x)$, free oscillations

Figure 6 shows the graph of the function $u(t, x)$ without a control action, when $f(t, x) = 0$. It can be seen that the string makes infinite fluctuations.

Let us see problem of demping this oscilation. The initial parameters is $x_0 = 0.5$,$\varepsilon = 0.005$, $T = 1.3$ and $N = 13$.

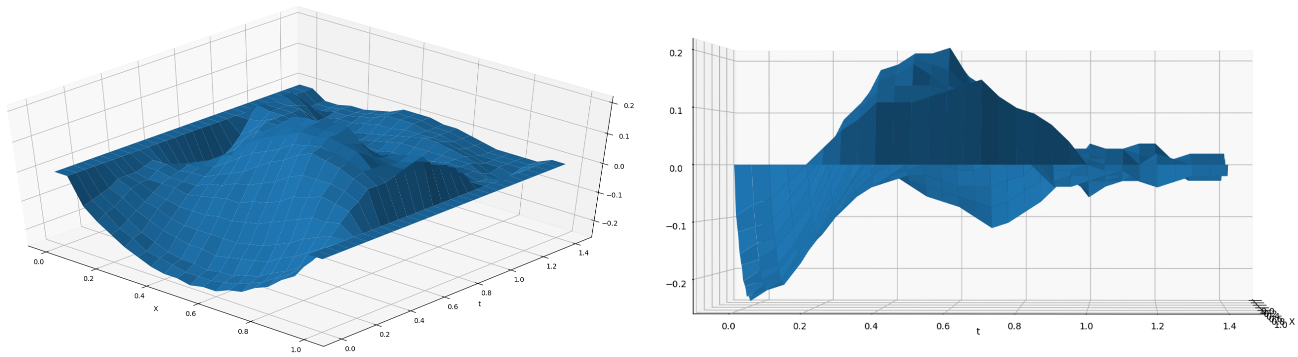

Figure 7: The function $u(t, x)$, with control

Figures 7 and 8 show process of damping for $u(t, x)$ and the control function $W(t)$ correspondingly.

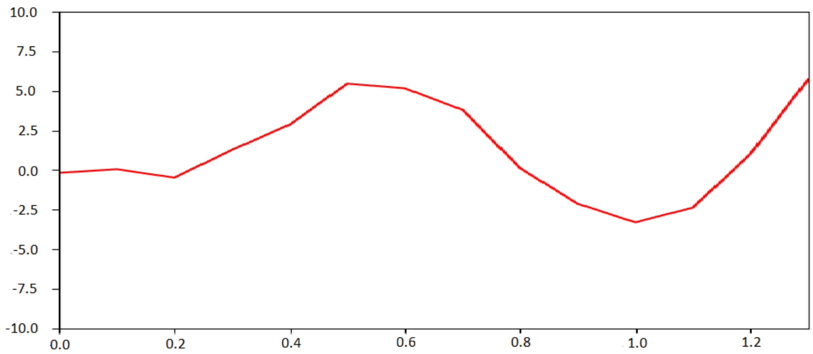

Figure 8: Control function W(t)

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References

[Archibald & Emslie, 1958] Archibald, F. R., Emslie, A. G. (1958). *ASME journal of Applied Mechanics*, 25, 347-348.

[Jeronen, 2011] Jeronen, J. (2011). *On the Mechanical Stability and Out-of-Plane Dynamics of a Travelling Panel Submerged in Axially Flowing Ideal Fluid.* Jyväskylä, Finland: University of Jyväskylä.