On a Statement of Problem of Control Synthesis the Process of Heating the Bar

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Abstract

In the work an approach to optimal synthesis of control actions in the systems with distributed parameters is investigated. As an example, we consider the problem of control of heating a bar at the expense of controlled source of energy, placed near to one of the ends of the bar. The current temperature of the controlled energy source is calculated with depend of temperatures on some defined points of the bar. It is required to define the optimal locations of controls and the controlled parameters of feedback, for which the value of the functional which determines the quality of the controlled heating process will be minimum. Are obtained the formulas for the gradient of the functional in the work, which allows the numerical solution of the problem.

1 Introduction

In the work an approach to optimal synthesis of control actions in the systems with distributed parameters is investigated. As an example, we consider the problem of control of heating a bar at the expense of controlled source of energy, placed near to one of the ends of the bar. The current value of temperature of the controlled source is calculated with depend of the observed values of the states (temperatures) on some defined points of the bar. It is required to define the optimal locations of controls and the controlled parameters of feedback, for which the value of the functional which determines the quality of the controlled heating process will be minimum.

Problems of synthesis of the control in the distributed parameter systems described by differential equations with partial derivatives are investigated significantly less than lumped parameter systems [Utkin, 1981], [Levine, 1996], [Vasilev, 2002], [Polyak & Sherbakov, 2002], [Ray, 1981], [Butkovskiy, 1984], [Yeqorov, 2004], [Sergienko, 2005], [Aida-zade, 2005]. Almost, is not studied the optimization problem of placements of the points of monitoring on the controlled object and optimization of the measurement moments of time, if the number of measurements is limited and they can be made only in some discrete timepoints.

In this work the problem of optimization with a feedback is solved on the example of synthesis of boundary control of the process of heating of the bar: 1) placement of the given number of measurement points; 2) measurement timepoints in discrete feedback in time; 3) parameters of a synthesizable linear relation of control from the current values of a state in measurement points. The considered problem is reduced to the problem

In: Yu. G. Evtushenko, M. Yu. Khachay, O. V. Khamisov, Yu. A. Kochetov, V.U. Malkova, M.A. Posypkin (eds.): Proceedings of the OPTIMA-2017 Conference, Petrovac, Montenegro, 02-Oct-2017, published at http://ceur-ws.org

of parametrical optimal control of the process described by a differential equation of parabolic type with the non-local boundary condition including non-separated intermediate states.

2 Problem Statement

Let's consider a problem of control of sequential heating of identical bars with same lengths of l [Tikhonov & Samarskii, 1977] on account of the one of their boundaries:

$$u'_t(x,t) = a^2 u''_{xx}(x,t) - \lambda_0[u(x,t) - \theta], (x,t)\in\Omega = [0,l]\times[0,T], \tag{1}$$

$$u'_x(0,t) = \lambda_1[u(0,t) - \vartheta(t)], t \in [0,T], \tag{2}$$

$$u'_x(l,t) = -\lambda_2[u(l,t) - \theta], t \in [0,T]. \tag{3}$$

Here $u(x,t)$ - is the temperature of the bar at point x on the moment of time t; θ - is the temperature of the external environment which is supposed a constant; $\vartheta(t)$ - is the temperature of controlled source, which is piecewise continuous function in time and satisfying to technological restrictions:

$$g(x,\vartheta(t)) = d_1 - |\tilde{\vartheta}_{d_0}(t)| \geq 0, t \in [0,T], \tag{4}$$

$$\tilde{\vartheta}_{d_0}(t) = d_0 - \vartheta(t).$$

the coefficients a, λ_0, λ_1, λ_2 and the parameters d_0, d_1 - are given.

It is supposed that the initial temperatures of the heated bar are not defined, but the set Φ of its possible values is defined as follows:

$$u(x,0) = \varphi = \text{const} \in \Phi, x \in [0,l], \tag{5}$$

with the density function $\rho_\Phi(\varphi)$. The set of possible initial conditions can be finite:

$$\Phi = \{\varphi_1, \varphi_2, \ldots, \varphi_{N_\varphi}\},$$

with the given values of probabilities:

$$p_i^\varphi = P(\varphi = \varphi_i) \in [0,1], i = 1, \ldots, N_\varphi.$$

Similarly, temperature of the external environment $\theta = \text{const}$ also may be not defined accurately, and is determined by a set of possible values Θ:

$$\theta \in \Theta, t \in [0,T], \tag{6}$$

with the given density function $\rho_\Theta(\theta)$. It is possible that the set of external temperature is finite:

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_{N_\theta}\},$$

with the given values of probabilities:

$$p_j^\theta = P(\theta = \theta_j) \in [0,1], j = 1, \ldots, N_\theta.$$

Duration of the heating process can be as given, and defined on a wide interval of time (as, for example, in problems of constructioning of regulators), or optimized, as in usual problems of optimal control. In this work we will assume that duration of the process T is given.

It is required to define the controlled action $\vartheta(t)$, at which the given functional reaches a minimum value, determining the root mean square deviation of the state of process from some given desirable distribution of temperature $U(x)$ on the bar on average by all admissible external temperatures θ and initial conditions φ, at the end of time:

$$J(\vartheta) = \int_\Theta \int_\Phi I(\vartheta;\varphi,\theta)\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta, \tag{7}$$

$$I(\vartheta;\varphi,\theta) = \int_0^l \mu(x)[u(x,T;\vartheta,\varphi,\theta) - U(x)]^2 dx + \varepsilon||\vartheta(t) - \vartheta^0||^2_{L_2[0,T]}. \tag{8}$$

Here $u(x,T;\vartheta,\varphi,\theta)$ - is the solution of an initial boundary value problem (1)-(3),(5) at admissible initial condition of $u(x,0)=\varphi$ and temperature of the external environment θ; ε, $\vartheta^0=\vartheta^0(t)$ - are the regularization parameters of the functional; $\mu(x)\geq 0$ - is the given weight function.

If the set of initial conditions Φ and the set of external temperatures Θ are finite, then instead of (7) will be used the functional:

$$J(\vartheta)=\sum_{i=1}^{N_\varphi}\sum_{j=1}^{N_\theta} I(\vartheta;\varphi,\theta)p_i^\varphi p_j^\theta.$$

Let measurements of temperature are in some L_x points $\xi_1,\ldots,\xi_{L_x}\in[0,l]$ of the bar in the heating process continuously in time

$$u_i(t)=u(\xi_i,t), i=1,\ldots,L_x, \tag{9}$$

or on the given L_t+1 discrete timepoints τ_s:

$$u_{is}=u(\xi_i,\tau_s), i=1,\ldots,L_x, s=0,\ldots,L_t, \tau_0=0. \tag{10}$$

In the controlled process we will determine the value of the controlled boundary action of $\vartheta(t)$ by the results of the current measurements in controlled points in form of the linear feedback by the state of process. In case of the continuous feedback (9) we will determine control by the formula [Aida-zade & Abdullayev, 2012]:

$$\vartheta(t,y)=\sum_{i=1}^{L_x} k_i[u(\xi_i,t)-z_i], t\in[0,T], \tag{11}$$

where z_i - rated value of temperature in i-th point of measurement from which the deviation of current state in this point influences value of the control; k_i - intensification coefficients, $i=1,\ldots,L_x$. In (11) and further are used these designations:

$$\xi=(\xi_1,\ldots,\xi_{L_x}), k=(k_1,\ldots,k_{L_x}), z=(z_1,\ldots,z_{L_x}), y=(\xi,k,z), \tau=(\tau_1,\ldots,\tau_{L_x}).$$

It is clear, that function of control (11) is continuous at $t\in[0,T]$.

Let's assume that the locations of measurement points $\xi_i\in[0,l]$, $i=1,\ldots,L_x$ are not set, also it is required to optimize the constant parameters z_i, k_i, $i=1,\ldots,L_x$ defining synthesizable control (11) with a feedback taking into account a functional (7).

Substituting (11) in a boundary condition (2) we will obtain the non-local (loaded) boundary condition with non-separated intermediate conditions:

$$u'_x(0,t)=\lambda_1\left[u(0,t)-\sum_{i=1}^{L_x} k_i[u(\xi_i,t)-z_i]\right], t\in[0,T]. \tag{12}$$

In case (10) of discrete observations in time for the state in controlled points ξ_i, $i=1,\ldots,L_x$, taking into account the feedback the boundary controlled action will be defined as follows:

$$\vartheta(t,y)=\sum_{i=1}^{L_x} k_i[u(\xi_i,\tau_s)-z_i], t\in(\tau_s,\tau_{s+1}], s=0,\ldots,L_t, \tau_{L_t}=T. \tag{13}$$

In this case synthesizable control is the step function, and its parameters k_i, z_i, ξ_i, $i=1,\ldots,L_x$, make the same sense like in (11). The boundary condition (2) in case of control (13) will take a form:

$$u'_x(0,t)=\lambda_1\left[u(0,t)-\sum_{i=1}^{L_x} k_i[u(\xi_i,\tau_s)-z_i]\right], t\in(\tau_s,\tau_{s+1}], s=0,\ldots,L_t.$$

Discrete timepoints τ_s, $s=1,\ldots,L_t$, both given, and also can be optimized.

In this case mathematical description of the process (1) as follows:

$$u'_t(x,t)=a^2u''_{xx}(x,t)-\lambda_0[u(x,t)-\theta], x\in[0,l], t\in(\tau_s,\tau_{s+1}], s=0,\ldots,L_t, \tag{14}$$

with a condition of a continuity of its state in timepoints of observations τ_s:

$$u(x,\tau_s) = u(x,\tau_s^-) = u(x,\tau_s^+), \tag{15}$$

where

$$\tau_s^- = \tau_s - 0, \tau_s^+ = \tau_s + 0.$$

In case (9) of the continuous observation of heating process the minimized functional (7), (8) will be written in this form:

$$J(y) = \int\limits_{\Theta}\int\limits_{\Phi} I(y;\varphi,\theta)\rho_{\Phi}(\varphi)\rho_{\Theta}(\theta)d\varphi d\theta, \tag{16}$$

$$I(y;\varphi,\theta) = \int\limits_0^l \mu(x)[u(x,T;y;\varphi,\theta) - U(x)]^2dx + \varepsilon||y - y^0||^2_{R^{3L_x}}. \tag{17}$$

Here $u(x,T;y,\varphi,\theta)$ - is the solution of an initial boundary value problem (1),(12),(3),(5) at randomly preset admissible values of parameters of the feedback $y = (\xi,k,z)$, initial condition $\varphi = \varphi(x)$ and temperature of the external environment θ; $\varepsilon > 0$, $y^0 \in R^{3L_x}$ - are the regularization parameters of functional.

Restriction (4) for the controlled actions $\vartheta(t)$ obviously will be replaced with the following restrictions for synthesizable parameters of feedback control:

$$0 \le \xi_i \le l, i = 1,\dots,L_x, \tag{18}$$

$$g(t,y) = d_1 - |\tilde{\vartheta}_{d_0}(t,y)| \ge 0, t \in [0,T], \tag{19}$$

$$\tilde{\vartheta}_{d_0}(t,y) = d_0 - \sum_{i=1}^{L_x} k_i[u(\xi_i,\tau_s) - z_i],$$

in case of the continuous observations, and in case of discrete observations instead of (19) restrictions will take place:

$$0 \le \tau_s \le \tau_{s+1} \le T, s = 0,\dots,L_t - 1,$$

$$g(\tau_s,y) = d_1 - |\tilde{\vartheta}_{d_0}(\tau_s;y)| \ge 0, s = 0,\dots,L_t.$$

So, synthesis of boundary control of heating process (attemperation) of the bar described by an initial boundary value problem with not precisely given initial condition and external temperature with the continuous in time, optimized the locations of points of the control (feedback) on the bar, is reduced to the problem (1),(12),(3),(5),(6),(16)-(19).

3 Formulas for the Numerical Solution of the Problem

For the solution of the problem (1),(12),(3),(16)-(19) syntheses of a finite-dimensional vector of the $y \in R^{3L_x}$ parameters of control like (11) at the continuous observation (9) for the accounting of restriction (19) will be used the method of penalty functions and the method of gradient projection for restriction (18) for minimization of penalty function. For concreteness will be used the method of an external penalty, and in this case the functional (17) will have the form:

$$\tilde{I}(y;\varphi,\theta) = I(y;\varphi,\theta) + rI_{\text{pnt}}(y),$$

$$I_{\text{pnt}}(y) = \int\limits_0^T \Big\{\min(0,g(t,y))\Big\}^2 dt,$$

where r - the penalty coefficient approaches $+\infty$ [Vasilev, 2002].

It will allow to build the iterative sequence

$$y^{n+1} = \mathcal{P}_{(18)}[y^n - \alpha_n \text{grad}J(y^n)], n = 0,1,\dots, \tag{20}$$

minimizing the functional $J(y)$ at the preset value of coefficient of the penalty. In (20) the following designations are used: $3L_x$ - dimensional vector of the gradient of the target functional (16):

$$\mathrm{grad} J(y) = \left(\frac{\partial J(y)}{\partial \xi}, \frac{\partial J(y)}{\partial k}, \frac{\partial J(y)}{\partial z} \right). \tag{21}$$

$\mathcal{P}_{(18)}$ - the operator of projection on restriction (18) having a prime appearance concerning each of parameters $\xi = (\xi_1, \ldots, \xi_{L_x})$:

$$\mathcal{P}_{(18)}[\xi_i] = \begin{cases} 0, & \xi_i < 0, \\ \xi_i, & 0 \le \xi_i \le l, i = 1, \ldots, L_x, \\ l, & \xi_i > l, \end{cases}$$

$\alpha_n \ge 0$ - the step in the direction of the projected anti-gradient determined in any known way, in particular, by methods of one-dimensional minimization, providing a monotonicity criterion of iterative process [Vasilev, 2002]:

$$J(y^{n+1}) \le J(y^n), n = 0, 1, \ldots.$$

Further, using the method of an increment of arguments [Vasilev, 2002],[Butkovskiy, 1984],[Yeqorov, 2004], the following formulas are received for the components of gradient (21) of the functional $J(y)$.

$$\frac{\partial J(y)}{\partial \xi_i} = \int\limits_{\Theta}\int\limits_{\Phi} \left\{ -\lambda_1 a^2 \int\limits_0^T \psi(0,t) k_i u'_x(\xi_i, t) dt + 2\varepsilon(\xi_i - \xi_i^0) \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta + \tag{22}$$
$$+ 2r \int\limits_{\Theta}\int\limits_{\Phi} \left\{ \int\limits_0^T k_i u'_x(\xi_i, t) \mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(t,y)\big) \min\big(0, g(t,y)\big) dt \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta, i = 1, \ldots, L_x,$$

$$\frac{\partial J(y)}{\partial k_i} = \int\limits_{\Theta}\int\limits_{\Phi} \left\{ -\lambda_1 a^2 \int\limits_0^T \psi(0,t)[u(\xi_i, t) - z_i] dt + 2\varepsilon(k_i - k_i^0) \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta + \tag{23}$$
$$+ 2r \int\limits_{\Theta}\int\limits_{\Phi} \left\{ \int\limits_0^T [u(\xi_i, t) - z_i] \mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(t,y)\big) \min\big(0, g(t,y)\big) dt \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta, i = 1, \ldots, L_x,$$

$$\frac{\partial J(y)}{\partial z_i} = \int\limits_{\Theta}\int\limits_{\Phi} \left\{ \lambda_1 a^2 \int\limits_0^T \psi(0,t) k_i dt + 2\varepsilon(z_i - z_i^0) \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta + \tag{24}$$
$$- 2r \int\limits_{\Theta}\int\limits_{\Phi} \left\{ \int\limits_0^T k_i \mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(t,y)\big) \min\big(0, g(t,y)\big) dt \right\} \rho_\Phi(\varphi)\rho_\Theta(\theta) d\varphi d\theta, i = 1, \ldots, L_x,$$

Here the function $\psi(x,t) = \psi(x,t; y, \varphi, \theta)$ is the solution of the following conjugate initial boundary value problem:

$$\psi'_t(x,t) = -a^2 \psi''_{xx}(x,t) + \lambda_0 \psi(x,t), x \in (\xi_i, \xi_{i+1}), i = 0, \ldots, L_x, t \in [0,T], \tag{25}$$

$$\psi(x,T) = -2\mu(x)[u(x,T) - U(x)], x \in [0,l], \tag{26}$$

$$\psi'_x(0,t) = \lambda_1 \psi(0,t), t \in [0,T], \tag{27}$$

$$\psi'_x(l,t) = -\lambda_2 \psi(l,t), t \in [0,T], \tag{28}$$

and satisfies following conditions in measurement points $\xi_i \in [0,l]$, $i = 1, \ldots, L_x$

$$\psi'_x(\xi_i^+, t) = \psi'_x(\xi_i^-, t) - \lambda_1 \psi(0,t) k_i + \frac{2rk_i}{a^2} \left\{ \mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(t,y)\big) \min\big(0, g(t,y)\big) \right\}, i = 1, \ldots, L_x, \tag{29}$$

$$\psi(\xi_i^+,t)=\psi(\xi_i^-,t), i=1,\ldots,L_x. \tag{30}$$

Conjugate equation (25), having included in it a condition (29) with use of a δ-function of Dirac, it is possible to consider on all length of the bar and to write in the following form:

$$\psi_t'(x,t)=-a^2\psi_{xx}''(x,t)+\lambda_0\psi(x,t)-a^2\lambda_1\psi(0,t)\sum_{i=1}^{L_x}k_i\delta(x-\xi_i)+$$
$$+2r\Big\{\mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(t,y)\big)\min\big(0,g(t,y)\big)\Big\}\sum_{i=1}^{L_x}k_i\delta(x-\xi_i),(x,t)\in\Omega.$$

An approach for numerical solution of the loaded initial boundary value problem (25)-(30) was offered and investigated in works [Aida-zade, 2004], [Abdullayev & Aida-zade, 2006], [Aida-zade & Abdullayev, 2014],[Abdullayev & Aida-zade, 2016], [Samarskii, 1989], [Aida-zade & Bagirov, 2006], [Aida-zade & Abdullayev, 2016].

It is simple to carry out similar calculations concerning the problem of synthesis of control of heating process (14),(15) of the bar at the discrete time feedback and the corresponding piecewise continuous control action like (13). In this case the integration by parts needs to be carried out with preliminary splitting an integration on x on a piece $[0,l]$ on pieces $[\xi_i,\xi_{i+1}]$, $i=0,\ldots,L_x$ and an integration on t on a piece $[0,T]$ into pieces $[\tau_s,\tau_{s+1}]$, $s=0,\ldots,L_t$. As a result we receive the following formulas for components of a gradient of the functional of $J(y)$ at the current values of the synthesizable parameters $y\in R^{3L_x}$ and for the optimized values of timepoints of measurements τ_s, $s=1,\ldots,L_t$:

$$\frac{\partial J(y)}{\partial\xi_i}=\int\limits_{\Theta}\int\limits_{\Phi}\Big\{-\lambda_1a^2\sum_{s=0}^{L_t}\int\limits_{\tau_s^+}^{\tau_{s+1}^-}\psi(0,t)k_iu_x'(\xi_i,\tau_s)dt+2\varepsilon(\xi_i-\xi_i^0)\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta \tag{31}$$
$$+2r\int\limits_{\Theta}\int\limits_{\Phi}\Big\{\sum_{s=0}^{L_t}k_iu_x'(\xi_i,\tau_s)\mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(\tau_s,y)\big)\min\big(0,g(\tau_s,y)\big)dt\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta, i=1,\ldots,L_x,$$

$$\frac{\partial J(y)}{\partial k_i}=\int\limits_{\Theta}\int\limits_{\Phi}\Big\{-\lambda_1a^2\sum_{s=0}^{L_t}\int\limits_{\tau_s^+}^{\tau_{s+1}^-}\psi(0,t)[u(\xi_i,\tau_s)-z_i]dt+2\varepsilon(k_i-k_i^0)\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta+ \tag{32}$$
$$+2r\int\limits_{\Theta}\int\limits_{\Phi}\Big\{\sum_{s=0}^{L_t}[u(\xi_i,\tau_s)-z_i]\mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(\tau_s,y)\big)\min\big(0,g(\tau_s,y)\big)dt\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta, i=1,\ldots,L_x,$$

$$\frac{\partial J(y)}{\partial z_i}=\int\limits_{\Theta}\int\limits_{\Phi}\Big\{\lambda_1a^2\sum_{s=0}^{L_t}\int\limits_{\tau_s^+}^{\tau_{s+1}^-}\psi(0,t)k_idt+2\varepsilon(z_i-z_i^0)\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta+ \tag{33}$$
$$-2r\int\limits_{\Theta}\int\limits_{\Phi}\Big\{\sum_{s=0}^{L_t}k_i\mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(\tau_s,y)\big)\min\big(0,g(\tau_s,y)\big)dt\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta, i=1,\ldots,L_x,$$

$$\frac{\partial J(y)}{\partial\tau_s}=\int\limits_{\Theta}\int\limits_{\Phi}\Big\{-\lambda_1a^2\int\limits_{\tau_s^+}^{\tau_{s+1}^-}\psi(0,t)\sum_{i=1}^{L_x}k_iu_t(\xi_i,\tau_s)dt+2\varepsilon(\tau_s-\tau_s^0)\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta+ \tag{34}$$
$$+2r\int\limits_{\Theta}\int\limits_{\Phi}\Big\{\sum_{i=1}^{L_x}k_iu_t(\xi_i,\tau_s)\mathrm{sgn}\big(\tilde{\vartheta}_{d_0}(\tau_s,y)\big)\min\big(0,g(\tau_s,y)\big)dt\Big\}\rho_\Phi(\varphi)\rho_\Theta(\theta)d\varphi d\theta, s=1,\ldots,L_t.$$

The function $\psi(x,t) = \psi(x,t;y,\varphi,\theta)$ in formulas (31)-(34) is the solution of the following conjugate initial boundary value problem:

$$\psi'_t(x,t) = -a^2\psi''_{xx}(x,t) + \lambda_0\psi(x,t) - a^2\lambda_1\psi(0,t)\sum_{i=1}^{L_x} k_i\delta(x-\xi_i, t-\tau_s) + \tag{35}$$

$$+2r\left\{\operatorname{sgn}\left(\tilde{\vartheta}_{d_0}(t,y)\right)\min\left(0, g(t,y)\right)\right\}\sum_{i=1}^{L_x} k_i\delta(x-\xi_i, t-\tau_s), (x,t)\in\Omega,$$

$$\psi(x,T) = -2\mu(x)[u(x,T) - U(x)], x\in[0,l], \tag{36}$$

$$\psi'_x(0,t) = \lambda_1\psi(0,t), t\in[0,T], \tag{37}$$

$$\psi'_x(l,t) = -\lambda_2\psi(l,t), t\in[0,T]. \tag{38}$$

Here two dimensional generalized function $\delta(x,t)$ such that

$$\int_0^l\int_0^T \delta(x,t)dtdx = 1,$$

and for arbitraries functions $f(x,t)$ integrated at $x\in[0,l]$, $t\in[0,T]$ satisfy a condition

$$\int_0^l\int_0^T f(x,t)\delta(x-\bar{x}, t-\bar{t})dtdx = f(\bar{x},\bar{t}),$$

for all $\bar{x}\in[0,l]$, $\bar{t}\in[0,T]$.

Solution of the conjugate initial boundary value problem (35)-(38) $\psi(x,t) = \psi(x,t;y,\varphi,\theta)$, $i = 1,\ldots,L_x$ satisfies the following conditions: 1) it is continuously differentiable on t and is twice continuously differentiable on x at $t\in(\tau_s,\tau_{s+1})$, $x\in(\xi_i,\xi_{i+1})$; 2) at $t=\tau_s$ is continuous on x at $x\in(\xi_i,\xi_{i+1})$; 3) at $x=\xi_i$ is continuous on t, and its first derivatives on x, $\psi'_x(\xi_i,t)$ have a finite gap; 4) points (ξ_i,τ_s) are continuity points on x and have gap t.

Were given numerous computer experiments. Reduction of their results would occupy volume in article. In general they confirmed the received formulas and effectiveness of the offered approach to numerical problem solving of synthesis of the concentrated controls in systems of distributed parameters.

4 Conclusion

On the example of synthesis of boundary control process of heating of the bar for control systems of objects with distributed parameters described by the equations with partial derivatives is considered. Unlike many other works in this direction, here is formulated the problem definition, in which it is optimized not only the controlled actions, but also are optimized both of control locations of the given number of points, and timepoints of measurements at restricted quantity of opportunities of the organization of a feedback. The objective is brought to a class of problems of parametrical optimal control of distributed parameter systems.

Are received formulas for components of a gradient of the target functional with the aim to applying of efficient numerical methods of optimization for determining of parameters of control depending on the state of the process and placement of points of control.

It is simple to extend the considered problem definition and approach to its decision, to other types of differential equations with partial derivatives and initial boundary conditions describing various evolutionary (technological, ecological, economic and others) processes.

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