Synthesis of Optimal Discrete Controller for Robotic Vibroprotective System Control

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Abstract

The article deals with the problem of constructing an optimal controller of an active vibration protection system under the action of low-frequency disturbances from the moving base. The relative movement of the object and the base, the accelerations of the object and the base are used as feedbacks. The problem is solved in a discrete domain, the controller is synthesized as multiply connected.The feedback coefficients of the stabilizing controller are obtained on the basis of the solution of the discrete Riccati equation. As a criterion of quality, the integral of the quadratic forms of control variables and control action is used. The possibility of using the observer of the system state to obtain an estimate of the absolute velocity of the object is considered. The observer works on a closed basis. The results of simulation modeling under the action of harmonic oscillations and "white noise" are presented. The results of the observer's work are presented.

1 Introduction

The problem of protection from vibration is very relevant at the present time. Medical research shows that elevated levels of vibration have an extremely unfavorable effect on operators of transport-technological equipment, expressed in changing the functional state of the body, efficiency reducing and morbidity increasing. Areas of frequencies are known in which vibration levels can affect the performance of production operations. The intensity of the vibration that causes the marked phenomena depends on the structure of the object, its mass, duration of exposure. The action of any vibration-proof device can be reduced to the formation of additional dynamic effects that provide the necessary change in the vibrational field. In this sense, the task of vibration protection can

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be considered as a task of controlling the movement of a protected mechanical system, and dynamic influences that cause the corresponding change in parameters - as control actions. This interpretation of the role of the vibration protection device is convenient for setting and solving various tasks of vibration protection; It allows the use of the theory of automatic control for the analysis and synthesis of vibration protection systems.

2 Synthesis of the Optimal Regulator

Consider the scheme of the active system of vibration isolation, shown in Fig. 1.

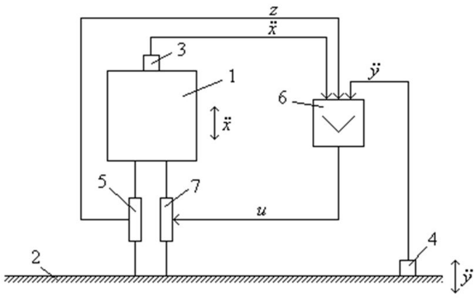

Figure 1: An active vibration isolation system of the kinematic principle of action: 1 - the object of vibration protection; 2 - base; 3, 4 - accelerometers; 5 - a sensor of the relative displacement; 6 - a regulator; 7 - a drive mechanism.

The system contains sensors, control amplifiers and actuators. As sensitive elements, sensors are used that record the kinematic parameters of the object - displacement, velocity, acceleration. The signals from the sensors characterize the quality of the vibration protection and are used to generate the control signals carried out by the elements of the feedback loop. After amplification, the signals are fed to the actuator creating the control action. Let us consider the problem of synthesis of optimal control using the example of a system with an electromechanical actuator with a “screw-nut” pair. To describe this entire system, including the actuator, in state space using the models obtained in [Rybak et al., 2014]. Given that $z = x - y$, add in another equation in the variable z, while also are conducting three state variables: $x_1 = \dot{x}$, $x_2 = z$, $x_3 = I_A$.We obtain the following system of equations:

$$\begin{cases} \dot{x}_1 = \frac{k_{em} \cdot r_1}{J_r + m \cdot r_1 \cdot r_2} \cdot x_3 + \ddot{y}, \\ \dot{x}_2 = x_1 - \dot{y}, \\ \dot{x}_3 = -\frac{k_{em}}{r_1 \cdot L} \cdot x_1 - \frac{R}{L} \cdot x_3 + \frac{1}{L} \cdot u + \frac{k_{em}}{r_1 \cdot L} \cdot \dot{y} \end{cases} \tag{1}$$

Or in vector-matrix form

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}u + \mathbf{GY} \tag{2}$$

The system of equations (2) fully describes the behavior of the system with an electromechanical actuator with the transmission of a “screw-nut” under the influence of perturbing and control actions. Each particular model of the electric motor corresponds to the matrix of the coefficients **A**, **B** input and the disturbance influence coefficient **G**. The problem of synthesis of an optimal regulator will be solved in a discrete domain, for which the system of equations (2) is transformed into a discrete form. As a result, we obtain a system of equations.

$$\mathbf{X}\left[i+1\right] = \mathbf{A}_{\Delta}\mathbf{X}\left[i\right] + \mathbf{B}_{\Delta}u\left[i\right] + \mathbf{G}_{\Delta}\mathbf{Y}\left[i\right], \tag{3}$$

where $\mathbf{A}_{\Delta} = exp(\mathbf{A}T) = \mathbf{I} + \sum_{i=1}^{\infty}(\frac{\mathbf{A}^i T^i}{i!})$, $\mathbf{B}_{\Delta} = (\int\limits_0^T(exp(\mathbf{A}T)dt)\mathbf{B} = (\mathbf{I}T + \sum_{i=1}^{\infty}(\frac{\mathbf{A}^i T^{i+1}}{(i+1)!}))\mathbf{B}$. In matrix equation (3) the symbols $\mathbf{A}_{\Delta}$, $\mathbf{B}_{\Delta}$ and $\mathbf{G}_{\Delta}$ are analogs of matrices **A**, **B** and **G** for a discrete system, and the symbol

T is entered to designate the sampling period. Because of the high complexity of mathematical calculations is not appropriate to display the general form of matrices $\mathbf{A}_\Delta$ and $\mathbf{B}_\Delta$. Preferred to calculate the matrix $\mathbf{A}_\Delta$ $\mathbf{B}_\Delta$ in numerical form for each specific pair of matrices $\mathbf{A}$ and $\mathbf{B}$. Next, to simplify the entries, we return to the notation without indices:$\mathbf{A} = \mathbf{A}_\Delta$;$\mathbf{B} = \mathbf{B}_\Delta$. We define the structure of the optimal discrete regulator in the form of a matrix relation

$$u\left[i\right] = -\mathbf{F}\mathbf{X}\left[i\right], \tag{4}$$

where $\mathbf{F} = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}$ is the matrix of coefficients of feedbacks on state variables. In fact, the optimal controller acts as a feedback on the state of the system[Rybak et al., 2016]. In order to solve the optimal control problem of synthesis will form a vector $\mathbf{Z}$ of controlled variables

$$\mathbf{Z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{D}\mathbf{X}, \tag{5}$$

where $\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is the matrix of communications coordinates state and controlled variables. As a criterion of quality will take the integral of quadratic forms of controlled variables and manipulated species:

$$J = \int_0^\infty (q_1 \cdot z_1^2(t) + q_2 \cdot z_2^2(t) + r \cdot u^2(t))dt \to min \tag{6}$$

where q_1,q_2,r are the weighting coefficients of the squares of the two controlled variables and manipulated respectively, that characterize the contribution of each term in the criterion function. The weights should be chosen empirically, based on the results of mathematical modeling. For discrete systems criterion function (6) in the form:

$$J = \sum_{i=0}^{\infty} (\mathbf{X}^T\left[i\right]\mathbf{D}^T \cdot \mathbf{Q} \cdot \mathbf{D} \cdot \mathbf{X}\left[i\right] + r \cdot u\left[i\right]) \to min \tag{7}$$

where $\mathbf{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$ is the matrix of weighting factors under controlled variables. Thus, we have formulated the task of management is to minimize the amplitude of the movement speed of the object (and hence the amplitude of its acceleration) and to limit the level of movement of the object relative to the base, thus limiting the available-resource management. Feedback gain matrix is given by

$$\mathbf{F} = (r + \mathbf{B}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{B}_\Delta)^{-1} \cdot \mathbf{B}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{A}_\Delta, \tag{8}$$

where $\mathbf{P}$ is a positive definite square auxiliary matrix with dimension 3×3. The matrix $\mathbf{P}$ satisfies the discrete Riccati equation

$$\mathbf{P} = \mathbf{D}^T \cdot \mathbf{Q} \cdot \mathbf{D} + \mathbf{A}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{A}_\Delta - \mathbf{A}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{B}_\Delta(r + \mathbf{B}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{B}_\Delta)^{-1} \cdot \mathbf{B}_\Delta^T \cdot \mathbf{P} \cdot \mathbf{A}_\Delta \tag{9}$$

This equation has a solution if the pair ($\mathbf{A}$, $\mathbf{B}$) is completely controllable. Let us investigate the controllability of the system for the continuous case (2). To do this, we form the matrix of control $\mathbf{W}$

$$\mathbf{W} = (\mathbf{B}|\mathbf{AB}|\mathbf{A}^2\mathbf{B}) = \begin{bmatrix} 0 & \frac{k_{em}\cdot r_1}{(J_r+m\cdot r_1\cdot r_2)L} & -\frac{k_{em}\cdot r_1 R}{(J_r+m\cdot r_1\cdot r_2)L^2} \\ 0 & 0 & \frac{k_{em}\cdot r_1}{(J_r+m\cdot r_1\cdot r_2)L} \\ \frac{1}{L} & -\frac{R}{L^2} & -\frac{k_{em}\cdot r_1 R}{(J_r+m\cdot r_1\cdot r_2)L^2} + \frac{R^2}{L^3} \end{bmatrix} \quad (10)$$

According to the Kalman theorem on controllability, the system is completely controllable if rank($\mathbf{W}$) $= n$, where n is the order of the system. In our case rank($\mathbf{W}$) $= 3$, Hence the system is completely controllable. Equation (9) has a unique positive definite solution in the form of a symmetric matrix $\mathbf{P}$, and such optimal closed-loop system is certainly stable, since all eigenvalues $(\mathbf{A} - \mathbf{BF})$ modulo less than one. In order not to depend on factors influencing the convergence of the solution, we will use a method that is based on the definition of eigenvalues and matrix vectors and is performed in the following sequence: 1. First we find the Hamiltonian of dimension $2n \times 2n$. In the particular case of regular matrix $\mathbf{A}$ (with a continuous sampling of the system it is guaranteed that this matrix is nonsingular) form the Hamiltonian matrix $\mathbf{H}$ by the expression

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} + \mathbf{B}r^{-1}\mathbf{B}^T(\mathbf{A}^T)^{-1}\mathbf{D}^T\mathbf{QD} & -\mathbf{B}r^{-1}\mathbf{B}^T(\mathbf{A}^T)^{-1} \\ -(\mathbf{A}^T)^{-1}\mathbf{D}^T\mathbf{QD} & (\mathbf{A}^T)^{-1} \end{bmatrix} \quad (11)$$

Next, we find all the eigenvalues of the matrix $\mathbf{H}$, satisfying the equation

$$\mathbf{H}\begin{bmatrix}\eta_i \\ \xi_i\end{bmatrix} = \mu_i \begin{bmatrix}\eta_i \\ \xi_i\end{bmatrix}, \quad (12)$$

where μ_i is the one of the eigenvalues of the matrix $\mathbf{H}$, $\begin{bmatrix}\eta_i \\ \xi_i\end{bmatrix}$ is the eigenvector corresponding to a given eigenvalue. 2. Among the obtained eigenvalues $\{\mu_1, \mu_2, ..., \mu_{2n}\}$, obviously, there are those whose absolute values are less than one, and the corresponding eigenvectors (assuming their independence) can be written as

$$\left\{\begin{bmatrix}\eta_1 \\ \xi_1\end{bmatrix}, \begin{bmatrix}\eta_2 \\ \xi_2\end{bmatrix}, ..., \begin{bmatrix}\eta_n \\ \xi_n\end{bmatrix}\right\}, \quad (13)$$

where η_i, ξ_i is the column vectors dimension $n \times 1$. Guaranteed there are n eigenvalues with absolute value less than one, because if μ_i is own a number, and $\begin{bmatrix}1 \\ \mu_i\end{bmatrix}$ is also an eigenvalue. 3. $\mathbf{P}$ values found from the expression $\mathbf{P} = [\xi_1|\xi_2|...|\xi_n]\,[\eta_1|\eta_2|...|\eta_n]^{-1}$. Hence it is not difficult to calculate the optimal feedback coefficients in expression (4). Thus, we obtain the control law of the optimal regulator in the form of feedback on the state of the system

$$u[i] = -f_1\dot{x}[i] - f_2 z[i] - f_3 I[i] \quad (14)$$

It should be noted that the current value of the armature of the motor I_A hardly accessible measurement, so it is advisable to coordinate the state expressed in terms of other quantities. From the first equation of the system (1) we express the armature current through the accelerations at the object and the base

$$I_A = \frac{J_r + m\cdot r_1\cdot r_2}{k_{em}\cdot r_1}\cdot\ddot{x} - \frac{J_r + m\cdot r_1\cdot r_2}{k_{em}\cdot r_1}\cdot\ddot{y} \quad (15)$$

Then the control law of the optimal regulator (14) takes the form

$$u[i] = -f_1\dot{x}[i] - f_2 z\dot{x}[i] - f_3{}'\ddot{x}[i] + f_3{}'\ddot{y}[i], \quad (16)$$

where $f_3{}' = \frac{f_3(J_r + m \cdot r_1 \cdot r_2)}{k_{em} \cdot r_1}$. In addition, the control signal must be limited from above and from below in accordance with expression (17)

$$u'[i] = \begin{cases} u[i], |u[i]| \le U_{max} \\ sgn(u[i])U_{max}, |u[i]| > U_{max} \end{cases} \tag{17}$$

where U_{max} is maximum allowable applied voltage of the DC motor. Fig. 2 is a block diagram of a vibration protection system with an optimal regulator, taking into account the control law (16), as well as constraints on the control action, in which the absolute value of the voltage at the motor armature can not exceed the maximum permissible value.

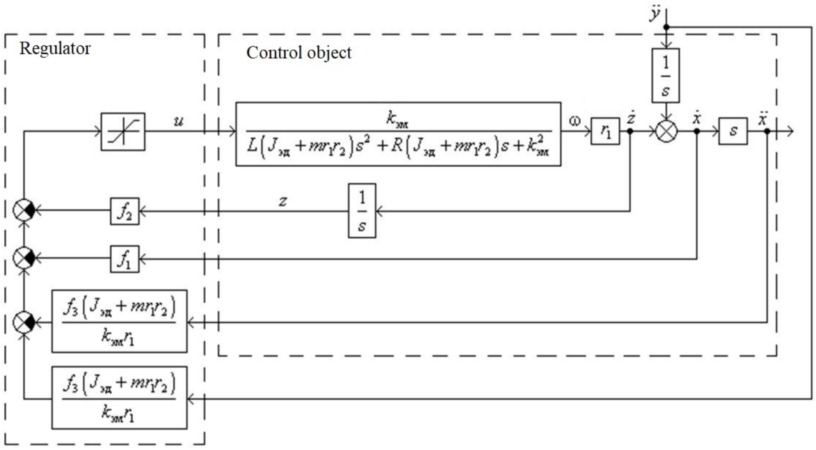

Figure 2: Structural diagram of AVZS with electromechanical actuator and optimal regulator: DPT - DC motor; PVG is the screw-nut transmission

For modeling we use a DC electric motor 2PN-90MU4 and synthesize the optimal regulator for a system with this engine model and the "screw-nut" transmission. The mass of the object of stabilization is assumed m=100 Kg. Going to the discrete task, taking the sampling period T=0.01s, to obtain the following matrix A_Δ and B_Δ of equation (3):

$$\mathbf{A_\Delta} = \begin{bmatrix} 0.967 & 0 & 1.271 \times 10^{-3} \\ 9.879 \times 10^{-3} & 1 & 7.489 \times 10^{-6} \\ -27.498 & 0 & 0.334 \end{bmatrix}, \mathbf{B_\Delta} = \begin{bmatrix} 6.241 \times 10^{-3} \\ 2.252 \times 10^{-6} \\ 0.513 \end{bmatrix}. \tag{18}$$

We set the following weighting factors:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, r = 10^{-6}. \tag{19}$$

After the procedure for finding the feedback coefficients of the optimal controller described by equations (8), (11) - (13), a matrix of feedback coefficients is obtained $F = \begin{bmatrix} 560.960 & 603.156 & 0.927 \end{bmatrix}$, for which mathematical modeling is performed. As a disturbing action from the base, a signal having the spectral characteristic shown in Fig. 3 and characteristic peaks at frequencies of 1.5; 3; 5; 8 Hz and a muffled noise background at other frequencies.

As a result of the simulation, the spectral characteristic of the resulting acceleration at the object is obtained, Fig. 4. Comparing the two characteristics, it can be seen that the characteristic peaks at 1.5 and 3 Hz were almost completely damped. The peaks at frequencies of 5 and 8 Hz also significantly decreased. This result

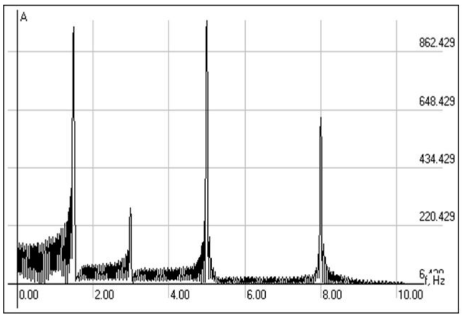

Figure 3: The spectral characteristic of the test disturbance signal

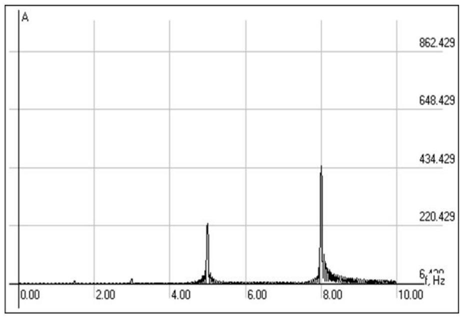

Figure 4: The spectral characteristic of acceleration at the site

clearly demonstrates the effectiveness of the system with an electromechanical actuator and an optimal regulator in suppressing perturbations of a complex species.

3 Synthesis of the Observer of States

Let us consider the possibility of using the observer of the state coordinates to obtain the magnitude of the absolute velocity of the object $\dot{x}$. This option has some advantages over direct integration of acceleration at the site $\ddot{x}$, the main of which is higher noise immunity [Fedosov et al., 2000]. Consider the principle of constructing an observing device. Let the controlled system be described by a vector equation

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}u. \tag{20}$$

To estimate the vector of state variables $\mathbf{X}$, construct observing apparatus according to the equation

$$\dot{\hat{X}} = \mathbf{A}\hat{\mathbf{X}} + \mathbf{B}u. \tag{21}$$

In this case, setting the manipulated variable u on the system and the monitored device, the output of the observer will have an accurate estimate $\hat{\mathbf{X}}$ system state variables $\mathbf{X}$ (20). The proposed calculator works by an open loop, and in real conditions, in the presence of unaccounted perturbations and an inaccurate specification of the initial condition $\hat{\mathbf{X}}(0)$ the values of the estimate and the vector of the state variables will diverge with time[Trofimov et al., 1997].To compensate for this discrepancy construct observing device according to the principle of a closed, using the measurement results $\mathbf{Y}$ part coordinates state vector $\mathbf{X}$ from the sensors according to the equation Image The difference between the measuring vector estimate $\mathbf{Y}$ and measurement $\hat{\mathbf{Y}} = \mathbf{C}\hat{\mathbf{X}}$ we use to improve the observer. For this purpose, monitored devices (21) introduce the feedback matrix $\mathbf{K}$. We obtain a closed observing device, described by equation $\dot{\hat{X}} = \mathbf{A}\hat{\mathbf{X}} + \mathbf{B}u + \mathbf{K}(\mathbf{Y} - \mathbf{C}\hat{\mathbf{X}})$, called the Lewinberger observer. Matrix $\mathbf{K}$ can be assigned arbitrarily, for example so as to provide stability and the observer damping transients in the observing device for the desired time. Closed observer typically has two inputs: u - Effects on the control and $\mathbf{Y}$ - from the measurements obtained from the sensors. Note that in our problem, it makes no sense to build a complete observer of all state coordinates. Instead, a simplified system of differential equations

$$\begin{cases} \dot{x}_1 = \ddot{x}, \\ \dot{x}_2 = x_1 - x_3, \\ \dot{x}_3 = \ddot{y}, \end{cases} \tag{22}$$

where 3 coordinates of state $x_1 = \dot{x}$, $x_2 = z$, $x_3 = \dot{y}$. In vector-matrix form the system (22) takes the form

$$\dot{\hat{X}} = \hat{\mathbf{A}}\hat{\mathbf{X}} + \mathbf{HM}, \tag{23}$$

where $\hat{\mathbf{X}}$ is the vector coordinate the state observer; $\hat{\mathbf{A}}$ is the matrix of observer coefficients; $\mathbf{M}$ is the vector of the values measured by the sensors; $\mathbf{H}$ is the matrix of the coefficients of the measured quantities. We will compose the observer's equation (24).

$$\dot{\hat{X}} = \hat{\mathbf{A}}\hat{\mathbf{X}} + \mathbf{HM} + \mathbf{K}(z - \hat{\mathbf{C}}\hat{\mathbf{X}}), \tag{24}$$

where $\hat{\mathbf{C}} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ is the matrix of the choice of the corrected value; $\hat{\mathbf{K}} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}^T$ is a correction matrix. The estimation of the absolute velocity of the object is given by the equation of the observer's exit: $\hat{\dot{x}} = \mathbf{C_o}\hat{\mathbf{X}}$, where $\mathbf{C}_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ is the matrix of the observer's output. It should be noted that in such a system, no control action is given to the observer[Mita et al., 1994]. The block diagram of the observing device constructed according to the expression (24) is shown in Fig. 5. Symbols η_x and $\eta_{\ddot{y}}$ is the noise in the acceleration measurement channels on the object and the base, respectively.

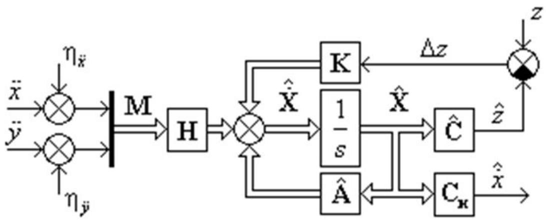

Figure 5: Block diagram of the observing device

We estimate the work of the observer. For modeling we use the system considered earlier without the introduction of a control action. The root-mean-square deviations in the acceleration measurement channels are accepted $\sigma(\eta_{\ddot{x}}) = \sigma(\eta_{\ddot{y}}) = 0.5$. To find the coefficients of the correction matrix K using coordinate descent procedure. During this procedure, a disturbing effect was applied to the system $f = 2$ Hz. As a result, a correction matrix

$$\mathbf{K} = \begin{bmatrix} 99.3 & 73.0 & 71.3 \end{bmatrix}^T. \tag{25}$$

The results of the simulation of the system with the observer (24), obtained by the correction matrix and the disturbing effect of the frequency are shown in Fig. 6.

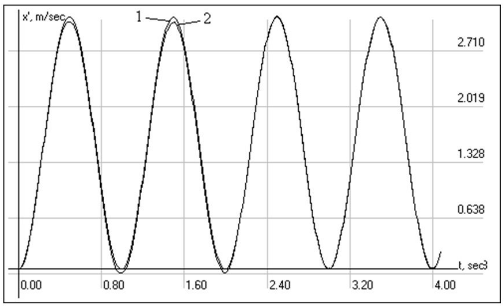

Figure 6: The results of simulation of the operation of the observing device at the frequency of the disturbing effect $f = 2$ Hz: *1* is the true value of the absolute speed of the object *2* is the estimate of the absolute velocity of the object

Acknowledgments.

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