Prime Cost Analysis in the Model of Gas Fields

Alexander K. Skiba
Federal Research Center "Information and Control", Russian Academy of Sciences
Vavilov st. 44,
119333 Moscow, Russia.
ipiran@ipiran.ru

Abstract

We consider continuous dynamic model of gas fields with interacting wells. The minimum prime cost of gas extraction is the main criterion for the construction project of the gas field. We analyze dependence of the minimum prime cost on the planning horizon. In our model at some turning point of time one gas field is selected for further development out of two gas fields on the basis of the available projects. The only selection criterion is the minimum prime cost of gas extraction. When the selection is done, any further revision of the choice is impossible. We set two problems about correctness of the made choice under various condition. We completely solve the first problem. We find the conditions when the choice of the gas field remains correct, although the planning horizon changes.

1 Introduction

Department of Methods for Designing Developing Systems explores the problem of oil, gas and condensate fields dynamic planning (individually and as a group) for many years. Based on the modern foundations of gas fields exploitation [Vyakhirev, 1998] a number of original mathematical models was developed and a large number of optimal control problems was solved [Margulov, 1992], [Skiba, 2009], [Skiba, 2012], [Khachaturov, 2015]. One of them is a problem of optimal economic growth [Skiba, 1978], whose solution is based on the propositions of K. Arrow [Arrow, 1974].

The Department pays special attention to the computational methods in the sphere of dynamic planning. Thus, a simulation model of the Gas Production Planning System (SPDG) was created and implemented on the computer. This system does dynamic calculation of indicators for any given horizon $T > 0$.

The economic indicators of gas production are the main subjects to investigation. Profit, income and other indicators related to the market price are usually taken as the main subjects to investigation. However, in the recent years it has become quite clear that the market price is a poorly predictable economic indicator, even given a small time horizon. That is why in this paper we consider prime cost is taken as a target economic indicator.

In recent years, several researches of the prime cost optimization have been made in the adjacent fields of study. For example, shale gas development was investigated in [Li, 2017] and optimal planning and infrastructure development for shale gas production was discussed in [Arredondo-Ramirez, 2016].

In: Yu. G. Evtushenko, M. Yu. Khachay, O. V. Khamisov, Yu. A. Kochetov, V.U. Malkova, M.A. Posypkin (eds.): Proceedings of the OPTIMA-2017 Conference, Petrovac, Montenegro, 02-Oct-2017, published at http://ceur-ws.org

In [Elsholkami, 2017] they consider the general optimization model for the energy planning of industries including renewable energy and prime cost optimization of biofuel production was elaborated in [Jong, 2017].

Increased interest in this issue confirms the importance of the topic under the study in this paper.

2 Prime Cost Analysis of the Gas Field

In the paper we consider a dynamic model of gas field with interacting wells [Margulov, 1992]. The following notations are used:

T is the planning horizon;

N is operating fund of producing wells;

$\bar{N}$ is general fund of producing wells;

q is average flow rate of producing wells;

V is the extracted gas reserve;

c is the natural gas market price.

All the variables are strictly positive. The relationship among them can be written in the form of differential equations:

$$\dot{V} = -Nq, \quad \dot{q} = -\alpha\ Nq, \quad \alpha = \frac{q_0}{V_0} \tag{1}$$

with initial conditions $V_0 > 0$, $q_o > 0$. The following constraint is imposed on the fund of producing wells

$$0 \leq N \leq \bar{N}. \tag{2}$$

Capital expenditures are described as a linear function $z + k\bar{N}$, where z and k are constant coefficients. Next, we write out two formulas that determine the prime cost of gas extraction and the profit obtained from its sale. The prime cost is defined as the ratio of capital expenditures during the construction of the field to the extracted volume of natural gas produced for the entire planning period.

$$\text{Cost} = \frac{z + k\bar{N}}{\int\limits_0^T qN\mathrm{d}t}. \tag{3}$$

Profit is the difference between the market cost of the products and all existing expenses.

$$Prib = c\int\limits_0^T qN\mathrm{d}t - z - k\bar{N}. \tag{4}$$

Expenses consist only of capital expenditures made in the pre-planned period. Total current expenses are small compared to all capital expenditures. Therefore total current expenses are not taken into account. We make the following change of variables:

$$N' = \alpha N, \quad \bar{N}' = \alpha\bar{N}, \quad V' = \alpha V = q, \quad z' = \alpha\frac{z}{k}, \quad c' = \frac{c}{k}. \tag{5}$$

We omit the strokes in the notation of these variables.

Double inequality (2) will be understood in the new variables. We rewrite the formula (1), (3) and (4) in a more convenient for further analysis:

$$\dot{q} = -Nq; \tag{6}$$

$$\text{Cost} = k\frac{z + \bar{N}}{\int\limits_0^T qN\mathrm{d}t}; \tag{7}$$

$$Prib = \frac{k}{\alpha}(c\int\limits_0^T qN\mathrm{d}t - z - \bar{N}). \tag{8}$$

We set two optimal control problems: the first problem is to minimize the prime cost of natural gas extraction, the second problem is to maximize the profit from its sale.

Problem 1. Minimize the functional (7) at differential connection (6), the initial condition $q^0 > 0$ and a constraint on control (2).

Problem 2. Maximize the functional (8) at differential connection (6) the initial condition $q^0 > 0$ and a constraint on control (2).

It is easy to show that the minimum of (7) and the maximum of (8) are achieved under the following control: $N(t) = \bar{N}$ throughout the whole planning period. Thus, solutions of these problems are to minimize the value of $q(T)$:

$$q(T) = q^0 e^{-\int\limits_0^T N(t)\mathrm{d}t} \geq q^0 e^{-\int\limits_0^T \bar{N}\mathrm{d}t} = q^0 e^{-\bar{N}T}.$$

By transforming (7) and (8), we obtain two functions that depend on $\bar{N}$ and T:

$$\mathrm{Cost}(\bar{N}, T) = k\frac{z + \bar{N}}{q^0(1 - e^{-\bar{N}T})}; \tag{9}$$

$$Prib(\bar{N}, T) = \frac{k}{\alpha}\left(cq^0(1 - e^{-\bar{N}T}) - z - \bar{N}\right). \tag{10}$$

Next we find the minimum of (9) given a fixed T. So, we take the partial derivative of (9) with respect to $\bar{N}$, that should be equaled to zero. After the transformations, we obtain the following equation:

$$e^{\bar{N}T} - 1 - \bar{N}T - zT = 0. \tag{11}$$

We denote the left-hand side of the equation (11) by $\Phi(T, \bar{N})$, where T and $\bar{N}$ are positive variables. In this case, the parameter z is assumed to be positive and fixed. The function $\Phi(T, \bar{N})$ is defined and continuous for $T > 0$ and for $\bar{N} > 0$, and this function has continuous partial derivatives $\partial\Phi/\partial T$ and $\partial\Phi/\partial\bar{N}$ for positive values of T and $\bar{N}$. Moreover, $\partial\Phi/\partial\bar{N} = T(e^{\bar{N}T} - 1) > 0$. It is easy to show that for any $T_0 > 0$ there exists a unique $\bar{N}_0 > 0$ such that $\Phi(T_0, \bar{N}_0) = 0$. Hence, by the implicit functional theorem for $T > 0$ there exists a unique positive-definite function $\bar{N} = \bar{N}^*(T)$ satisfying the equation (11).

We substitute the function $\bar{N}^*(T)$ into the equation (11). As a result, we arrive at the identity

$$e^{\bar{N}^*(T)T} - 1 - \bar{N}^*(T)T - zT = 0. \tag{12}$$

Using the last identity, we simplify the function (9)

$$\mathrm{Cost}(\bar{N}^*(T), T) = \frac{k}{q^0 T}e^{\bar{N}^*(T)T}. \tag{13}$$

It is interesting to study the behavior of the functions $\bar{N}^*(T)T$, $\bar{N}^*(T)$ and $\mathrm{Cost}(\bar{N}^*(T), T)$ depending on parameter $T > 0$. The study of the behavior of these functions will be useful to us in the future. The following three statements are given with proof.

Statement 1. The function $\bar{N}^*(T)T$ is defined and continuous for $T > 0$ and it is strictly increasing from 0 to ∞.

We expand the exponent into the identity (12) in the Maclaurin series. After the transformations, we obtain the following double inequality:

$$0 < \bar{N}^*(T)T < \sqrt{2zT}. \tag{14}$$

In the double inequality (14), we pass to the limit, directing the parameter T go to zero. As a result, we obtain $\lim_{T\to 0}(\bar{N}^*(T)T) = 0$. We differentiate both sides of the identity (12) with respect to T and after the transformations we obtain

$$(\bar{N}^*(T)T)' = \frac{z}{e^{\bar{N}^*(T)T} - 1} > 0. \tag{15}$$

Then the function $\bar{N}^*(T)T$ is strictly increasing. The following result follows from the identity (12): $\lim_{T\to\infty}(\bar{N}^*(T), T) = \infty$. The statement 1 is proved.

Statement 2. The function $\bar{N}^*(T)$ is defined and continuous at $T > 0$ and is strictly decreasing from ∞ to 0.

Dividing both sides of the identity (12) by T, we differentiate it with respect to the parameter T. After the transformations, we obtain

$$(\bar{N}^*(T))' = \frac{e^{\bar{N}^*(T)T} - 1 - \bar{N}^*(T)Te^{\bar{N}^*(T)T}}{T^2(e^{\bar{N}^*(T)T} - 1)}. \tag{16}$$

The denominator of (16) is positive for $T > 0$. The numerator of (16) is zero for $T = 0$. To show that the numerator of this fraction is negative for $T > 0$ we differentiate it by T. Taking into account statement 1 for $T > 0$, we obtain

$$-\bar{N}^*(T)Te^{\bar{N}^*(T)T}(\bar{N}^*(T)T)' < 0.$$

So, the numerator is negative for $T > 0$. The function $\bar{N}^*(T)$ is strictly decreasing. We divide all the parts of the double inequality (14) by T

$$0 < \bar{N}^*(T) < \sqrt{\frac{2z}{T}}. \tag{17}$$

Passing to the limit as $T \to \infty$, we arrive at the following result: $\lim_{T\to\infty} \bar{N}^*(T) = 0$. We expand the function $e^{\bar{N}^*(T)T}$ using the Maclaurin formula of the second order with the remainder term in the Peano form and substitute the resulting expansion into the identity (12). As a result, we obtain

$$\frac{(\bar{N}^*(T)T)^2}{2} + o(\bar{N}^*(T)T)^2 - zT = 0. \tag{18}$$

We divide both sides of the identity (18) into $\frac{(\bar{N}^*(T)T)^2}{2}$ and taking into account $\lim_{T\to 0}(\bar{N}^*(T)T) = 0$, we pass to the limit as $T \to 0$

$$1 - \lim_{T\to 0}\left(\frac{2z}{(\bar{N}^*(T))^2T}\right) = 0. \tag{19}$$

The result is $\lim_{T\to 0} \bar{N}^*(T) = \infty$. The statement 2 is proved.

Statement 3. The function $\mathrm{Cost}(\bar{N}^*(T), T)$ that describes the prime cost of gas extraction, defined and continuous for $T > 0$ and it is strictly decreasing from ∞ to $\frac{kz}{q^0}$.

We substitute the function $\bar{N}^*(T)$ into the function (9). Next, we differentiate the function $\mathrm{Cost}(\bar{N}^*(T), T)$ with respect to the parameter T and, using the relation (11), we obtain

$$(\mathrm{Cost}(\bar{N}^*(T), T))' = -k\bar{N}^*(T)e^{-\bar{N}^*(T)T}\frac{z + \bar{N}^*(T)}{q^0(1 - e^{-\bar{N}^*(T)T})^2} < 0. \tag{20}$$

Hence the function $\mathrm{Cost}(\bar{N}^*(T), T)$ strictly decreases for $T > 0$. Using statement 1 and 2, we calculate the limits:

$$\lim_{T\to 0} \mathrm{Cost}(\bar{N}^*(T), T) = k\frac{z + \bar{N}^*(T)}{q^0(1 - e^{-\bar{N}^*(T)T})} = \infty;$$

.

$$\lim_{T\to\infty} \mathrm{Cost}(\bar{N}^*(T), T) = k\frac{z + \bar{N}^*(T)}{q^0(1 - e^{-\bar{N}^*(T)T})} = k\frac{z}{q^0}.$$

The statement 3 is proved.

Let us now turn to the main part of the present paper.

3 Comparative Analysis of Two Gas Fields Prime Costs

Suppose one of two gas fields must be selected for further production during the planning period T_0. The main criterion for selection is the minimum prime cost of gas extraction. The choice is made on the basis of the submitted projects of field construction. The choice of only one field could be done due to various reasons, for example, the possibility to connect the gas pipeline with the only one out of the two fields.

Obviously, in this situation, every owner of the field wants to win the tender. Therefore, each field is projected taking into account the planning horizon T_0 and the minimum prime cost of gas extraction. Thus, the producing wells fund $\bar{N}^*(T_0)$ and minimum prime cost $\mathrm{Cost}(\bar{N}^*(T_0), T_0)$ are determined. The winner of the tender is determined without any possibility of further change.

Let the planning horizon changes from T_0 to another value T. There is a natural question of the correctness of the choice made. This section of the paper is devoted to the answer to this question.

When choosing one out of two fields for further gas extraction, two cases are considered.

In the first case when there is no possibility to reorganize the field construction, the fund of producing wells $\bar{N}^*(T_0)$ and the capital costs $k(z + \bar{N}^*(T_0))$ remain the same. However, the accumulated gas extraction $q^0(1 - e^{-\bar{N}^*(T_0)T})$ and the prime cost of gas extraction $\mathrm{Cost}(\bar{N}^*(T_0), T)$ are changed.

In the second case, the field construction reorganization is possible. Another field construction project is being created with the new planning horizon T. Thus, we get a new value of the fund of producing wells $\bar{N}^*(T)$ and the minimum prime cost of gas extraction $\mathrm{Cost}(\bar{N}^*(T), T)$

Statement 4. The following inequality holds:

$$\mathrm{Cost}(\bar{N}^*(T), T) \le \mathrm{Cost}(\bar{N}^*(T_0), T),$$

moreover, the equality holds only when $T = T_0$. For any positive quantity N_0 there exists a unique value $T_0 > 0$ such that $\bar{N}^*(T_0) = N_0$. For any positive $N \ne \bar{N}^*(T_0)$, a strict inequality holds:

$$\mathrm{Cost}(\bar{N}^*(T_0), T_0) < \mathrm{Cost}(N, T_0).$$

Problem 3. We have a set of five positive numbers: $\bar{N}_1^*(T_0)$, $\mathrm{Cost}_1(\bar{N}_1^*(T_0), T_0)$, $\bar{N}_2^*(T_0)$, $\mathrm{Cost}_2(\bar{N}_2^*(T_0), T_0)$ and T_0. The first two numbers of the set refer to the first field. The next two numbers of the set refer to the second field. The last number is the horizon of gas production planning. It is assumed that inequality

$$\mathrm{Cost}_1(\bar{N}_1^*(T_0), T_0) < \mathrm{Cost}_2(\bar{N}_2^*(T_0), T_0) \tag{21}$$

It is necessary to find values of T such that the inequality

$$\mathrm{Cost}_1(\bar{N}_1^*(T_0), T) < \mathrm{Cost}_2(\bar{N}_2^*(T_0), T) \tag{22}$$

holds and such values of T for which it does not hold.

A set of five numbers is formed from field construction projects. This set is enough to calculate the prime costs of gas extraction for any planning horizon. We show how this is done.

According to (9), the prime costs of gas extraction are calculated by the formulas:

$$\mathrm{Cost}_i(\bar{N}_i^*(T_0), T) = k_i \frac{z_i + \bar{N}_i^*(T_0)}{q_i^0(1 - e^{-\bar{N}_i^*(T_0)T})}, \qquad i = 1, 2. \tag{23}$$

Based on the initial data, we determine the concrete values of two numbers:

$$a_i = \mathrm{Cost}_i(\bar{N}_i^*(T_0), T_0)(1 - e^{-\bar{N}_i^*(T_0)T_0}), \qquad i = 1, 2. \tag{24}$$

Then the prime costs of gas extraction are calculated by the formulas:

$$\mathrm{Cost}_i(\bar{N}_i^*(T_0), T) = \frac{a_i}{1 - e^{-\bar{N}_i^*(T_0)T}}, \qquad i = 1, 2. \tag{25}$$

The ratio of the prime costs of gas extraction of $R(T)$

$$R(T) = \frac{\mathrm{Cost}_2(\bar{N}_2^*(T_0), T)}{\mathrm{Cost}_1(\bar{N}_1^*(T_0), T)} = \frac{a_2(1 - e^{-\bar{N}_1^*(T_0)T})}{a_1(1 - e^{-\bar{N}_2^*(T_0)T})}.$$

Using the new notation, from the inequality (ref eq21) we obtain a restriction on the value of the function $R(T)$ at the point T_0:

$$1 < R(T_0) = \frac{\mathrm{Cost}_2(\bar{N}_2^*(T_0), T_0)}{\mathrm{Cost}_1(\bar{N}_1^*(T_0), T_0)} = \frac{a_2(1 - e^{-\bar{N}_1^*(T_0)T_0})}{a_1(1 - e^{-\bar{N}_2^*(T_0)T_0})}. \tag{26}$$

Statement 5. The function $R(T)$ is defined and continuous for $T > 0$ and it monotonically changes from $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)}$ to $\frac{a_2}{a_1}$. The function $R(T)$ is strictly increasing when $\bar{N}_1^*(T_0) < \bar{N}_2^*(T_0)$ and is strictly decreasing when $\bar{N}_1^*(T_0) > \bar{N}_2^*(T_0)$.

Next, we explore the function

$$f(T) = \frac{1 - e^{-\bar{N}_1^*(T_0)T}}{1 - e^{-\bar{N}_2^*(T_0)T}}, \tag{27}$$

which is different from $R(T)$ by a constant value. It is easy to show that:

$$\lim_{T\to 0} f(T) = \frac{\bar{N}_1^*(T_0)}{\bar{N}_2^*(T_0)}; \qquad \lim_{T\to\infty} f(T) = 1. \tag{28}$$

Next we differentiate $f(T)$ with respect to T:

$$f'(T) = \frac{\bar{N}_1^*(T_0)e^{-\bar{N}_1^*(T_0)T}(1 - e^{-\bar{N}_2^*(T_0)T}) - \bar{N}_2^*(T_0)e^{-\bar{N}_2^*(T_0)T}(1 - e^{-\bar{N}_1^*(T_0)T})}{(1 - e^{-\bar{N}_2^*(T_0)T})^2}$$

$$= \frac{\bar{N}_2^*(T_0)e^{-(\bar{N}_1^*(T_0)+\bar{N}_2^*(T_0))T}}{(1 - e^{-\bar{N}_2^*(T_0)T})^2}\left[\frac{\bar{N}_1^*(T_0)}{\bar{N}_2^*(T_0)}(e^{\bar{N}_2^*(T_0)T} - 1) - (e^{\bar{N}_1^*(T_0)T} - 1)\right]. \tag{29}$$

At $T = 0$ the expression in square brackets is zero. Differentiate with respect to T the expression in square brackets in the ratio (29). The result is $\bar{N}_1^*(T_0)(e^{\bar{N}_2^*(T_0)T} - e^{\bar{N}_1^*(T_0)T})$. It follows that $f'(T) > 0$ for $\bar{N}_1^*(T_0) < \bar{N}_2^*(T_0)$ and $f'(T) < 0$ for $\bar{N}_1^*(T_0) > \bar{N}_2^*(T_0)$. In both cases, $T > 0$. The statement 5 is proved.

Let us restate the problem 3 in terms of the functions $R(T)$.

Problem 3′. Under the assumption (27) it is necessary to find the values of T when the function $R(T)$ is greater than 1, and when it is less than 1.

The following theorem (the solution of problem 3) follows from the inequality (27) and the statement 5.

Theorem 1. Case 1. Let $\bar{N}_1^*(T_0) < \bar{N}_2^*(T_0)$. In this case, $\frac{a_2}{a_1} > 1$, then:

a) the function $R(T)$ is strictly increasing on the interval $(0, \infty)$ from $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)}$ to $\frac{a_2}{a_1}$;

b) if $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)} < 1$, then there exists a unique positive number T_1^* less T_0 such that $R(T_1^*) = 1$, and when $T > T_1^*$ inequality $R(T) > 1$, and when $0 < T \le T_1^*$, it is not executed.

c) if $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)} \ge 1$, then the inequality $R(T) > 1$ occurs for all positive values of T.

Case 2. Let $\bar{N}_1^*(T_0) > \bar{N}_2^*(T_0)$. In this case, $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)} > 1$, then:

a) the function $R(T)$ is strictly decreasing on the interval $(0, \infty)$ from $\frac{a_2\bar{N}_1^*(T_0)}{a_1\bar{N}_2^*(T_0)}$ to $\frac{a_2}{a_1}$;

b) if $\frac{a_2}{a_1} < 1$, then there exists a unique positive number T_1^* greater T_0 such that $R(T_1^*) = 1$, and when $0 < T < T_1^*$ inequality $R(T) > 1$, and if $T \ge T_1^*$, it is not executed.

c) if $\frac{a_2}{a_1} \ge 1$, then the inequality $R(T) > 1$ occurs for all positive values of T.

Case 3. Let $\bar{N}_1^*(T_0) = \bar{N}_2^*(T_0)$. In this case, $\frac{a_2}{a_1} > 1$, then:

a) the function $R(T)$ on the interval $(0, \infty)$ takes a constant value, equal to $\frac{a_2}{a_1}$;

b) inequality $R(T) > 1$ for all positive values of T.

Problem 4. The conditions of the problem 4 such as in problem 3 with the exception of its goals. The goal of the problem consists in finding such values of T for which the inequality

$$\mathrm{Cost}_1(\bar{N}_1^*(T), T) < \mathrm{Cost}_2(\bar{N}_2^*(T), T) \tag{30}$$

is performed and, accordingly, in which it fails.

4 Conclusion

In this paper, we considered a continuous dynamic model of a gas field. Within the framework of the model, formulas for calculating profit and prime cost of gas extraction were written out. Profit is considered as the most important economic indicator. Only profit is fully able to reflect the level of efficiency of any enterprise. However, unlike the prime cost of gas extraction , the market price of gas (which is included in the profit calculations) is a poorly predicted dynamic value. This fact greatly complicates the prediction of total profit. The prime cost is defined as the ratio of capital costs during the construction of the field to the volume of natural gas extracted during the planned period.

Capital costs are described as a linear function of one variable with a constant part and a variable part. The variable part depends on the fund of producing wells. For each planning horizon, we find the minimum prime cost of gas extraction. It is proved that the minimum exists. Thus, for each planning horizon, we unambiguously determine the value of the producing wells fund. It is proved that the optimal value of the fund of producing wells is strictly decreasing from infinity to zero with a change in the planning horizon. The minimum value of the prime cost of gas extraction strictly decreases from infinity to a certain positive value. These properties of the fund of producing wells and prime costs of gas extraction do not contradict the basic property of the model. Over infinite time, the entire natural gas reserve can be completely extracted by a single well.

The following situation is considered. Two fields participate in the tender. One field is selected with the lowest prime cost of gas extraction. Revision of the choice is impossible. The planning horizon is changing. Two cases are considered. In one case, producing wells remain unchanged. In another case, the prime cost of gas extraction for each field is minimized with a new planning horizon. For each case, the problem is posed with the following question. Is the right choice made? Both problems are solved.

However, the paper gives a complete description of the solution of only the first problem. The solution of the second problem is limited only to its formulation. The solutions of the two problems are structurally similar to each other. The existence of no more than two areas is proved. If the area is one, then the choice is made correctly. If there are two areas, then in one area the choice is made correctly, and in another it is wrong.The sequence of areas can be any.

In practice, a conflict situation may arise with the organizers of the tender in the event of the negative answer to the question posed in the problems. Sometimes a protracted conflict can be resolved only through the courts.

References

[Vyakhirev, 1998] Vyakhirev, R.I.,Korotaev, Yu.P.,& Kabanov, N.I. (1998) *The Theory and Experience of Gas Recovery.* Moscow: Nedra

[Margulov, 1992] Margulov, R. D.,Khachaturov, V. R.,& Fedoseev, A.V.. (1992) *System Analysis in Long-term Planning of Gas Production.* Moscow: Nedra

[Skiba, 2009] Skiba, A. K. (2009). Maximum Principle in a Problem of Maximization of the Income for Model of a Gas Deposit. *Bulletin of Peoples Friendship University of Russia. Series Mathematics. Information Sciences. Physics, 1*, 14-22

[Skiba, 2012] Skiba, A. K. (2012). An Optimal Control Applied Problem with a Mixed Constraint.*Bulletin of Peoples Friendship University of Russia. Series Mathematics. Information Sciences. Physics, 4*, 31-43

[Khachaturov, 2015] Khachaturov, V. R., Solomatin, A. N., Zlotov, A. V., Bobylev, V.N., Veselovsky, V. E., Kovalenko, A.G., Kosachev, Yu. V., Krylov, I. A, Livanov, Yu. V., Skiba, A. K., & Cherepanov, V. V. (2015) *Planning and Design of Development of Oil and Gas Producing Regions and Deposits: Mathematical Models, Methods, Application.* Moscow: URSS: LENAND

[Skiba, 1978] Skiba, A. K. (1978). Optimal growth with a convex-concave production function. *Econometrica, 3(46)*, 527-539

[Arrow, 1974] Arrow, K. (1974). The Application of Control Theory to Economic Growth. *Mathematical Economics*, 7-45, Moscow: Mir

[Arredondo-Ramirez, 2016] Arredondo-Ramirez K., Ponce-Ortega J. M.,& El-Halwagi M. M.(2016). Optimal planning and infrastructure development for shale gas production. *Energy Conversion and Management. 119*, 91-100

[Li, 2017] Li H., & et al.(2017). A theoretical cost optimization model of reused flowback distribution network of regional shale gas development. *Energy Policy. 100*, 359-364.

[Elsholkami, 2017] Elsholkami M., & Elkamel A. (2017). General optimization model for the energy planning of industries including renewable energy: A case study on oil sands. *AIChE Journal. 2(63)*, 610-638.

[Jong, 2017] Jong S., & et al. (2017) Cost optimization of biofuel productionThe impact of scale, integration, transport and supply chain configurations. *Applied Energy. 195*, 1055-1070