Sensitivity Analysis of Steady State Reliability Characteristics of a Repairable Cold Standby Data Transmission System to the Shapes of Lifetime and Repair Time Distributions of its Elements

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Continuous development of computer networks and data transmission systems emphasizes the increasing need for adequate mathematical models and tools that allow for the study of their performance and reliability.

We consider the problem of sensitivity analysis of reliability characteristics of a repairable cold standby data transmission system with exponential life time and general repair time distributions to shapes of the input distributions. The simulation model based on the discrete-event approach is introduced to obtain results in case of a general (non-exponential) distribution of repair time of the elements.

There are several approaches to model the reliability of systems with general life- and repair times distributions. Anyhow all of them are reduced to markovization of the process that describes the system behavior [1].

The proposed analytical methodology allows to assess system-level reliability in case of failures of system elements. Explicit analytical expressions were obtained for the stationary probability distribution of system states, which enable to analyze other operational variables of the system with respect to the redundant element's performance.

The demonstrated analytical and simulation results show excellent asymptotic insensitivity of the stationary reliability of the system under "fast" recovery of its elements to the type of repair time distribution. Comparison of numerical and graphical results obtained using both analytical and simulation approaches, shows that they have close agreement, so the elaborated simulation model can be used in cases when explicit analytical solution can not be achieved, or as a part of a more complex simulation model.

Key words and phrases: system reliability, steady state probabilities, sensitivity, mathematical modeling and simulation.

1. Introduction

Continuous development of computer networks and data transmission systems emphasizes the increasing need for adequate mathematical models, and tools that allow for the study of their functioning. You must have the help of both the design stage (for comparison-making) and exploitation (Service Quality Management) network systems. Indeed, the development of complex technical system requires not only high-quality simulation to check how well it is constructed logically, but also required a priori verification of system performance during the design phase. The aim is to conduct analytical and simulation of the system $\langle M_3|GI|1\rangle$.

2. The Model and Analytical Results

Consider a random process $v(t)$ — the number of failed elements at time t, the set of states of the system $E = 0, 1, 2, 3, 4$. To describe the behavior of the system using the Markov process, we introduce an additional variable $x(t) \in R_+^2$ — time spent at time

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t, for the repair of the failed element. We obtain a two-dimensional process $(v(t), x(t))$, with an extended state space $\epsilon = (0), (1, x), (2, x), (3, x)$.

We denote $p_0(t)$ — the probability that at time t the system is in the state $i = 0$, $p_i(t; x)$ — density distribution (in continuous component) the probability that at time t the system is in state $i (i = 1, 2, 3)$, and the time taken to repair the failed element is in the range $(x, x + dx)$.

$$
\begin{gathered}
p_0(t) = p\{v(t) = 0\},\\
p_1(t, x)dx = p\{v(t) = 1, x < x(t) < x + dx\},\\
p_2(t, x)dx = p\{v(t) = 2, x < x(t) < x + dx\},\\
p_3(t, x)dx = p\{v(t) = 3, x < x(t) < x + dx\}.
\end{gathered}
$$

With the help of the formula of total probability we move to a system of Kolmogorov differential equations

$$
\begin{cases}
p_0(t + \Delta) = p_0(t) \cdot (1 - \alpha\Delta) + \int\limits_0^t p_1(t, x)\delta(x)\Delta dx,\\
p_1(t + \Delta, x + \Delta) = p_1(t, x) \cdot (1 - \alpha\Delta) \cdot (1 - \delta(x)\Delta),\\
p_2(t + \Delta, x + \Delta) = p_2(t, x) \cdot (1 - \alpha\Delta) \cdot (1 - \delta(x)\Delta),\\
p_3(t + \Delta, x + \Delta) = p_3(t, x) \cdot (1 - \delta(x)\Delta) + p_2(t, x) \cdot \alpha\Delta,\\
p_1(t + \Delta, 0)dx = p_0(t) \cdot \alpha\Delta + \int\limits_0^t p_2(t, x)\delta(x)\Delta dx,\\
p_2(t + \Delta, 0)dx = \int\limits_0^t p_1(t, x)\alpha\Delta dx + \int\limits_0^t p_3(t, x)\delta(x)\Delta dx,
\end{cases}
$$

and a passage to the limit $\Delta \to 0$, we get:

$$
\begin{cases}
\alpha \cdot p_0 = \int\limits_0^\infty p_1(x)\delta(x)dx,\\
\dfrac{dp_1(x)}{dx} = -(\alpha + \delta(x)) \cdot p_1(x),\\
\dfrac{dp_2(x)}{dx} = -(\alpha + \delta(x)) \cdot p_2(x),\\
\dfrac{dp_3(x)}{dx} = -\delta \cdot p_3(x) + \alpha p_2(x),\\
p_1(0)dx = \alpha \cdot p_0 + \int\limits_0^\infty p_2(x)\delta(x)dx,\\
p_2(t0)dx = \int\limits_0^\infty p_1(x)\alpha dx + \int\limits_0^\infty p_3(x)\delta(x)dx,
\end{cases}
$$

and under assumption that the process has a stationary distribution when $t \to \infty$, we get:

$$\begin{cases} \alpha \cdot p_0 = \int\limits_0^\infty p_1(x)\delta(x)dx, \\ \dfrac{dp_1(x)}{dx} = -(\alpha + \delta(x)) \cdot p_1(x), \\ \dfrac{dp_2(x)}{dx} = -(\alpha + \delta(x)) \cdot p_2(x), \\ \dfrac{dp_3(x)}{dx} = -\delta \cdot p_3(x) + \alpha p_2(x), \\ p_1(0)dx = \alpha \cdot p_0 + \int\limits_0^\infty p_2(x)\delta(x)dx, \\ p_2(t0)dx = \int\limits_0^\infty p_1(x)\alpha dx + \int\limits_0^\infty p_3(x)\delta(x)dx. \end{cases}$$

From here, we move on to the solution obtained system of Kolmogorov differential equations using the method of variation of constants, and obtain the stationary probabilities of the system states

$$p_0 = \frac{\tilde{b}^2(\alpha)}{\rho^-1(1-\tilde{b}(\alpha)) + \tilde{b}(\alpha)}, \quad p_1 = \frac{\tilde{b}(\alpha)(1-\tilde{b}(\alpha))}{\rho^-1(1-\tilde{b}(\alpha)) + \tilde{b}(\alpha)},$$

$$p_2 = \frac{(1-\tilde{b}(\alpha))^2}{\rho^-1(1-\tilde{b}(\alpha)) + \tilde{b}(\alpha)}, \quad p_3 = \frac{(1-\tilde{b}(\alpha))(\rho^-1 - 1 + \tilde{b}(\alpha))}{\rho^-1(1-\tilde{b}(\alpha)) + \tilde{b}(\alpha)}, \text{ where } \rho^{-1} = b \cdot \alpha.$$

Obviously, there is a dependency of stationary probabilities of states of the system on the type of distribution of repair time.

Figure 1 shows plots of the stationary probability of failure-free operation of the system on the relative speed of recovery.

Evidently, this dependence becomes vanishingly small with a “quick” recovery.

Explicit analytic expressions for the stationary distribution of the system under consideration cannot always be obtained. Therefore, the problem arose of constructing a simulation model that would adequately approximate the analytic model of the system.

3. Comparison and Analysis of the Results of Mathematical and Simulation Modeling

Define the following states of the modeled system:

- state 0: one (main) element is running, the second — in cold reserve;
- state 1: one device failed and is under repair, the second — works;
- state 2: one device refused, one — in repair, the second is waiting for its turn for repair, the third is running;
- state 3: all appliances refused, one — in repair, the others wait their turn for repair.

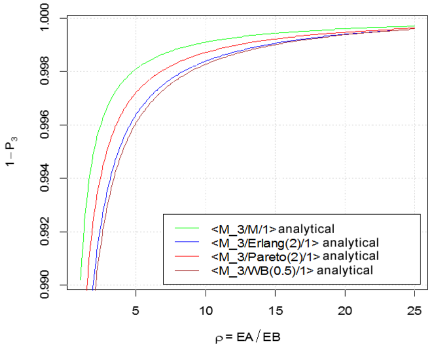

Figure 1. Plots of the stationary probability of failure-free operation $1 - p_3$ of ρ the various functions of the distribution of repair time

For clarity, the simulation model is represented graphically in Figure 2 in the form of a block diagram.

The criterion for stopping the main cycle of the model is to achieve the maximum model execution time T. The simulation was carried out with the limitation for the maximum model time $T = 10000$ runs.

Figure 3 shows plots of the probability Failure-free operation of the system from the model parameter ρ, constructed from the results of simulation modeling.

Evidently, the differences between the curves with growth become vanishingly small. The constructed simulation model well approximates the analytical model.

Table 1 shows the values of the stationary state probabilities calculated by a simulation and the analytical formulas.

Evidently, the simulation results are in good agreement with the results obtained by explicit analytical formulas. It also evidently that with increasing ρ differences in the values of p_i disappear.

4. Conclusions

Explicit analytical expressions for the stationary probability distribution of states of the system and a fixed probability of failure of the system were obtained in the general case, and for some special cases of distributions. These formulas show the clear dependence of these characteristics on the form of the distribution function. However, numerical research and analysis charting shown that this dependence becomes vanishingly small for a "fast" recovery, that is, relative to the growth rate of recovery ρ.

It was conducted simulation system $\langle M_3|GI|1\rangle$ based on discrete-event approach. Numerical and graphical comparison of results obtained using both approaches, shows a high degree of similarity, it can be used as the analytical solution and the simulation model (e.g., as part of a more complex simulation models).

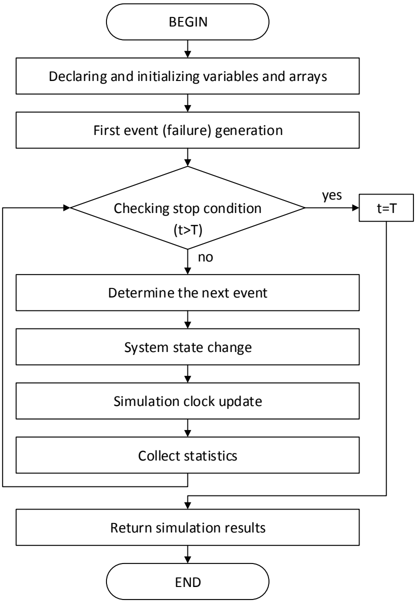

Figure 2. Scheme of the simulation system

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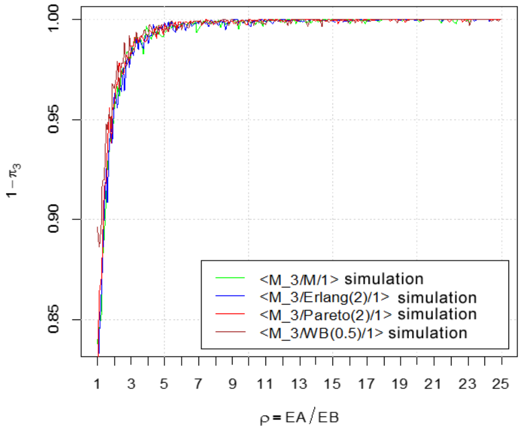

Figure 3. Graphs of the dependence of the probability of failure-free operation of the system $1 - p_3$ on the model parameter ρ per the results of simulation

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Table 1

Stationary probabilities p_i of system states $< M_3/GI/1 >$, imitation calculated analytically for different values of the model parameter $\rho = 1, 10, 100$

M_2 \ GI		$EXP\left(\beta = \frac{1}{EB}\right)$		$Erlang\left(\beta = \frac{k=2}{EB}\right)$		$Pareto\left(k, \frac{k}{k-1}\frac{1}{EB}\right)$		$WB\left(\beta = \frac{k=2}{EB}, \frac{1}{2}\right)$	
$EXP\left(\alpha = \frac{1}{EA}\right)$	p_i	Simul.	Analyt.	Simul.	Analyt.	Simul.	Analyt.	Simul.	Analyt.
$\rho = 1$; $EA = 5$ $EB = 5$	p_0	0.21603	0.25	0.19947	0.19753	0.13503	0.19643	0.47748	0.80219
	p_1	0.22039	0.25	0.25341	0.24691	0.26411	0.24678	0.21202	0.09346
	p_2	0.25692	0.25	0.31883	0.30864	0.35479	0.31002	0.094906	0.01089
	p_3	0.30666	0.25	0.22829	0.24691	0.24607	0.24678	0.21559	0.09346
$\rho = 10$; $EA = 50$ $EB = 5$	p_0	$8.9307 \cdot 10^{-1}$	0.90009	$9.0553 \cdot 10^{-1}$	0.89783	$9.0791 \cdot 10^{-1}$	0.90091	$9.332 \cdot 10^{-1}$	0.98502
	p_1	$9.5768 \cdot 10^{-1}$	0.09001	$8.9362 \cdot 10^{-2}$	0.09203	$8.0737 \cdot 10^{-2}$	0.08928	$5.845 \cdot 10^{-2}$	0.0136
	p_2	$1.0167 \cdot 10^{-2}$	0.00900	$5.1098 \cdot 10^{-3}$	0.00943	$1.0331 \cdot 10^{-2}$	0.00885	$6.7836 \cdot 10^{-3}$	0.00019
	p_3	$9.9769 \cdot 10^{-4}$	0.00090	0	0.00071	$1.0263 \cdot 10^{-3}$	0.00097	$1.574 \cdot 10^{-3}$	0.00119
$\rho = 100$; $EA = 500$ $EB = 5$	p_0	$9.8974 \cdot 10^{-1}$	0.99	$9.9202 \cdot 10^{-1}$	0.98998	$9.872 \cdot 10^{-1}$	0.99006	$9.9481 \cdot 10^{-1}$	0.99857
	p_1	$1.0261 \cdot 10^{-2}$	0.0099	$7.9853 \cdot 10^{-3}$	0.00993	$1.28 \cdot 10^{-2}$	0.00984	$5.190 \cdot 10^{-3}$	0.00141
	p_2	0	$9.9 \cdot 10^{-5}$	0	$9.949 \cdot 10^{-5}$	0	$9.7867 \cdot 10^{-5}$	0	$1.995 \cdot 10^{-6}$
	p_3	0	$9.9 \cdot 10^{-7}$	0	$7.4682 \cdot 10^{-7}$	0	$1.5464 \cdot 10^{-6}$	0	$1.214 \cdot 10^{-5}$