Digital signal processing under uncertainty conditions. Interval Approach

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Abstract. Digital signal processing under uncertainty conditions is considered. The signal represents an experimental chemical process, whose parameters have to be estimated. Measurments both of the process and its argument contain errors of bounded values. Sample of the process measurements is very short and there is uncertainty of probability characteristics of the errors. So, it is difficult to validate application of standard statistical methods to estimating the process parameters. An alternative is in application of the Interval Analysis methods. In the work, these methods are used for constructing the information set of admissible values of the process parameters and admissible tube of its dependencies.

Keywords: Digital signal, processing, algorithms, noised measurements, two-dimensional uncertainies, interval analysis methods, set of admissible parameters, tube of admissible dependencies

1 Introduction

As a rule, in investigations of experimental chemical processes, data are obtained with measuring errors. It is used to process such data by the standard methods that are based on the mathematical statistics ideology [1], [2], [3].

But in practice, a sample of measurements is very short and the errors' probability characteristics are unknown. Moreover, the errors can be not only in the process measurements, but, also, in ones of the process' argument, and uncertainty of each measurement becomes *two-dimensional*. So, under these conditions, it is difficult (or impossible) to validate application of standard methods.

As an alternative, application of the statistical methods can be completed by using the Interval Analysis ones.

To do this, the process is described by some model fuction with a vector of parameters; in our investigation, the linear dependence is used to describe the process. The problem is formulated for estimation of admissible set of these parameters. Such a set is used to call the *Information Set*. It comprises only such parameters of the model that are consistent with its description, accumulated sample of measurements, and the given bounds on the measuring errors in the process values and values of its argument. On the basis of the determined Information Set, corresponding *tube* of the admissible dependencies of the process is built.

The paper has the following structure. In Section 2, specific properties of experimental data and difficulties of application of standard methods for their procession are discussed, the typical model of a chemical process is introduced, short description of the main Interval Analysis procedures used for constructing the Information Set of process parameters is given, and problem of estimation is formulated. In Section 3, results of processing real experimental data are given. Here, results obtained by the interval approach are compared with ones calculated by formal aplication of standard least square means method. In Section 4, conclusions are given on abilities of the suggested Interval Analysis approach and its applications in addition to known estimation procedures [1], [2], [3].

2 Specific properties of experimental data. Interval approach to estimating the process parameters. Problem formulation

In practice of chemical experiments, a sample of measurements can be very short and the errors' probability characteristics are unknown. Moreover, the errors can be not only in the process measurements, but, also, in ones of the process' argument, and uncertainty of each measurement becomes *two-dimensional*. So, it becomes difficult or even impossible to validate application of standard methods to estimation of the process parameters. As an alternative, application of the statistical methods can be completed by using the Interval Analysis ones.

Foundations of Interval Analysis and its applications to processing observations under uncertainty conditions were developed on the basis of the fundamental pioneering work of L.V. Kantorovich [4].

Nowadays, effective theoretical, applied, and numerical methods of the Interval Analysis have been created both abroad [5] and by Russian researchers [6]. Special interval algorithms and software were developed for solving applied problems of estimation of parameters for experimental chemical processes [8], [9], [10], [11], [12].

Remind that the essence of the interval methods is in estimating the process parameters under conditions of a short measurement sample, uncertainty of the measuring errors probability characteristics, and under only interval bounding onto the error values.

Introduce the following necessary definitions with using the standard on notations in Interval Analysis [7].

The function describing the process has the form

$$F(x, A, B) = A + Bx, \tag{1}$$

where x is the process' argument; A, B are parameters to be estimated.

The **uncertainty set** of each measurement. Since absence of probability characteristics of measuring errors, uncertainty of each measurement x_n, F_n is formalized as a rectangle $\boldsymbol{H}_n$ with the left $\underline{\boldsymbol{x}}_n$ and right $\overline{\boldsymbol{x}}_n$, lower $\underline{\boldsymbol{F}}_n$ and upper $\overline{\boldsymbol{F}}_n$ boundaries

$$\begin{array}{l} \text{2a) } \{x_n, F_n\}, n = \overline{1,N},\ \boldsymbol{H}_n : \\ \text{2b) } \boldsymbol{F}_n = [\underline{\boldsymbol{F}}_n, \overline{\boldsymbol{F}}_n], \\ \qquad \text{where } \underline{\boldsymbol{F}}_n = F_n - e_{\max},\ \overline{\boldsymbol{F}}_n = F_n + e_{\max}; \\ \text{2c) } \boldsymbol{x}_n = [\underline{\boldsymbol{x}}_n, \overline{\boldsymbol{x}}_n], \\ \qquad \text{where } \underline{\boldsymbol{x}}_n = x_n - b_{\max},\ \overline{\boldsymbol{x}}_n = x_n + b_{\max}, \end{array} \tag{2}$$

where $e_{\max}$, $b_{\max}$ are bounds onto maximal (on modulus) values of errors in the process and its argument measurements.

The **admissible value of the parameters vector** (A, B) for the **linear model** (1) is a pair

$$(A, B) : \ F(x, A, B) \in \boldsymbol{F}_n, \text{for } x \in \boldsymbol{x}_n, \text{for all } n = \overline{1,N}. \tag{3}$$

and corresponding dependence $F(n, A, B)$ is also called **admissible**.

The **Information Set** is a totality of all admissible values of the parameter vector for model (1) satisfying the following system of interval inequalities:

$$\boldsymbol{I}(A, B) = \{(A, B) : \ F(x, A, B) \in \boldsymbol{F}_n, \text{for } x \in \boldsymbol{x}_n, \text{for all } n = \overline{1,N}\}. \tag{4}$$

The input sample (2a) is called **consistent** in the interval sense if by (4) there exists at least one admissible value of the parameter vector and corresponding admissible dependence.

The **tube** of admissible dependencies $\{\boldsymbol{Tb}(x)\}$, $n = \overline{1,N}$ is a totality of all admissible values of the dependence at the nth measuring of the process. For the linear model (1) and the information set $\boldsymbol{I}(A, B)$, the tube boundaries are calculated with taking into account the boundaries of the uncertainty sets $\boldsymbol{F}_n \times \boldsymbol{x}_n$ (3)

$$\begin{array}{l} \underline{\boldsymbol{Tb}}(x) = \min_{(A,B)\in \boldsymbol{I}(A,B)} F(x, A, B), \\ \overline{\boldsymbol{Tb}}(x) = \max_{(A,B)\in \boldsymbol{I}(A,B)} F(x, A, B). \end{array} \tag{5}$$

Problem of estimation for the linear model (1) is formulated as follows: *by means of the Interval Analysis methods to construct the Information Set* (4) *of the process parameters consistent with the given data* (2) *and to build the tube* (5) *of the admissible dependencies of the process.*

For the linear model (1), fast procedures for constructing the Information Set (4) with exact description of its boundaries have been elaborated and applied to solving many practical problems [8], [9], [10], [11], [12]. Due to direct using the linearity of model (1), these procedures are more fast and give exact description of the Information Set in comparison with even very powerful procedures of the SIVIA-type [5].

Remark. As it will be shown below, formal application of the standard Least Square Means method (LSQM) [2] and corresponding point-wise estimate of the parameters $(A_{\mathrm{SQ}}, B_{\mathrm{SQ}})$ for the linear model demonstrate to be useful for analysis of the input sample (2) and for qualitative comparison with the results on the basis of Interval Analysis.

3 Results of processing experimental data

The first example (Fig. 1) of experimental data is joined with investigating the heat of fusion of cryolites (Institute of High Temperature Electrochemistry UrB RAS, Ekaterinburg).

Figure 1a shows the sample of measurements (circles) and their two-dimensional uncertainty sets built for bounds $e_{\max} = 2$ kJ mole-1 and $b_{\max} = 4$ K onto measuring errors of the process and its temperature argument. Remark the difficult situation: there are only 7 measurements and their uncertainty sets cross each other. At the left, there are four measurements with practically coinciding values of temperature. Point–dash lines correspond to the LSQM-line and its rough standard tolerances $\pm 3\sigma$. The shadowed fragment is the tube of admissible dependencies. The thick central line (Fig. 1a) marks the dependence corresponding to the central point of the information set of admissible parameters (Fig. 1b), where, the cross marks the LSQM-point obtained by formal application of standard LSQM-method.

In the next example (Fig. 2), the experimental data was obtainted in investigation of relative electric potential between Uranium and Gallium chlorides in the process of treatment of nuclear waists (Institute of High Temperature Electrochemistry UrB RAS, Ekaterinburg).

Figure 2a shows the sample of measurements (crosses) and their two-dimensional uncertainty sets built for bounds $e_{\max} = 0.0075$ V and $b_{\max} = 5$ K onto measuring errors of the process and its temperature argument. Remark the difficult situation: there are only 10 measurements with coinciding values of the temperature measurements and their uncertainty sets crossing each other. Moreover, two measurements (at the right) and their uncertainty sets practicically coincide. Point–dash lines correspond to the LSQM-line and its rough standard tolerances $\pm 3\sigma$. The shadowed fragment is the tube of admissible dependencies. The thick central line (Fig. 2a) marks the dependence corresponding to the central point of the information set of admissible parameters (Fig. 2b), where, the cross marks the LSQM-point obtained by formal application of standard LSQM-method.

In the last example (Fig. 3), the experimental data was also obtainted in investigation of relative electric potential between Uranium and Gallium chlorides in the process of treatment of nuclear waists (Institute of High Temperature Electrochemistry UrB RAS, Ekaterinburg).

Figure 3a shows the sample of measurements (crosses) and their two-dimensional uncertainty sets built for bounds $e_{\max} = 0.0075$ V and $b_{\max} = 5$ K onto measuring errors of the process and its temperature argument. Again remark

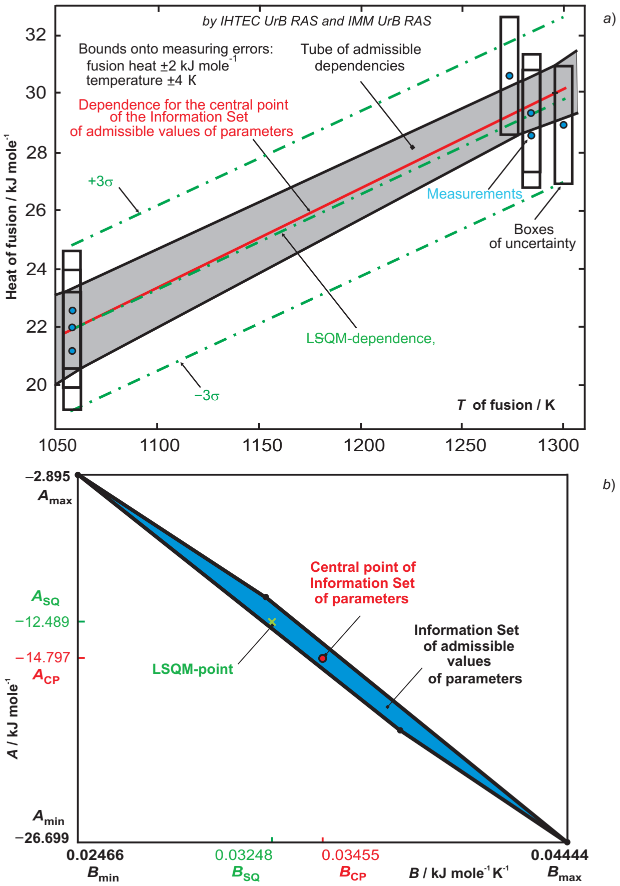

Fig. 1. Processing data on heat of fusion of cryolites; a) input data and tube of admissible dependencies; b) information set of admissible parameters

the difficult situation: there are only 10 measurements with coinciding values of the temperature measurements and their uncertainty sets crossing each other. Moreover, two measurements (at the right) and their uncertainty sets practicically coincide. Point–dash lines correspond to the LSQM-line and its rough standard tolerances $\pm 3\sigma$. The shadowed fragment is the tube of admissible dependencies. The thick central line (Fig. 3a) marks the dependence corresponding to the central point of the information set of admissible parameters (Fig. 3b), where, the cross marks the LSQM-point obtained by formal application of standard LSQM-method.

Underline the sophisticated character of information sets of admissible values of parameters (Figs. 1b, 2b, and 3b). Such their detailed structure *can not be calculated* by any standard method of processing the input data with two-dimensional uncertainties in measurements.

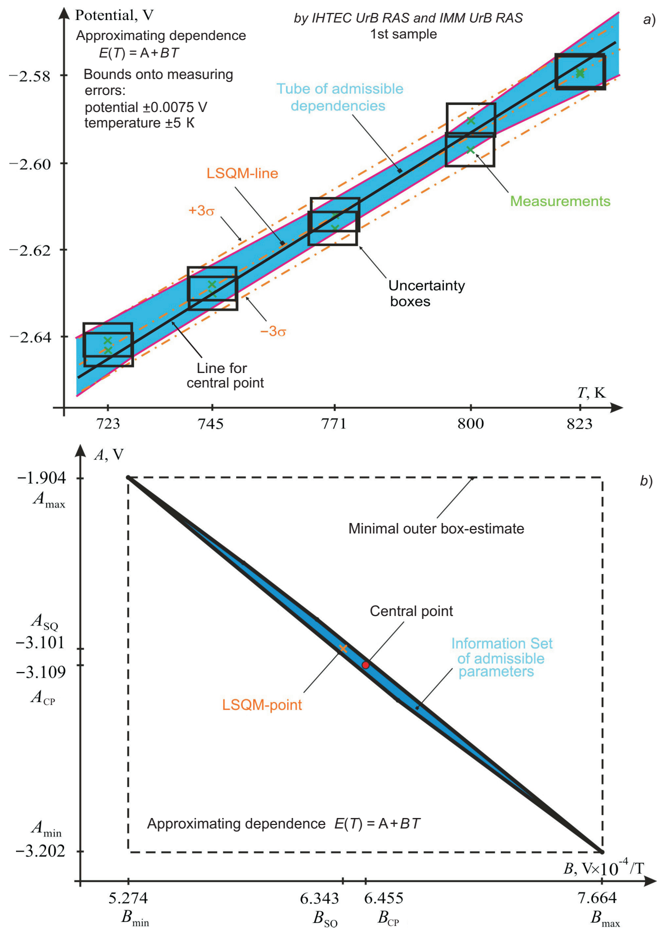

Fig. 2. Processing data on potential of Uranium–Gallium chlorides; the 1st sample; a) input data and tube of admissible dependencies; b) information set of admissible parameters

Besides the information set and the tube of admissible dependencies, the described interval approach gives for practical using the following useful information: the central "calibration" dependence (Figs. 1a, 2a, and 3a, the central thick lines), the central estimate point $(A_{\text{cp}}, B_{\text{cp}})$, the minimal outer unconditional intervals $[\boldsymbol{A}_{\min}, \boldsymbol{A}_{\max}]$ and $[\boldsymbol{B}_{\min}, \boldsymbol{B}_{\max}]$ on parameters (Figs. 1b, 2b, and 3b, the rectangles with boundaries in dashes), and LSQM estimate point $(A_{\text{SQ}}, B_{\text{SQ}})$.

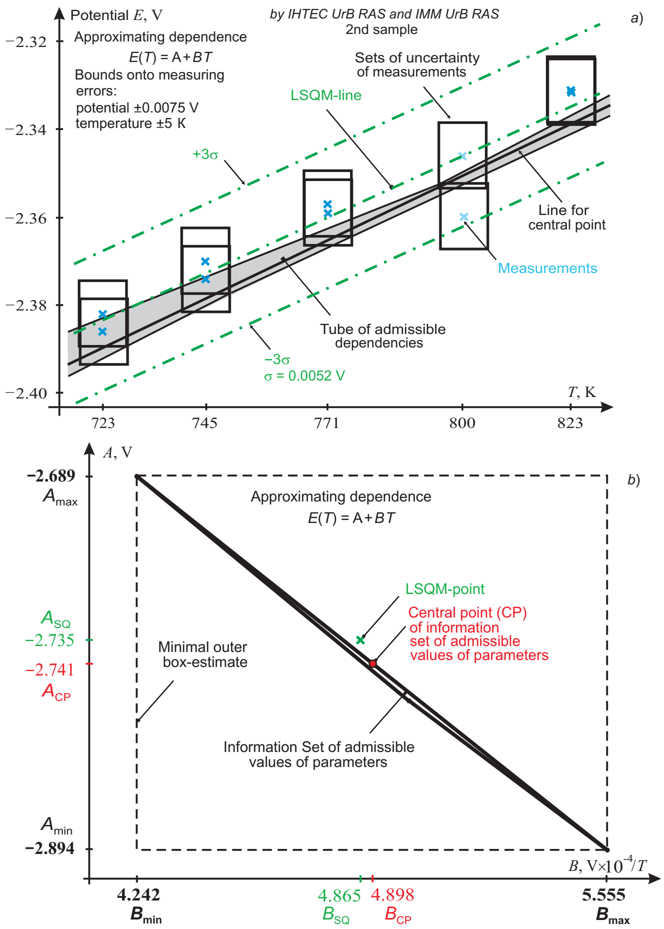

Fig. 3. Processing data on potential of Uranium–Gallium chlorides; the 2nd sample; a) input data and tube of admissible dependencies; b) information set of admissible parameters

4 Conclusions

Digital signal procession is implemented on the basis of the Interval Analysis methods. Parameters of noised chemical processes are estimated under conditions of absence of probability data for the measuring errors and specific two-dimensional uncertainty of each measurement. Such a case can not be treated by any standard methods.

It was shown that under mentioned conditions Interval Analysis approach gives sophisticated guaranteed estimation of the process parameters set and better estimation of the tube of admissible dependencies.

Moreover, simulation results show that simultaneous using the interval and standard statistical approaches complement each other and allows one to perform more detailed analysis and qualitative comparison of the estimation results.

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