Agents' Interaction Algorithm in a Strongly Coupled System with a Transferable Utility

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Abstract. The problem of the utility distribution mechanism analyzing in a strongly coupled system with the utility (profit) transfer possibility by the aggregated utility criterion is considered. The utility distribution algorithm in the system with complementary demand functions is developed. The strategic behavior algorithm under stability and the individual rationality conditions as well as Pareto efficiency fulfillment is confirmed by numerical simulation of the mechanism for the "retailer-bank-insurer" system. Experimental modeling proves that the application of the proposed algorithm with the increasing risk rates claimed by agents leads to a decrease in the profit of the meta-agent while preserving the profits of the agents. The situations of the environment's domination and the meta-agent's domination with similar initial parameters of the system lead to the same dynamics of the agent's profits with an increase in the risk rate. Thus, distortion of information about the risk rate towards the increase is an advantage for agents and a disadvantage for the meta-agent.

Keywords: distribution mechanism, tightly coupled system, anonymous agent, aggregated utility, complementary demand, transferable utility, nash equilibrium, retailer, bank, insurer

1 Introduction

The strongly coupled organizational and economic systems are formed under the subjective factors influence, for example, as a result of some legal entities with other affiliation by virtue of a controlling stake disposal. Also, these systems are formed under the objective factors influence of some agents' prevailing economic activity in comparison with others. A typical microeconomic problem that illustrates the agents' heterogeneity in economic activity levels is the agents' interests reconciliation in integrated systems with complementary demand. This situation arises when conditionality of the buyer's need in one commodity by the fact of acquiring other goods. Agents, whose goods initiate the demand for other agents' goods, are characterized by predominant economic activity. The complementary demand effect is often observed for specific goods. Under these conditions, either of the agents or the agents' association in the case of affiliation may have the status further defined as "meta-agents". Other agents may delegate to the meta-agent the right to redistribute (transfer) the integration effect in the system. The status “meta-agents” is realized in the form of possessing information about the true utility functions or utility values of other agents, as well as the right to choose the distribution mechanism of the system integration effect. Therefore, the agents’ utilities in this case can be considered as transferable.

Transferable utility distribution algorithms were developed for systems in which agents were anonymous, that is, they had equivalent criteria. In this case, the priorities of the criteria were not taken into account when aggregating. In particular, Pareto efficiency was justified [3, 10, 21] for an algorithm in which a minimum between the agent’s optimum and the average undistributed system utility was determined. If the agents’ criteria [5, 6, 19] have different priorities, then the distribution algorithms [14, 15, 18] were reduced to a median multicriteria choice [4, 7, 16].This solution was not Pareto efficient in the general case, however, mechanisms for anonymous symmetric coalitions [1, 2] were Pareto efficient in particular cases. The distribution mechanisms were constructed for the utility distribution problem represented as a multicriteria choice problem: studies with additive aggregation of agents’ utilities were made for nontransferable utilities [11, 13, 17] and transferable utility [12, 20].

Further we consider the distribution mechanism, optimal by the multiplicative utility criterion, which under certain conditions leads to Nash equilibrium (compatibility with stimuli) and Pareto efficiency. The problem is to develop an algorithm for distributing the transferable utility between the agents of the “retailer-bank-insurer” system [9] based [8] on the results obtained for anonymous agents.

Thus, this article’s subject is the study of the utility distribution mechanism from the standpoint of resistance to the agents’ strategic behavior and the individual rationality conditions and Pareto efficiency.

2 Methods and materials

We introduce the following assumptions about the price’s functions and agents’ costs functions.

1. Agents operate in the monopolistic competition’s markets, which causes the decreasing demand curves, simulated in the form of power functions (“inverse functions of demand”),

$$p_{k(n)} = a_{k(n)} Q_k^{b_{k(n)}}, \quad a_{k(n)} > 0,\ b_{k(n)} < 0,\ |b_{k(n)}| < 1,\ k \in K,\ n = 0, 1, 2, \tag{1}$$

where $p_{k(n)}$is k-th agent’s goods price, $a_{k(n)}$, $b_{k(n)}$ are the price function coefficients for n-th variant of the system organization ($n = 0$ is lack of integration, $n = 1$ is the integrated system for $Q^*_{-l} = Q^*_l$, $l \in K$, $n = 2$ is the integrated system for $Q^*_{-l} < Q^*_l$, $l \in K$); K is the agents’ set; $|K|$ is the amount of elements of set K; the meta agent is indicated by the symbol $l \in K$, the environment is indicated by the «$-l$» symbol, the agent’s optimum is indicated by the «$*$» symbol.

2. Sales growth occurs with a constant return expansion, i.e. the agent's marginal costs are constant $c_k = \text{const}$, $k \in K$, agents have risky costs $\rho_k = \text{const}$, $k \in K$ and integration costs $u_k = \text{const}$, $k \in K$. Risk costs characterize the share of the agent's average proceeds probable losses from the goods price. The ones for the retailer mean the banks overdue debt on loans for goods sold. The ones for the bank mean the overdue debts on loans granted, which are taken into account by the discount factor. The ones for the insurer mean the payments on insurance cases, which take into account the probability of their occurrence. Integration costs $u_k > 0$, $k \in K$ are interpreted as price discounts or commissions in favor of agents with greater economic activity. If integration costs $u_k < 0$, $k \in K$, then they represent the income of a more active agent as a transfer in the form of price premiums or commissions from other agents for participating in an integrated system.

3. The system is characterized by complementary demand. We consider the sales volumes of all agents expressed in one measure and assume that the volume of the meta-agent's sales through the coefficient of its demand function depends on the volume of the environment's sales in the following form (in this case, the relationship between demands for environment goods is neglected):

$$a_l = \alpha_{lk} a_{l0} Q_k^*, \quad k \in K \backslash l, \quad \alpha_{lk} = \begin{cases} \alpha_{lk(1)}, & n = 1, \\ \alpha_{lk(2)}, & n = 2, \end{cases} \tag{2}$$

where α_{lk} is the coefficient of complementarity of the k-th and l-th goods, $\alpha_{lk(1)} > \alpha_{lk(2)} > 0$ are constants; the complementarity effect is expressed in the fact that for a retailer the growth in lending at a low interest rate ($n = 1$) leads to faster growth in goods turnover, and at a high interest rate ($n = 2$) leads to a slow growth in turnover. In other words, in the piecewise constant model α_{lk}, the property of growth of the complementarity effect is simplified with a decrease in the complement's price.

Based on these assumptions, we present models for the optimal actions choice in the following form:

$$\begin{cases} Q_k^* = \arg \max\limits_{Q \in A_{Q_k}} \pi_k(Q_k), \\ \pi_k(Q_k) = \bar{p}_{k(n)} Q_k^{b_{k(n)}+1} - c_k Q_k, \quad k \in K, \end{cases} \tag{3}$$

where $\bar{p}_k = a_k - u_k - \rho_k > 0$, $k \in K$; $\pi_k(Q_k)$ is the function of the agent's profit. The criteria for the agents (3) are obviously strictly concave, twice continuously differentiable, and the optima of the agents are finite, that is, the solutions of problems (3) are internal.

We introduce the following utility distribution mechanism:

$$\pi_k^0 = \begin{cases} \pi_k^{\max} - \mu \bar{\pi}, & k \in K_1, \\ \pi_k^{\max}, & k \in M, \end{cases} \quad \mu = \frac{|K|}{|K| - |M|} \geq 1, \tag{4}$$

where π_k^0 is the agents utility after distribution; M is the minority agents' set for which the profit unconditional maximum is lower than the average profit loss in the system; $|M|$ is the amount of elements of set M; K_1 is non-minority agents' set; these sets are defined in the form

$$M = \{k \in K : \pi_k^{\max} < \bar{\pi}\}, \quad K_1 = \{k \in K \backslash M : \pi_k^{\max} \geq \bar{\pi}\}.$$

The notation (4)

$$\bar{\pi} = \frac{1}{\kappa}\left(\sum_{k\in K}\pi_k^{\max} - \sum_{k\in K}\pi_k^*\right) \geq 0$$

is an average profit loss in the system; $\pi_k^* = \pi_k(Q_k^*)$ is conditional maximum of agent's profit by criterion

$$Q_k^* = \arg\max_{Q_k\in A_{Q_{k\backslash l}}}\pi_k(Q_k), \quad k \in K\backslash l, \quad A_{Q_{k\backslash l}} = \{Q_k \in R_+^k, Q_k \leq Q_l^*, k \in K\backslash l\},$$

which is found taking into account that the meta-agent has already chosen its optimum; $\pi_k^{\max}$ is the unconditional maximum of the agent's profit according to the criterion

$$\pi_k^{\max} = \max_{Q_k\in A_{Q_k}}\pi_k(Q_k), \quad A_{Q_k} = \{Q_k \in R_+^k, k \in K\}.$$

The problem is to analyze the influence of agent-type parameters c_k, u_k, ρ_k, the parameters of the markets under consideration a_k, b_k and the parameters of the market's relationship α_{lk} on the resulting utility distribution (4).

3 Results

The necessary optimality condition written for the k-th agent has the following form

$$\pi'_{kQ_k} = \bar{p}_{k(n)}(b_{k(n)}+1)Q_k^{b_{k(n)}} - c_k = 0, \quad k \in K. \tag{5}$$

The optimum of the agent taking into account (5) for the agent's optimum has the form:

$$Q_k^* = \left[\frac{c_k}{\bar{p}_{k(n)}(b_{k(n)}+1)}\right]^{1/b_{k(n)}}, \quad k \in K,\ n = 0, 1, 2, \tag{6}$$

which for $n = 0$ characterizes the individual optimum of the non-integrated agent $Q_{k(0)}^*$.

A sufficient maximum condition has the form

$$\pi''_{kQ_k} = \bar{p}_{k(n)}(b_{k(n)}+1)b_{k(n)}Q_k^{b_{k(n)}-1} < 0, \quad k \in K,\ n = 0, 1, 2. \tag{7}$$

The analysis of condition (7) shows that, taking into account the restrictions on the coefficients of the price functions (1), it is satisfied under condition when $\bar{p}_{k(n)} > 0$, $k \in K$.

The algorithm for analyzing the influence of agent's type parameters, market parameters, and the parameters of the markets relationship to the resulting utility distribution includes the following steps.

1. Input of market environment parameters a_k, b_k and cost parameters c_k, u_k, as well as the complementarity parameters α_{lk}, the accuracy parameter of the iterative process ε.
2. Setting the step number $t = 1$.
3. Entering agent messages about the type parameters ρ_{kt}.
4. The agents' optimal sales volume calculation of Q_{kt}^* by (6).
5. The agents' profit functions calculation (profits $\pi_{kt}^{\max}$) according to (3).
6. The determination of meta-agent by the condition of the maximum value of the agents' profit
$$l_t = \arg\max_{k\in K} Q_{kt}^*.$$
7. The system organization determination (the meta-agent's domination $n = 1$ or the environment domination $n = 2$) by the rule
$$n_t = \begin{cases} 1, & \max\limits_{-l\in K} Q_{-lt} > Q_{lt}^*, \\ 2, & \max\limits_{-l\in K} Q_{-lt} \le Q_{lt}^*. \end{cases}$$
8. The demand function coefficient a_{lt} determination for the meta-agent product according to (2).
9. The meta-agent conditional optimum calculation $\bar{Q}_{lt}^*$ by (2) with the value of a_{lt} found in step 8.
10. The meta-agent's conditional maximum profit calculation $\pi_{lt}^* = \pi_l(\bar{Q}_{lt}^*)$ by (3).
11. The calculation of profit in the integrated system according to the mechanism (4) as a vector π_{kt}^0, $k \in K$.
12. The termination condition verification: if $\forall t > 1$
$$\sum_{k\in K} |\pi_{kt}^0 - \pi_{kt-1}^0| \le \varepsilon, \quad k \in K,$$
then iteration ends.
13. The distribution analysis by agents π_{kt}^0, $k \in K$.
14. The step number setting $t = t + 1$.
15. The agent's messages about the type parameters ρ_{kt} and proceeding to step 4 entering.

4 The effect distribution simulation according to the algorithm

The simulation is carried out on the basis of integrated system parameters sets, indicated $k = 1, 2, 3$, respectively. The choice of the meta-agent is made according to the maximum profit criterion. The prices are presented as inverse demand functions (1). The situations of the meta-agent's domination and the environment's domination are considered.

The initial data for the first variant of the simulated systems are presented in Table 1.

Table 1. Parameters of the model and criteria vector

Parameter of the model	Element		
	1	2	3
a_k	5000.00	0.75	0.35
b_k	-0.06	-0.19	-0.20
c_k	2250.00	0.06	0.03
u_k	-1000.00	0.03	0.02
Q_k^*	214744.64	195682.87	176234.17
α_{lk}	$4.5 \cdot 10^{-6}$		

It is obvious from the table data that the environment domination situation is realized.

The figures below show the agents' profit and the total (Σ) profit graphics depending on the declared risk of the agent.

The declared agents' risk rate p_k vary in steps in the range from 0 to 0.23 with step of 0.01. If the agent's risk rate does not change, then it is set at the base level.

In Fig. 1 π_1^0 is retailer's profit, after profit distribution in the integrated system, reduced by 10,000 times, π_2^0 is the bank's distributed profit, π_3^0 is the insurer's distributed profit in the integrated system.

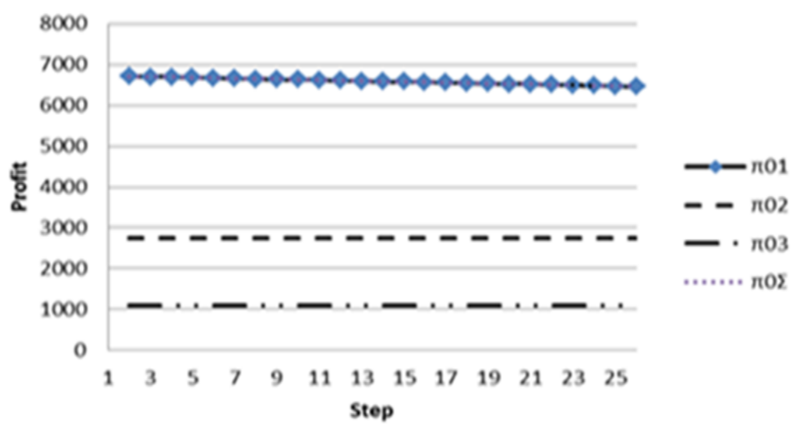

Fig. 1. The agents' distributed profit dynamics in the integrated system with the increase in the declared risk rate of all agents simultaneously

Figure 1 shows that the bank's and the insurer's profits are incomparably less than the retailer's one. The system's total profit is approximately equal to the retailer's profit. The retailer acts as a meta-agent and transfers part profit among agents who lost the profit in the integration. Thus, in the integrated system after the profits' transferring the bank's and the insurer's profits are constant, regardless of the change in the risk rate. Changes in the declared values of all agents' risk rates will only affect the retailer's profit.

The effect of an increase in the individual declared agents' risk on the meta-agent profit is reflected in Fig. 2.

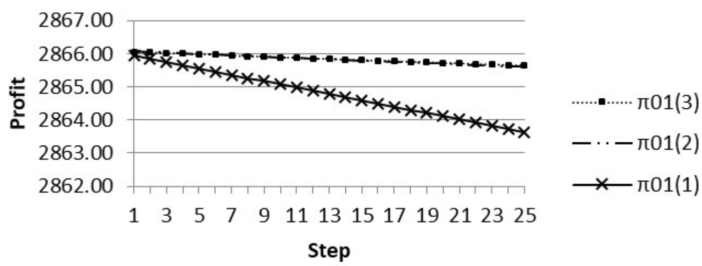

Fig. 2. The meta-agent's distributed profit dynamics in the integrated system

In the Fig. 2 $\pi^0_{1(1)}$ is the retailer's profit with the increase in the risk rate declared by the retailer; $\pi^0_{1(2)}$ is the retailer's profit with the increase in the risk rate declared by the bank; $\pi^0_{1(3)}$ is the retailer's profit with the increase in the risk rate declared by the insurer; the profit's amount in the integrated system is reduced by 10,000 times.

When declared the risk rate is increased by one of the agents, the retailer's profit as a meta-agent and, accordingly, the system profit is reduced. The greatest impact on the system profit is the change in the declared retailer risk rate. Insignificant impacts on the system profit are the change in the declared bank's and insurer's risk rates.

In Fig. 3 $\pi^0_{\Sigma 1}$ is the system total profit with an increase in the risk rate declared by the retailer; $\pi^0_{\Sigma 2}$ is the system total profit with an increase in the risk rate declared by the bank; $\pi^0_{\Sigma 3}$ is the system total profit with an increase in the risk rate declared by the bank; the profit amount in the integrated system is reduced by 10,000 times.

The system total profit shows the dynamics similar to the retailer's profit.

The agents' profit received at integration before transferring differs from the declared profit (Fig. 4).

In Fig. 4 π^*_1 is the retailer's profit conditional maximum reduced by 10,000 times; π^*_2 is the bank's profit conditional maximum; π^*_3 is the insurer's profit conditional maximum; $\pi^*_{2(0.03)}$ is the bank's profit conditional maximum with real risk rate (0.03); $\pi^*_{3(0.02)}$ is the insurer's profit conditional maximum with real risk rate (0.02).

In the integrated system the agents receive a part of the profit independently, and the meta-agent then replenishes the amount to the value of the agents' profit

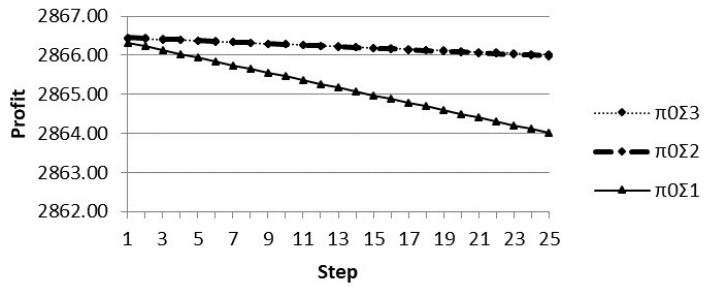

Fig. 3. The integrated system profit dynamics when the declared one of the agents' risk rate changes

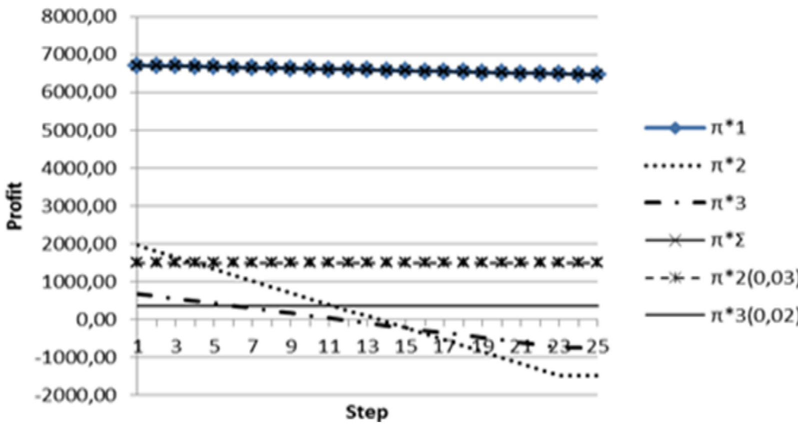

Fig. 4. Agents' profit dynamics in the integrated system

without integration. It is stimulus for the agents to overestimate the values of the declared risk rates in order to obtain additional profit from the meta-agent.

Let's consider a model similar to the previous one, but in the situation of the meta-agent's domination (Table 2).

A significant amount of retailer's profit compared to other agents makes inefficient change of the meta-agent in this model.

The meta-agent's profit dynamics (Fig. 5) shows that an increase in the risk rate declared by one of the agents leads to a stable decrease in the meta-agent profit.

In Fig. 5 $\pi^0_{1(1)}$ is the retailer's profit with the increase in the risk rate declared by the retailer; $\pi^0_{1(2)}$ is the retailer's profit with the increase in the risk rate declared by the bank; $\pi^0_{1(3)}$ is the retailer's profit with the increase in the risk rate declared by the insurer; the profit's amount in the integrated system is reduced by 1,000 times.

Table 2. Parameters of the model and criteria vector

Parameter of the model	Element		
	1	2	3
a_k	5500.00	0.75	0.35
b_k	-0.06	-0.19	-0.20
c_k	2520.00	0.06	0.03
u_k	-200.00	0.03	0.02
Q_k^*	159037.68	195682.87	176234.17
α_{lk}	$4 \cdot 10^{-6}$		

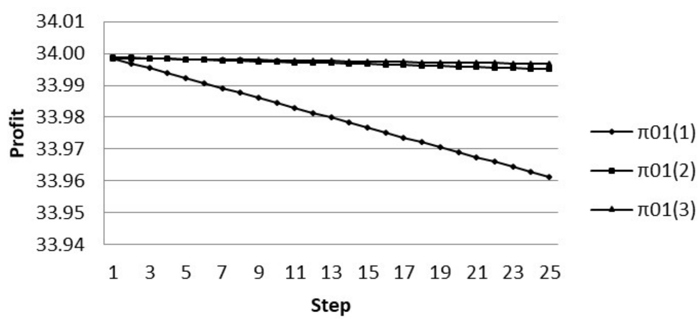

Fig. 5. Meta-agent's distributed profit dynamics in the integrated system

When the declared risk rate is increased by one agent, the retailer's profit as a meta-agent and, accordingly, the total profit is reduced. The greatest impact on the system's profit is the change in the declared retailer risk. Insignificant impact on the total profit is the change in the bank's and the insurer's declared risk rates.

The transferred profit dynamics in the system is the same for both the environment's domination and the meta-agent's domination. Thus, there is no need to demonstrate other similar graphics.

5 Conclusion

The algorithm of simulation of the agents' interaction in a strongly coupled system with a transferable utility is developed. The algorithm makes it possible to simulate the states of the system for specified parameters. The optimizing of the utility function taking into account the priorities leads to a distribution based on the minimax principle (guaranteed result), therefore, determines the Pareto efficient equilibrium.

It is shown that an increase in the any agents declared risk rate leads to a decrease in the meta-agent's profit in any case and does not change the transfer profit of the agents themselves. Thus, the overestimation of the risk rates is beneficial to all agents of the system in addition to the meta-agent.

References

1. Bierbrauer, F., Hellwig, M.: Robustly coalition-proof incentive mechanisms for public good provision are voting mechanisms and vice versa. Review of Economic Studies 83(4), 1440–1464 (2016)
2. Bondarik, V., Corgin, N.: Mechanisms of resource allocation based on non-manipulative, symmetric, anonymous voting procedures with delegation. Problemy upravleniya 5, 26–32 (2012)
3. Burkov, V., Gorgidze, I., Novikov, D.: Models and mechanisms of distribution of costs and incomes in a market economy. IPP RAS, Moscow (1997)
4. Burkov, V., Iskakov, M., Corgin, N.: Application of generalized median schemes for constructing a non-manipulative mechanism for multi-criterial active expertise. Problemy upravleniya 4, 38–47 (2008)
5. Chumak, V., Ramzaev, V., Khaimovich, I.: Challenges of data access in economic research based on big data technology. CEUR Workshop Proceedings pp. 327–337 (2015)
6. Ehlers, L., Majumdar, D., Mishra, D., Sen, A.: Continuity and incentive compatibility in cardinal voting mechanisms. https://papyrus.bib.umontreal.ca, accessed: 03.08.2017
7. Floyd, M.K., Barker, K., Rocco, C.M., Whitman, M.G.: A multi-criteria decision analysis technique for stochastic task criticality in project management. Engineering Management Journal 29(3), 165–178 (2017)
8. Geraskin, M.: Optimal mechanism for the distribution of the effect in an integrated strongly coupled system of anonymous agents with a transferable utility. Problemy upravleniya 2, 27–41 (2017)
9. Geraskin, M., Manakhov, V.: Optimization of interactions in a multi-agent, tightly linked “retailer-bank-insurer” system. Problemy upravleniya 4, 9–18 (2015)
10. Johnson, M.: Wicksell on pareto: Efficiency, welfare, and distribution. http://papers.ssrn.com, accessed: 03.08.2017
11. Kawamori, T., Miyakawa, T.: Nash bargaining solution under externalities. Mathematical Social Sciences 84, 1–7 (2016)
12. Korgin, N., Korepanov, V.: An efficient solution of the resource allotment problem with the Groves–Ledyard mechanism under transferable utility. Automation and Remote Control 77(5), 914–942 (2016)
13. Korgin, N., Korepanov, V.: Experimental gaming analysis of ADMM dynamic distributed optimization algorithm. IFAC-PapersOnLine 49(12), 574–579 (2016)
14. Moulin, H.: One dimensional mechanism design. Theoretical Economics 12(2), 587–619 (2017)
15. Neumann, G., Cairns, D.: A targeted estimation of distribution algorithm compared to traditional methods in feature selection. In: Madani, K., Dourado, A., Rosa, A., Filipe, J., Kacprzyk, J. (eds.) Computational Intelligence: Revised and Selected Papers of the International Joint Conference, IJCCI 2013, Vilamoura, Portugal, September 20–22, 2013. pp. 83–103. Springer International Publishing (2016)
16. Pattanaik, P., Salles, M.: Social choice and welfare. Elsevier science publishing company, inc, New York (2016)
17. Pęski, M.: Large roommate problem with non-transferable random utility. Journal of Economic Theory 168(Supplement C), 432–471 (2017)
18. Ramzaev, V., Khaimovich, I., Chumak, P.: Models for forecasting the competitive growth of enterprises due to energy modernization. Studies on Russian Economic Development 26, 47–52 (01 2015)

19. Shao, R., Zhou, L.: Voting and optimal provision of a public good. Journal of Public Economics 134, 3–41 (2016)
20. Smirnova, N., Tarashnina, S.: Properties of solutions of cooperative games with transferable utilities. Russian Mathematics 60(6), 63–74 (2016)
21. Ventura, A., Cafiero, C., Montibeller, M.: Pareto efficiency, the coase theorem, and externalities: A critical view. Journal of Economic Issues 50, 872–895 (2016)