Hedging and Risk Aversion on Russian Stock Market: Strategies Based on MGARCH and MSV Models

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Abstract. The paper studies the problem of dynamic hedge ratio calculation for the portfolio consisted of two assets – futures and the underlying stock. We apply the utility based approach to account for the degree of risk aversion in the hedging strategy. Seventeen portfolios, consisted of Russian blue-chip stocks and futures, are estimated in the paper. In order to estimate the conditional covariances of hedged portfolio returns, such multivariate volatility models as GO-GARCH, copula-GARCH, asymmetric DCC and parsimonious stochastic volatility model are applied. The hedging efficiency is estimated on the out-of-sample period using the maximum attainable risk reduction, the financial result and the investor's utility. It's shown that for 60% of portfolios ADCC surpasses the other models in hedging. Including the degree of risk aversion in the investor's utility function together with above-mentioned volatility models allows to reach hedging efficiency of 88%.

Keywords: dynamic hedge ratio, stock futures, multivariate volatility models, risk aversion, hedging efficiency, copula, expected utility

1 Introduction

Hedging is one of the most common tasks in finance. It requires knowing both hedging and hedged asset returns' distribution. In other words, one should be aware of the multivariate distribution of returns, or, at least, the first two moments of this distribution. Our paper is focused on modeling the second moment because the variance-covariance matrix is needed to solve the aforementioned financial task.

There are two main approaches to modeling volatility: generalized autoregressive conditional heteroskedasticity or GARCH (a survey on multivariate GARCH models see in [2]) and models of stochastic volatility or SV (a review of multivariate SV models see in [1]). The latter take into account the volatility uncertainty directly by including the random term in the volatility equation. This assumption seems to be closer to the empirical evidence. But estimation of multivariate

SV poses a challenge both due to dimensionality problem and the lack of closed-form likelihood function in general case.[1] It's worth mentioning that the first problem is also acute for GARCH models and implies that the number of parameters grows quadratically (sometimes even faster as in VECH model, [8]) related to the number of assets in the portfolio.

The second issue, arising from the use of SV for building the hedging strategy, is estimation. In contrast to GARCH, SV contains two sources of uncertainty and in most cases it is not possible to derive the likelihood function analytically. However, there are a number of ways to estimate multivariate SV by the maximum likelihood method. For example the likelihood function can be approximated by a Gaussian density (see, e.g., [19]), or simulated (see, e.g., [4, 13]).

In this paper we attempt to propose a multivariate SV model in which both the aforementioned problems are remedied and apply it to stocks hedging. The model suggests that the demeaned returns follow Student's t-distribution, whereas the volatility matrix is also random. This property follows from the fact that Student's t-distribution can be represented as a mixture of normal distributions. As a matter of fact, if the demeaned returns are distributed normally conditionally on volatility matrix and the volatility matrix itself has inverse Wishart distribution, then, according to [5], the volatility matrix can be marginalized out from the returns' distribution, which results in Student's t-distribution for the returns. Consequently, the demeaned returns distribution is known in contrast to the majority of other multivariate SV models.

The paper contains the estimation of the described model via Markov chain Monte Carlo algorithm implemented in Stan software [18]. The parameters are obtained for seventeen stocks listed on Moscow Exchange futures market [14]. The sample covers the period from January 2006 to December 2016. All the positions are hedged with futures and the dynamic hedging coefficients are calculated using multivariate SV as well as several multivariate GARCH models. The resulted hedging strategies are compared via different criteria of hedging efficiency.

The rest of the article is organized as follows. Section 2 describes the methodology. Section 3 contains the estimation results and their discussion. Section 4 concludes.

2 Methodology

2.1 Hedge Ratio

The aim of hedging is the reduction of portfolio value fluctuations. This can be achieved by opening the opposite position on the hedging instrument, usually a futures. The main task of building the hedging strategy is finding the optimal

[1] The likelihood function for multivariate SV could be obtained under certain conditions, see [11].

amount of futures in the portfolio, i. e. calculation of optimal hedge ratio, which shows the relation of the hedged asset value to the hedging asset value.

The return of the hedged position at time t is denoted by r_t and equals to (1).

$$r_t = r_{S,t} - hr_t^* \cdot r_{F,t}, \tag{1}$$

where $r_{S,t}$ – stock returns at time t; $r_{F,t}$ – futures returns at time t; hr_t^* – optimal hedge ratio at time t.

We use the utility approach to implement the investor's attitude to risk in building the hedging strategy. We obtain the optimal hedge ratio from maximization of the investor's expected utility $\mathrm{E}\,U\,(r_t)$, (2).

$$\mathrm{E}\,U\,(r_t) = \mathrm{E}(r_t) - \tau \frac{\mathrm{V}\,(r_t)}{2}, \tag{2}$$

where τ – positive parameter for risk aversion (large values indicate that investor dislikes risk), $\mathrm{E}(r_t)$ – expected portfolio returns, $\mathrm{V}(r_t)$ – portfolio variance. The optimal hedge coefficient hr_t^* is defined in (3).

$$hr_t^* = \frac{\mathrm{cov}(r_{S,t}, r_{F,t})}{\mathrm{V}(r_{F,t})} - \frac{\mathrm{E}(r_{F,t})}{2\tau\,\mathrm{V}(r_{F,t})}, \tag{3}$$

where $\mathrm{cov}(\cdot)$ – covariance. Evidently, if $\tau \to \inf$, then hr_t^* coincides with traditional optimal hedge ratio, based on minimization of portfolio variance.

2.2 Multivariate Volatility

Historically the first methods of hedge ratio calculation assumed that the ratio is constant. Further hedging strategies based on the dynamic hedge ratio appeared. They allow to implement heteroskedasticity of returns in the model. In this study, four volatility models have been taken to estimate the dynamic hedge ratio.

There are two main approaches to the covariance matrix modeling: GARCH models and stochastic volatility models (MSV). Since the empirical evidence shows that volatility is volatile itself, it seems more appropriate to use SV, while modeling volatilty. SV has two sources of uncertainty – in mean and volatility equations. This fact results in challenging estimation procedure of SV, because it's impossible to derive the likelihood function analytically in general case. Thus SV models are usually estimated within the Bayesian framework.

Let x_t, $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})^\intercal$ be a portfolio consisted of n assets at time moment t. x_t is represented as a sum of its mathematical expectation $\mathrm{E}\,(x_t|\mathcal{F}_t)$, conditional on all available at $t-1$ information, and innovations y_t, (4).

$$x_t = \mathrm{E}\,(x_t|\mathcal{F}_{t-1}) + y_t,\ t = 1, \ldots, T,\ x_t - (n \times 1)\text{-vector}, \tag{4}$$

Innovations y_t conditional on volatility Σ_t are distributed normally with zero mean, (5).

$$y_t|\Sigma_t \sim N\,(0, \Sigma_t) \tag{5}$$

At the same time, Σ_t is a random process itself, generated by inversed Wishart distribution, (6).

$$\Sigma_t \sim IW(\nu, H_t) \tag{6}$$

Using properties of compound distributions, we obtain (7).

$$y_t \sim t\left(\nu, 0, H_t\right), \tag{7}$$

where $t_\nu\left(0, H_t\right)$ – multivariate Student's t-distribution with ν degrees of freedom and variance H_t, [5].

It's worth mentioning that, since the distribution of y_t is known in the model, the conjugate priors for the parameters could be derived. For univariate case conjugate priors for parameters with fixed ν are derived in [21], where the conjugate prior for volatility is Fisher distribution with $n-k$ (k – number of parameters) and ν degrees of freedom. According to [6], multivariate beta distribution is the generalization of Fisher distribution, which is a prior for H_t in (6).

We also use several multivariate GARCH models, namely general orthogonalized GARCH (GO-GARCH), copula-GARCH and asymmetric dynamic conditional correlations (ADCC).

The initial setup is analogous to MSV model, (see (4), (5)), except that the conditional distribution of innovations could be different.

For the GO-GARCH model (8) holds.

$$\Sigma_t^{GO-GARCH} = XV_tX^\intercal,\ V_t = \mathrm{diag}(v_t), \tag{8a}$$

$$v_t = \mathbb{C} + A(y_t \odot y_t) + Bv_{t-1}, \tag{8b}$$

where X is the matrix whose parametrization is based on the singular decomposition of the unconditional variance of returns (for details, see [20]), V_t – a diagonal matrix, which nonzero elements are portfolio assets volatilities, given by any one-dimensional GARCH model. For example, in (8b) A, B are diagonal matrices of parameters, $\mathbb{C}$ is a $n \times 1$ parameter vector, $\odot$ is an element-wise multiplication. As a result, each row of v_t represents a standard univariate GARCH.

ADCC volatility is modeled as in (9).

$$\Sigma_t^{ADCC} = D_tR_tD_t, \tag{9a}$$

$$D_t = \mathrm{diag}(d_t),\ d_t \odot d_t = v_t \tag{9b}$$

$$R_t = \mathrm{diag}\left(q_{11,t}^{-1/2} \dots q_{nn,t}^{-1/2}\right) Q_t\, \mathrm{diag}\left(q_{11,t}^{-1/2} \dots q_{nn,t}^{-1/2}\right), \tag{9c}$$

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha y_{t-1}y_{t-1}^\intercal + \beta Q_{t-1} + \gamma\widetilde{y}_{t-1}\widetilde{y}_{t-1}^\intercal, \tag{9d}$$

where R_t is conditional correlation matrix of returns, α, β, γ – parameters and γ is responsible for asymmetry effects in volatility, $\widetilde{y}_{t-1}$ are the zero-threshold innovations which are equal to y_t when less than zero and are equal to zero otherwise. More details are in [3].

Copula-GARCH model is similar to (9), but differs by the fact that joint distribution of returns are modeled via Student's t copula function, see [12, 16].

To sum up, our hedging strategies are based on four multivariate volatility models, three GARCH and one SV. We obtain the dynamic optimal hedge ratio from conditional covariance matrices, estimated by these models. Utility approach allows to account for investor's risk aversion, while building the hedging strategy.

3 Empirical Results

3.1 Data Description

For our empirical study we take 17 companies, listed on Moscow Exchange [14], which stocks are also traded on the futures market. The companies included in the sample are presented in Table 1.

Table 1. Companies under consideration

Ticker	Company name	Ticker	Company name
CHMF	Severstal	ROSN	Rosneft Oil Company
FEES	Federal Grid Company	RTKM	Rostelecom
GAZP	Gazprom	SBER	Sberbank of Russia
GMKN	Norilsk Nickel	SNGS	Surgutneftegas
HYDR	RusHydro	TATN	Tatneft
LKOH	Lukoil	TRNF	Transneft
MGNT	Magnit	URKA	Uralkali
NLMK	Novolipetsk Steel	VTBR	Bank VTB
NVTK	Novatek		

It's worth mentioning that there are about 100 participants on the stock section of MOEX futures market, but reasonable prices' history is available only for stocks in Table 1. Each bivariate "stock-futures" price series has its own length and we do not take stocks with historical prices, which amount less than 200 observations. The longest series, belonging to LKOH, has 3586 observations and covers the period from the 9th of August 2002 till the 30th of December 2016. The rest of the series are within this period. The source of the data is Finam investment company website [9]. Short descriptive statistics of the data under consideration is presented in Table 2.

3.2 Estimation Results

To calculate the dynamic hedge ratio, the following volatility models are evaluated in this paper: ADCC, GO-GARCH, cop-GARCH and MSV. Conditional mean of returns is modeled using ARMA. For each asset the whole sample is divided into two parts: in-sample period includes the first 80% of the series, the

Table 2. Descriptive statistics. N – number of observations, Mean – mean of daily logarithmic returns, St.dev. – standard deviation, Skew. – skewness coefficient, Kurt. – kurtosis coefficient

		Stocks				Futures			
Ticker	N	Mean	St.dev.	Skew.	Kurt.	Mean	St.dev.	Skew.	Kurt.
CHMF	1395	0.040	2.215	-0.366	6.237	0.040	2.402	-0.592	10.494
FEES	1318	-0.050	2.941	-0.324	9.748	-0.048	3.184	0.189	8.993
GAZP	2730	-0.013	2.458	-0.084	19.300	-0.013	2.591	0.213	24.388
GMKN	3041	0.058	2.747	-1.001	20.524	0.058	2.907	-1.112	26.078
HYDR	1404	-0.032	2.172	0.199	6.239	-0.031	2.273	-0.092	6.818
LKOH	3586	0.055	2.317	-0.055	16.122	0.055	2.393	-0.335	25.889
MGNT	587	0.036	2.027	-0.089	5.140	0.040	2.202	-0.042	5.573
NLMK	231	0.244	2.030	0.229	3.906	0.242	2.879	0.936	12.178
NVTK	1973	0.084	2.925	-1.362	31.895	0.084	3.712	-0.530	14.097
ROSN	2547	0.025	2.610	0.929	36.043	0.025	2.770	0.536	47.209
RTKM	3020	0.030	2.284	0.293	12.500	0.030	3.031	-0.676	26.798
SBER	2753	0.068	2.995	0.129	17.146	0.068	3.122	0.210	17.946
SNGS	3583	0.031	2.692	0.963	24.704	0.031	2.799	2.272	54.004
TATN	1402	0.065	2.170	-0.020	4.212	0.067	2.145	-0.407	7.613
TRNF	2367	0.065	3.082	0.023	18.438	0.066	3.150	0.039	9.705
URKA	1383	-0.028	2.136	-1.621	23.653	-0.024	2.537	-0.649	13.852
VTBR	2357	-0.026	2.919	0.576	45.755	-0.027	3.286	2.680	80.832

rest 20% are for out-of-sample period. The number of lags is chosen by minimizing Schwartz information criterion, according to which ARMA(1,0) for mean and GARCH(1,1) for all volatility models give the best fit.

The parameters of MSV model are obtained by one of the Markov chain Monte Carlo methods – Hamiltonian Monte Carlo algorithm, also known as Hybrid Monte Carlo [7, 15]. The convergence of Markov chains is checked by Geweke Z-test [10]. The test is based on the idea that the means, calculated on the first and the last parts of a Markov chain (usually 10% and 50% correspondingly), are equal. In that case the parameter samples are drawn from the stationary distribution of the Markov chains and Geweke's statistics has an asymptotically standard normal distribution. The Markov chain converges under the null. For some important parameters (namely, covariance of stock and futures returns, futures variance and conditional return at a specific time point, see (3)) p-values are presented in Table 3. Evidently that Geweke Z-test reveals the convergence for the parameters under consideration.

Mean hedge ratios for $\tau = 4$ are presented in Table 4. The average hedge ratios range from 19% for LKOH to 94% for SNGS.

In order to compare hedging strategies, obtained from different volatility models, we calculate three measures of hedging efficiency – maximum risk reduction, financial result (or profit) and investor's utility. The first measure is

Table 3. Geweke Convergence Z-testtext

Ticker	$\text{cov}(r_{S,t}, r_{F,t})$	$\text{V}(r_{F,t})$	$\text{E}(r_{F,t})$
CHMF	0.612	0.554	0.505
FEES	0.832	0.829	0.937
GAZP	0.844	0.929	0.557
GMKN	0.896	0.897	0.562
HYDR	0.630	0.545	0.790
LKOH	0.914	0.944	0.759
MGNT	0.834	0.545	0.984
NLMK	0.709	0.548	0.822
NVTK	0.534	0.644	0.803
ROSN	0.931	0.887	0.666
RTKM	0.664	0.598	0.611
SBER	0.876	0.849	0.551
SNGS	0.933	0.933	0.674
TATN	0.610	0.569	0.898
TRNF	0.885	0.913	0.580
URKA	0.591	0.690	0.969
VTBR	0.872	0.821	0.637

Table 4. Mean Hedge Ratios

Ticker	ADCC	GO-GARCH	cop-GARCH	MSV
CHMF	0.812	0.778	0.799	0.768
FEES	0.836	0.837	0.843	0.820
GAZP	0.862	0.810	0.851	0.889
GMKN	0.893	0.817	0.897	0.905
HYDR	0.867	0.874	0.861	0.844
LKOH	0.934	0.821	0.192	0.935
MGNT	0.914	0.825	0.911	0.805
NLMK	0.612	0.611	0.608	0.644
NVTK	0.673	0.732	0.678	0.567
ROSN	0.883	0.815	0.863	0.864
RTKM	0.562	0.618	0.609	0.785
SBER	0.891	0.824	0.899	0.883
SNGS	0.910	0.816	0.910	0.939
TATN	0.935	0.903	0.246	0.896
TRNF	0.885	0.820	0.879	0.797
URKA	0.555	0.559	0.514	0.679
VTBR	0.871	0.753	0.864	0.835

defined as in (10).

$$E = 1 - \frac{\text{var}(r)}{\text{var}(r_S)} \tag{10}$$

Financial result is calculated as the sum of the logarithmic returns of the portfolio for the forecast period [17]. The formula for investor's utility is described in (2).

The comparison is conducted for the out-of-sample period and summarized in Table 5. We compute the measures of hedging performance for various levels of risk aversion τ. They vary from very small values, almost equal to zero, till ten and are presented in the first column of Table 5, denoted by "ra". Maximum risk reduction is abbreviated by "mrr". The rest of the column labels are self-explanatory. Table 5 reveals the numbers of assets, for which the model in the corresponding column maximizes the corresponding criterion. According to

Table 5. Summary of hedging efficiency

	ADCC			GO-GARCH			cop-GARCH			MSV		
ra	mrr	profit	util	mrr	profit	util	mrr	profit	util	mrr	profit	util
0.00	4	0	1	4	2	2	7	6	8	2	9	6
1.11	10	0	13	7	2	3	0	3	1	0	12	0
2.22	10	2	10	7	1	6	0	5	1	0	9	0
3.33	10	2	10	7	1	6	0	6	1	0	8	0
4.44	10	2	10	6	5	6	1	5	1	0	5	0
5.56	10	2	10	6	5	6	1	5	1	0	5	0
6.67	10	2	10	6	5	6	1	5	1	0	5	0
7.78	10	2	10	6	6	6	1	5	1	0	4	0
8.89	10	2	10	6	6	6	1	5	1	0	4	0
10.00	10	2	10	6	6	6	1	5	1	0	4	0

Table 5, ADCC model clearly outperforms the other models by the maximum risk reduction and investor's utility. MRR in average amounts to 74% for this model and ranges from 47% to 88%. GO-GARCH, copula-GARCH and MSV reach their maximum MRR at the levels of 84%, 85% and 83% respectively.

The performance level of ADCC model seems to be stable and remains the same for τ larger than 2. The dynamics of GO-GARCH hedge efficiency criteria values also stabilizes for higher risk aversion levels. GO-GARCH performance is the same among different efficiency measures and is relatively lower than in ADCC case. Copula-GARCH demonstrates even lower performance by all criteria except the profit of the hedged position. Stochastic volatility clearly provides the highest financial result, if the investor prefers risk. At the same time, with the growth of τ performance level of MSV declines.

It's also worth mentioning that on small values of τ ADCC and MSV reach their maximum performance by utility and profit correspondingly.

4 Conclusion

The article considers the development of a hedging strategy based on maximizing investor's expected utility, taking into account the level of risk aversion. The optimal hedge ratio is time-dependent and is calculated using four multivariate volatility models ADCC, GO-GARCH, copula-GARCH with the Student's copula and multivariate stochastic volatility. The calculation is conducted for seventeen portfolios consisted of stocks and futures of seventeen Russian companies. The efficiency of hedging strategies is assessed by maximum risk reduction, financial result of hedged position and investor's utility with risk aversion parameter varying from zero to ten. The most stable performance of hedging strategies according to the chosen criteria demonstrates ADCC model, which provides the highest maximum risk reduction and utility for about 60% of portfolios. MSV maximizes profit of the hedged position for small values of risk aversion in 70% cases. To summarize, ADCC and MSV models are recommended to use for constructing hedging strategies on Russian stock market according to maximum risk reduction and utility for the former and profit for the latter. MSV gives better results for risk-lovers and ADCC outperforms the other models if risk aversion parameter is larger than 2.

The possible directions of the future research include implementing time-varying degree of risk aversion, introducing the heterogeneity of investors by their attitude to risk and using other hedging instruments.

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