# MDR Analysis Technique for a Metallic Sphere in the Rectangular Waveguide

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#### Abstract

The Morphology-dependent resonance (MDR) for a sphere follows form Mie theory. Analytics gives the solution only for a free-space situation. This work describes the initial principles of microwave discernibility conditions for a sphere, placed at the center of the rectangular waveguide. Having previously obtained data for this position, extreme frequencies of the range of transmission line are of interest. Obtained formulations, describe the behavior of reflection coefficient in this diapason. The main interest aroused for the Rayleigh region. The caustic analysis, between sphere surface and waveguide walls, also gives the desired results. Numerical and natural experiments at the frequency range 7-12 GHz, confirms the findings.

## 1 Introduction

In [1] empirically obtained reflection coefficient for the metallic sphere, that touches the broad wall of the waveguide was obtained. This position have the approximate solution, but in that formulation, there is no any frequency dependence. Work [2] specify this omission and also gives numerical and experimental results for central sphere position. In this form, the solution becomes closed, but low frequencies excluded from the analysis, however, on certain electrical sizes of a sphere, MDR may be observed. Work [3] gives the detailed explanations, basically for Rayleigh diapason, which is considered in this work. This work also provides the developed theory for metallic sphere MDR. Extensive generalizations of the problem are given in [4]. Estimation of minimal displacement can be obtained in the terms of object tracking by setting the sensitivity of measurement instruments and observational error. This formulation may be examined in [5]. The discernibility of the object determines by using this provisions. This work also gives the main formulations for MDR analysis of dielectric spheres.

This work chases several aims. First is to confirm the recombination algorithm (this algorithm can be used in wide range of propagation problems). Second - determines the divergence between free space Mie formulation and the corresponding situation in the waveguide (as limited space). This point leads to the main aim - general method of sphere discernibility prediction.

Denote topic-related problems. The first as the most obvious – radio diagnostics of transmitting lines. This problem is also connected with optical lines particles detection algorithms. Semianalytical boundaries for the reflection coefficient of the metallic sphere can serve as a gauge measure for line calibration process. Based on this algorithm, a new method for materials parameters measurements is being developed. The single granule of powdered material can be approximated by a sphere for the further analysis. Some experimental data are presented in [6]. The next problem is a small effects detecting. Diffraction on small bodies, as well as other effects can be reliably detected only when reconciled with expected values. It also can be connected with optimization process for numerical simulation.

#### 2 Formulations

Analytical findings for the metallic sphere reflection in free space, obtained with a special approximation, in the form [7]. In Mie strict solution boundaries ([8] and [9]), MDR appears when two waves (direct and enveloped) adding up in phase. Current density on the shadow side of the sphere not equal to zero anyway. In the case of a dielectric sphere, MDR appearance also connected with the index of material refraction. Some useful formulations can be found in [10].

The generalized complex metallic sphere reflectivity, derived by propagation condition recombination for rectangular waveguide, is as in author's conference thesis [11]. This formulation allows getting the resonance Mie curve for perfect conducting sphere, placed at the center of rectangular waveguide cross section. Coefficient expression corresponds to the wave summation (S = S0+S1) as it presented on fig.1. Radar cross section of a sphere included in S0. For the dielectric sphere also refraction waves should be taken into account. Recombination algorithm for the case of broad or narrow wall touching not developed, due to new geometry accounting necessity. However, in this extreme situation, the caustic numerical analysis provides focused filed distribution ([12] and [13]).



Figure 1: A sphere MDR appearance approximation

Propagation condition recombination affects three aspects: first - the characteristic type of wave (fundamental mode only). Second - dephasing, due to wave dispersion, makes some corrections to the phase factor. And, third – incidence at the certain angle (also due to dispersion). In this case, works [11] and [13] gives the necessary explanations. Equations for wave reflection from [2] and [11] will be compared for propagation recombination algorithm verification. At the same time, natural experiment results reconciled with theoretical data.

#### 3 Experiment

As the object under test the metallic sphere with r = 2.25 mm was used. Structure of the laboratory setup is common for waveguides measurements (fig. 2 and 3). Object placed at the center of the line cross section, by using radio transparent

holders. Waveguide transmission line for operating frequencies f = (7 - 12) GHz, with a = 23 mm and b = 10 mm. Electrical sizes in Rayleigh region: (0.12 - 0.475). Vector circuit analyzer - Rohde and Schwarz ZVA 24. Two port calibration method by TRM algorithm gives the observational error (the maximum deviation from the zero level for thought line) equal to 0.03 dB.

Stages of setting the experiment with the key factors of analysis are similar to standard technologies of waveguide measurements ([6], [15]). Fig. 4 gives the complete findings of the pointed formulation. Objective functions from [2] and [11] were converted to dB for easy comparison with experimental data. Circuit reflection  $S_{11}$  as the output data from analyzer also shown in fig. 4 for the lower bound of the frequency range.

Estimation of center displacement was not made. In the case of sphere discernibility such a procedure is unnecessary. In more exact positioning situation this evaluation should be carried out as in [5]. To analyze the field inhomogeneities in the caustic regions numerical simulation was hold up.

#### 4 Results

Analysis of fig. 4 gives the following results. First, the previous method as in [2], excluding the initial operating frequencies of transmission line. Free space to line recombination equation [11] accurately describes the behavior of reflection process

in all frequency band and most importantly gives the lost information about possible MDR of the sphere. This formulation gives all necessary data to predict the discernibility of the target in waveguides. Electrical radius 0.15 corresponds to the minimum of reflectivity. Presents of this resonance processes let to assume that Mie recombination algorithm may be saved in such a form for other limited spaces situations. The explanation of theory and experiment line significant deviation (2 dB) at point 10.5 GHz, lies in increasing of coefficient oscillation at high frequencies (start from electrical radius equal to 0.3) due to multiple sphere-walls reflections. This point corresponding to the maximum magnitude of this effect.



Figure 2: The structure of the laboratory stand



Figure 3: Carrying out an experiment

Numerical analysis at this frequency point confirms such assumptions. On fig.5 shown the CAD geometry for calculation, and on fig. 6 normalized near field at the sphere position cross-section. The focused field areas strongly correspond to non-linear diffraction and wave propagation distortion. The numerical coefficient for refraction coefficient for this frequency should be changed.



Figure 4: Theoretical and experiment results for r=2.25mm: (solid line) - reflection coefficient as in [11]; (dotted line) - reflection coefficient as in [2]; (crosses) - experiment output reflection coefficient.



Figure 5: Modeling geometry

By virtue of obtained results from strict solution approximation with specific recombination, extremely small spheres in waveguides may be described by free space classic equations. But in this case, dispersion and incident angle also will make the certain impact. Fundamental convergence of initial and recombination expression determines by the limits of the line wavenumber (to zero) and frequency (to infinity). Obviously, such conditions, besides that, not determines the possible discernibility of the sphere, as well as exceeds the waveguide frequency boundaries together with Rayleigh region approximation. It should be noted that problem of invisibility of heterogeneous in transmission line may also be solved by the recombination equations. General algorithms cause also specified anisotropic medium.



Figure 6: Electric field distribution at the sphere position for the caustic analysis

# 5 Conclusion

The main result of this article is expressed in the meaning of a metallic sphere reflection coefficient in limited space. For a dielectric sphere the same approach can be applied with the exception of wave summation approximation. This procedure can be replaced by direct calculation of Mie coefficient how it was done in [5].

Any specific conclusions about the use of such reconstruction can be made on the basis of data from [3]. It should also specify that all pointed limitations of waveguide measurement method in [14] are confirmed by this work. Permittivity and permeability estimation of materials in a medium with wave dispersion should take into account these characterisations. The task is complicated by the fact that the appearance of caustics is unpredictable and can not be taken into account while standard line calibration.

It is logical to assume, that corresponding measurement for a dielectric sphere with a comparison with the Mie reformulation for limited space, can be used to determine it material parameters. This is the topic for the next studies.

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