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ЗАДАЧА H∞-ОПТИМАЛЬНОГО СИНТЕЗА С НЕЕДИНСТВЕННЫМ РЕШЕНИЕМ*

Аннотация

Данная статья посвящена задаче синтеза H∞-оптимального управления для объектов, заданных линейной стационарной системой со скалярными управляющим воздействием и внешним возмущением, и невозмущенным измерением по нескольким переменным. Решение этой задачи не единственно, что дает возможность выбирать структуру наблюдателя и обеспечивать его дополнительные свойства. Отсутствие шумов в измерениях делает невозможным решение данной задачи стандартными средствами теории H-оптимизации, основанными на решении уравнений Риккати или линейных матричных неравенств. Вместо этого применяется спектральный подход, основанный на факторизации полиномов, что снижает вычислительную сложность синтеза. Продемонстрированы теоретическое обоснование подхода и пример его практического применения.

Ключевые слова

H∞-оптимизация; параметризация; закон управления; устойчивость.

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H∞-OPTIMAL SYNTHESIS PROBLEM WITH NONUNIQUE SOLUTION

Abstract

This paper is devoted to H∞-optimization problem for LTI systems with scalar control and external disturbance signals and with no noisy multivariate measurement signal. The solution of this problem is not unique, that provides possibility to construct the controller with desired structure and additional properties. Besides, the absence of the measurement noise makes solution of this problem with implementation of standard H-theory methods, such as various modifications of 2-Riccati or LMI technique, impossible. A special analytical spectral approach in frequency domain based on polynomial factorization can be used that increases computational efficiency. Its theoretical description and example of practical implementation are presented in this paper.

Keywords

H∞-optimization; parametrization; control law; stability.

Введение

Problem of H_∞ optimal controller synthesis is a hot research area, paid serious attention in both papers and monographies. Various algorithms of the optimal control design are proposed in much-quoted monographs such as [1, 2]. Let us note that most of them are based on solution of Riccati matrix equations («2-Riccati» approach) or linear matrix inequalities («LMI» technique).

However, there are situations, where implementation of these well-known algorithms is non-effective or even impossible. For example, such problem with no-noisy measurement is irregular is directly unsolvable by «2-Riccati» approach. This difficulty can be overcome can be solved by using a special approach, based on polynomial factorization and parametrization of the set of stabilizing controllers. The proposed method is close to ones, proposed in the papers [3, 4]. Its additional serious advantage is computational efficiency, outbalancing 2-Riccati or LMI techniques one. It is not very significant the stationary laboratory conditions, but can be crucial for the

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plants, operated in real-time regime, especially with adoptive changeover of control laws, mentioned in [4].

This work is a development of concepts, proposed in [5-7]. The considered LTI system is affected a scalar control, scalar external disturbance and measurement signal is noiseless. The similar problem of synthesis of H_2 – optimal controller has already been considered in [6] and the paper [7] is devoted to solving SISO (Single Input-Single Output) H_∞ – synthesis problem with implementation of interpolation technique.

The paper is organized as follows. In the next section, equations of a controlled plant are presented and the problem of H_∞-optimal synthesis is posed. Section 3 is devoted to computation of the transfer functions of H_∞-optimal closed-loop system. In Section 4, we consider calculation of the optimal controller. In section 5, we give illustrative examples of synthesis. Finally, Section 6 concludes this paper by discussing the overall results and future perspective of research in this area.

2. Mean-square Optimization Problem

Let us introduce a linear time invariant plant

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{c}d(t), \tag{1}$$

where $\mathbf{x} \in R^n$ is the state space vector, which can be measured, u and $d(t)$ are the scalar control and external disturbance respectively. All components of the matrices $\mathbf{A}, \mathbf{b}, \mathbf{c}$ are known constants, the pair $\{\mathbf{A}, \mathbf{b}\}$ is controllable.

External disturbance $d(t)$ for the system (1) is treated as an output of the following system

$$d(s) = S_1(s) i_d(s)\,,\; S_1(s) = N(s)/T(s)\,, \tag{2}$$

where the polynomials N and T are Hurwitzб $i_d(s)$ is an auxiliary signal with unknown structure. Let us use the following notation в дальнейшем

$$S_d(s) = S_1(s) S_1(-s)| \,.$$

The controller is to be designed in the form

$$u = \mathbf{W}(s)\mathbf{x}\,, \tag{3}$$

where $\mathbf{W}(s) \equiv \mathbf{W}_1(s)/W_2(s)$, $\mathbf{W}_1(s) = (W_{11}(s) \quad W_{12}(s) \quad \ldots \quad W_{1n}(s))$, $W_{1i}, (i = \overline{1,n})$, W_2 are polynomials. The choice of the transfer matrix function should provide minimum of functional

$$I = I(\mathbf{W}) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[\mathbf{x}'(t) \mathbf{R} \mathbf{x}(t) + k^2 u^2(t) \right] dt\,. \tag{4}$$

Note that the exact value of the functional (4) cannot be calculated, because structure of the external disturbance is not completely known, but it is possible to minimize its upper bound. Now we rewrite the expression (4) in frequency domain: firstly, introduce the transfer function $H(s)$, such as

$$H(s)H(-s) = F_x^T(-s)\mathbf{R}F_x(s) + k^2 F_u(-s)F_u(s)\,, \text{ where} \tag{5}$$

$$\mathbf{F}_x(s) = (\mathbf{E}s - \mathbf{A} - \mathbf{b}\mathbf{W})^{-1}\mathbf{c},$$
$$F(s) = \mathbf{W}(\mathbf{E}s - \mathbf{A} - \mathbf{b}\mathbf{W})^{-1}\mathbf{c}. \tag{6}$$

Integrand in the formula (4) can be presented in the frequency domain as $H(s)H(-s)S_d(s)$. Let introduce the following functional to be minimized:

$$J = \|H(s,W)S_1(s)\|_\infty^2 = \max_{\omega \in [0,\infty)} |H(j\omega, W)S_1(j\omega)|^2 \to \min_{W \in \Omega}\,, \tag{7}$$

where is the set of controllers $\mathbf{W}(s)$, such that the characteristic polynomial of the closed – loop system (1)-(3)

$$\Delta(s) = A(s)W_2(s) - \mathbf{W}_1(s)\mathbf{B}(s)\,, \tag{8}$$

is stable.

3. H∞-Optimal Transfer Matrices

Solving of the stated problem directly is significantly obstructed by the fact that dependency of the functional J of the transfer function $W(s)$ is nonlinear. This difficulty can be avoided by using of the parametrization technique, proposed in [3]. Let introduce the function parameter

$$\Phi(s) = \boldsymbol{\alpha}(s)\mathbf{F}_x(s) + \beta(s)F_u(s)\,, \tag{10}$$

where $\boldsymbol{\alpha}(s) = (\alpha_1(s) \quad \alpha_2(s) \quad \ldots \quad \alpha_n(s))$, $\alpha_i(s), (i = \overline{1,n})$, $\beta(s)$ where $\boldsymbol{\alpha}$ and β are polynomials such that
$Q(s) = A(s)\beta(s) + \boldsymbol{\alpha}(s)\mathbf{B}(s)$
is Hurwitz polynomial. Transfer functions $F_u(s)$, $F_x(s)$ can be expressed as functions of the parameter $\Phi(s)$,

$$\mathbf{F}_x = \mathbf{F}_x(\Phi) = \mathbf{P}^{-1}\mathbf{c} + \mathbf{B}Q^{-1}\tilde{\Phi},$$
$$F_u = F_u(\Phi) = AQ^{-1}\tilde{\Phi},\ \tilde{\Phi} = \left(\Phi - \alpha\mathbf{P}^{-1}\mathbf{c}\right), \tag{11}$$

The auxiliary transfer f

$$H(s)H(-s) \equiv (\overline{T}_1 + \overline{T}_2\overline{\tilde{\Phi}})(T_1 + T_2\tilde{\Phi}) + T_3 \text{, где}$$
$$T_1 = \left(\mathbf{B}_s^*\mathbf{RC}_s\right)/(A_s G),\ T_2 = \overline{G}/Q, \tag{12}$$
$$T_3 = \mathbf{C}_s^*\mathbf{RC} - (\mathbf{C}_s^*\mathbf{RB}_s\mathbf{B}_s^*\mathbf{RC}_s)/(A_s\overline{A}_s G\overline{G}).$$

The initial problem (8) can be transformed to minimization of the functional

$$J_2 = J_2(\Phi) = \left\|H(\tilde{\Phi})S_1\right\|_2^{\infty},\ J_2 \to \min_{\Phi\in\Omega_\Phi}, \tag{13}$$

where $\Omega_{\tilde{\Phi}}$ is set of rational fractions $\tilde{\Phi}(s)$ with Hurwitz denominators. In accordance with the formulae (12)

$$J(\tilde{\Phi}) = \left\|H(s,\tilde{\Phi})S_1(s)\right\|_\infty^2 = \sup_{\omega\in[0,\infty)} |H(j\omega)S_1(j\omega)|^2 = \ldots$$
$$\ldots = \sup_{\omega\in[0,\infty)} \left[\left|(T_1 - T_2\tilde{\Phi})S_1\right|_{s=j\omega}^2 + T_3(j\omega)S_d(j\omega)\right] \to \min_{\Phi\in\Omega_{\tilde{\Phi}}}. \tag{14}$$

It is evident, that $T_3(s)$ is independent of the parameter $\tilde{\Phi}(s)$ and the value

$$J_a = \sup_{\omega\in[0,\infty)} (T_3(j\omega)S_d(j\omega)), \tag{15}$$

Is the lower bound of the functional $J(\tilde{\Phi})$. So we can consider search of the parameter $\tilde{\Phi} \in \Omega_\infty^{\tilde{\Phi}}$ such that:

$$J(\tilde{\Phi}) \le \rho^2,\ \rho^2 \ge J_a, \tag{16}$$

instead solving of the initial problem (7). Nevanlinna-Pick interpolation can be implemented to guarantee this inequality.

Let formulate the necessary and sufficient conditions of problem solvability, using the Pick's theorem.

Theorem 1. The problem (16) is solvable if and only if the value ρ is such that Hermitian matrix $L_h(\rho^2) = \{l_{ij}(\rho^2)\}$ is non-negative, where

$$l_{ij} = \left(1 - d_i\overline{d}_j\right)/(g_i + \overline{g}_j),$$
$$d_i = \mathbf{B}_s^T(-g_i)\mathbf{RC}_s(g_i)N(g_i)/[A_s(g_i)R(g_i)],\ i, j = \overline{1,n}, \tag{17}$$

Here g_i $(i = \overline{1,n})$ are the roots of polynomial $G(-s)$ (we assume that they all for simplicity) and the polynomial $R(s)$ is result of the following factorization

$$\rho^2 - T_3(s)S_d(s) \equiv R(s)R(-s)/[G(s)G(-s)].$$

Proof. It is necessary and sufficient to design function $\Phi \in \Omega_\Phi$ such that

$$|H(j\omega)\overline{H}(j\omega)S_d(j\omega)| \le \rho^2,\ \omega\in[0,\infty), \text{ or}$$
$$\left|(T_1 - T_2\tilde{\Phi})(\overline{T}_1 - \overline{T}_2\overline{\tilde{\Phi}})S_d(j\omega)\right| \le \rho^2 - T_3 S_d(j\omega). \tag{18}$$

Let us execute the following factorization

$$\rho^2 - T_3(s)S_d(s) \equiv R_\rho(s)R_\rho(-s)/[T(s)T(-s)\,G(s)G(-s)], \tag{19}$$

where $R_\rho(s)$ is a Hurwitz polynomial and rewrite the expression (18) as

$$\left|\left(\frac{\overline{G}}{Q}\tilde{\Phi} + \frac{\mathbf{B}_s^*\mathbf{RC}_s}{A(s)G(s)}\right)\frac{N(s)G(s)}{R(s)}\right|^2 \le 1, \tag{20}$$

Now we consider a transfer function

$$Z(s) = \left(\frac{\overline{G}}{Q}\tilde{\Phi} + \frac{\mathbf{B}_s^*\mathbf{RC}_s}{A(s)G(s)}\right)\frac{N(s)G(s)}{R(s)}, \tag{21}$$

and calculate the values $Z(g_i)$. It can be seen that

$Z(g_i) = d_i,\ i = \overline{1,n}$,

where the complex values d_i are determined by the formulas (17). The initial problem can be solved as NP –one

$$\|Z(s)\|_\infty \le 1,\ Z(g_i) = d_i,\ i = \overline{1,n}, \tag{22}$$

where $Z(s)$ is the transfer function to be designed. Non-negativity of the Pick matrix $L_h(\rho) = \{l_{ij}(\rho)\}$ necessary and sufficient condition of solvability of the problem (18) that proves the theorem.

Theorem 2. The dynamics of the optimal closed-loop system (1), (2) can be described by the following transfer functions

$$\mathbf{F}_u = \frac{[m_1(s)A(s)R_\rho(s) - m_2(s)N(s)\mathbf{B}_s^*\mathbf{R}\mathbf{C}_s]}{G(s)G(-s)m_2(s)N(s)}, \quad (23)$$

$$F_x = \frac{[\mathbf{C}_s m_2(s)N(s)G(s)G(-s) + \mathbf{B}_s m_1(s)A(s)R_\rho(s) - \mathbf{B}_s m_2(s)N(s)\mathbf{B}_s^*\mathbf{R}\mathbf{C}_s]}{G(s)G(-s)A(s)m_2(s)N(s)} = \dots$$
$$= \frac{\mathbf{B}_s m_1(s)R_\rho(s) + m_2(s)N(s)[k^2 A(-s)\mathbf{C}_s + \boldsymbol{\gamma}(s)\mathbf{R}\mathbf{B}(-s)]}{G(s)G(-s)m_2(s)N(s)}, \quad (24)$$

where the polynomial matrix $\gamma(s)$ is result of division

$\boldsymbol{\gamma}(s) = (\mathbf{C}(s)\mathbf{B}^T(s) - \mathbf{B}(s)\mathbf{C}^T(s))/A(s)$,

and division is done totally [6].

Proof. Let us consider, that the condition of the theorem 1 выполняется is and the transfer function $Z(s) = m_1(s)/m_2(s)$ is solution of the Nevanlinna-Pick interpolation problem (20). As a result

$$\left(\frac{\overline{G}G}{Q}\tilde{\Phi}_0 + \frac{\mathbf{B}_s^*\mathbf{R}\mathbf{C}_s}{A_s}\right)\frac{N(s)}{R_\rho(s)} = \frac{m_1(s)}{m_2(s)}, \text{ and} \quad (25)$$

$$\tilde{\Phi}_0 = \frac{Q(s)[m_1(s)A(s)R_\rho(s) - m_2(s)N(s)\mathbf{B}_s^*\mathbf{R}\mathbf{C}_s]}{G(s)G(-s)A(s)m_2(s)N(s)}, \quad (26)$$

One can check that the expression in the square brackets in (23)-(24) is equal to zero in the points $s = g_i$: this follows that division to $G(-s)$ is done totally. Substitution (25) to the formulae (11) results in (23)-(24).

Remark. Note that transfer functions of the optimal control process can be improper, that is undesirable. One of ways to overcome this difficulty is to deform the spectral power density, using $\tilde{S}_1(s) = \tilde{N}(s)/T(s)$, where $\tilde{N}(s) = \hat{N}(s)N_d(s)$ and polynomial $\hat{N}(s)$ is Hurwitz.

4. Transfer Matrix of the Optimal Controller

As a result, the optimal transfer functions (23), (24) of the closed loop system are calculated, but it is necessary to design controller (3), providing such dynamics of the closed – loop system. Optimum condition, proved in [6], can be used for its computation.

Theorem 3 [6].: The controller (3) provides the optimal transfer functions (37), (38) for the closed loop system (1), (3) if and only if its transfer matrix $\mathbf{W}(s) = \mathbf{W}_1(s)/W_2(s)$ satisfies the following main polynomial equation (MPE):

$$\mathbf{W}_1(s)\mathbf{f}_x(s) - W_2(s)f_u(s) = 0, \quad (26)$$

where polynomial column $\mathbf{f}_x$ and polynomial f_u represent the numerators of the optimal transfer functions (23), (24).

Now we specially note that, the polynomial equation (39) has infinitely many solutions, if there are no common roots of the items $f_{xi}(s)$ of the column $\mathbf{f}_x(s)$ and the polynomial $f_u(s)$, which implies that the optimization problem has no unique solution. Also, it is necessary to mention that the functions $\tilde{\mathbf{F}}_x$, $\tilde{F}_u$ can have the common multiplier $C_0(s)$. It can be seen that $C_0(s)$ is divider of the characteristic polynomial $\Delta(s)$ of the closed-loop system and it must be a Hurwitz polynomial to provide solvability of the problem.

Let us implement one way of construction of the optimal controller (3), described in [6], accepting the following particular structure of the controller (3) transfer matrix:

$$\mathbf{W}(s) \equiv \mathbf{W}_1(s)/W_2(s), \ \mathbf{W}_1(s) \equiv \mathbf{k}_w W_0(s), \quad (27)$$

where $W_0(s)$ is the polynomial and $\mathbf{k}_w$ is n-dimensional real raw such that the polynomial $C_0(s)$

$C_0(s) = \mathbf{k}_w \mathbf{C}(s)$,

is Hurwitz. The polynomials $W_0(s)$ and $W_2(s)$ can be calculated as follows

$$W_0(s) \equiv f_u(s),$$
$$W_2(s) \equiv \mathbf{k}_w[\mathbf{B}(s)f_u(s) + N(s)m_2(s)G(s)\mathbf{C}(s)]/A(s). \quad (28)$$

Substitution of (28) to the expression of the characteristic polynomial $\Delta(s)$ (8) results in

$\Delta(s) = N(s)G(s)C_0(s)$,

i.e. stability of the closed-loop systems is guaranteed. It is easy to verify that the designed controller satisfies the optimality conditions (26).

5. Examples of Synthesis

Let us use Theorems 3 – 5 to design the optimal controller with the model (1) of a control plant, having the following matrices:

$$\mathbf{A}=\begin{pmatrix}-0.0936 & 0.634 & 0\\ 0.048 & -0.0717 & 0\\ 0 & 1 & 0\end{pmatrix},\ \mathbf{b}=\begin{pmatrix}0.0196\\ 0.0160\\ 0\end{pmatrix},\ \mathbf{c}=\begin{pmatrix}0.41\\ 0.0076\\ 0\end{pmatrix}.$$

The functional (4) is determined by the parameters

$$\mathbf{R}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix},\ k=0.005,$$

to minimize external disturbance effect to the coordinate x_3 of the state vector. The external disturbance has the spectrum (2) with the parameters $N_d = s^3+0.3s^2+0.03s+1$, $T_d(s)=20s^4+17.3s^3+11.9s^2+3.87s+1.08$. It will be deformed by the polynomial $\widehat{N}(s)=s^3+9s^2+24.75s+20.25$.

Firstly, we consider presentation of this system in the frequency domain:

$$B(s)=\begin{pmatrix}0.019\,s^2+0.0238\,s\\ 0.016\,s^2+0.0024\,s\\ 0.016\,s+0.0024\end{pmatrix},\ C(s)=\begin{pmatrix}0.019\,s^2+0.0238\,s\\ 0.016\,s^2+0.0024\,s\\ 0.016\,s+0.0024\end{pmatrix},\ A(s)=s^3+0.811s^2+0.037s.$$

Then calculate the polynomial $G(s)=0.005s^3+0.014s^2+0.018s+0.0024$, the value $J_a=0.0000063$ and the polynomial $R(s)=\ 0.0054s^7+0.0189s^6+0.0326s^5+0.024s^4+0.0135s^3+0.005s^2+0.0015s+0.0001$. Now let us receive the values d_1=0.0166 + 0.0394j, d_2=0.0166 – 0.0394j d_3=-0.0007 and construct the Pick matrix

$$L_h(J_a)=\begin{pmatrix}0.3784 & 0.2062-0.1893\mathrm{i} & 0.4061-0.3338\mathrm{i}\\ 0.2062+0.1893\mathrm{i} & 0.3784+0.0000\mathrm{i} & 0.4061+0.3338\mathrm{i}\\ 0.4061+0.3338\mathrm{i} & 0.4061-0.3338\mathrm{i} & 3.3182+0.0000\mathrm{i}\end{pmatrix},$$

and calculate its eigenvalues: 0.076, 0.49, 3.5090, i.e. NP-interpolation problem (20) has a solution. It is notable that there are infinitely many solutions of this problem because all eigenvalues of the Pick matrix are positive. Then we compute the solution of the interpolation problem

$$Z(s)=\frac{m_1(s)}{m_2(s)}=\frac{0.055s^2-0.077s+0.010}{0.49s^2+0.678s+0.090},$$

and receive the transfer functions of the optimal closed-loop system:

$$F_u(s)=\frac{-24.53s\text{^}9-139.8s\text{^}8-382.4s\text{^}7-589.4s\text{^}6-537.5s\text{^}5-275.9s\text{^}4-70.22s\text{^}3-9.17s\text{^}2-0.6s-0.015}{\Delta(s)},$$

$$F_x(s)=\begin{pmatrix}\dfrac{0.41s^{10}+5.49s^9+29.4s^8+84.84s^7+145.1s^6+147.3s^5+82.32s^4+22.1s^3+2.967s^2+0.19s+0.005}{\Delta(s)}\\ \dfrac{0.0076s^{10}+0.117s^9+0.35s^8+0.6s^7+0.49s^6+0.36s^5+0.175s^4+0.06s^3+0.01s^2+0.0006s}{\Delta(s)}\\ \dfrac{0.0076s^9+0.117s^8+0.35s^7+0.6s^6+0.49s^5+0.36s^4+0.175s^3+0.06s^2+0.01s+0.0006}{\Delta(s)}\end{pmatrix},$$

where $\Delta(s)$ is the characteristically polynomial of the closed-loop system

$$\Delta(s)=s^{11}+13.47\,s^{10}+73.91\,s^9+219.6\,s^8+390.1\,s^7+418.9\,s^6+258.7\,s^5+$$
$$+85.1\,s^4+15.43\,s^3+1.56\,s^2+0.083s+0.002.$$

It can be seen that roots of the polynomial $\Delta(s)$ coincides with ones of the polynomials $G(s)$, $N_d(s)$, $m_2(s)$.

Let us choose the vector $\mathbf{k}_w$, such that the polynomial $C_0(s)=\mathbf{k}_w\mathbf{C}(s)$ is Hurwitz $C_0(s)=s+2$. As a result, we receive $\mathbf{k}_w=(-0.318\ \ 17.141\ \ 98.081)$ and the optimal controller (28), where

$$W_0(s)=\ -24.53s^9-139.8s^8-382.4s^7-589.4s^6-537.5s^5-275.9s^4-70.22s^3-9.17s^2-0.6s-0.015,$$

$$W_2(s)=s^9+8.1s^8+17.45s^7+20.5s^6+7.81s^5+12.53s^4+11.13s^3+5.01s^2+0.91s+0.056.$$

Let us demonstrate the frequency response: $A_d(\omega)=\|H(j\omega)S_1(j\omega)\|^2$ on the Figure 1. It can be seen that

$A_d(\omega) \le J_a$ it possesses the maximal value J_a on the frequency $\omega_0 = 0.46$.

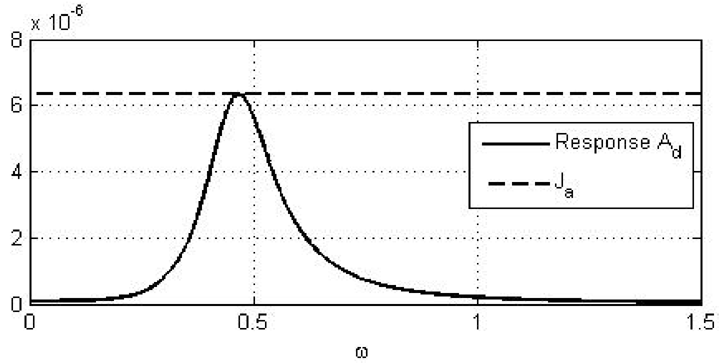

Figure 1. Frequency response $A_d(\omega)$ and the bound J_a

The dynamics of minimized variable $x_3(t)$ before and after activation of the controller at 250 s is shown on the figure 2. It can be seen that oscillation is successfully suppressed.

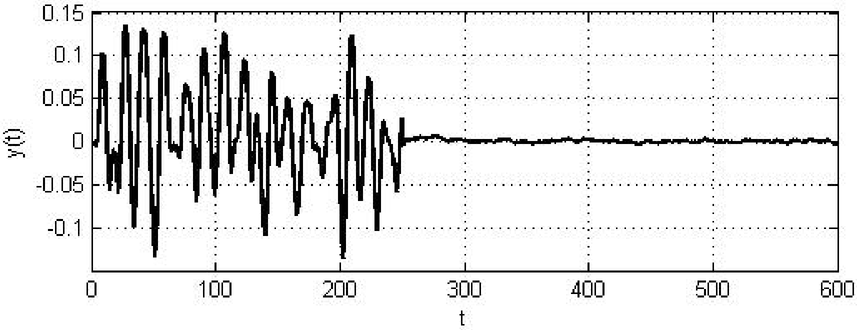

Figure 2. Dynamics of the $x_3(t)$ before and after activation of the controller at 250 s

6. Conclusion

A novel special approach in frequency domain to *H∞*-optimization for LTI controlled plants is proposed and described in details. The demonstrated method is not absolutely universal and can be implemented only for systems with scalar control. Despite this flaw, the mentioned approach is of importance for a wide range of practical control applications, such as marine autopilots, tokamak plasma control, mobile robots etc.

The solution of the presented problem is not unique, because there is no measurement noise, i.e. it is irregular. Irregularity is the main property of the stated problem, making its solving by popular methods impossible and defines suitability to design an alternative special spectral approach in frequency domain that is free from this disadvantage and can be successfully implemented for this problem investigation. Model of the plant is presented in a polynomial form and transfer functions of the closed loop system are parameterized in accordance to the special method. Then we use Nevanlinna-Pick interpolation and obtain transfer functions of the *H∞* – optimal system. Finally, we can compute transfer function of the optimal controller, providing dynamics.

Using approach, based on the polynomial factorization instead implementation of the well-known methods, makes an optimal synthesis procedure significantly easier, especially for the plants with small dimension. This property is very important for onboard control systems.

The working capacity and effectiveness of the proposed method is demonstrated with numerical example: *H∞* optimal controller is designed for the control plant of 3-th order in one of the easiest variants of synthesis

Finally, let us note that the proposed approach has one serious disadvantage: it cannot be used for plants with multiple control signals. Overcoming of this difficulty is the problem of the future research. Also the polynomial presentation is very suitable for investigation of the additional properties of the control system, which means that robust features of the controller, disturbance with non-fractional representation of the specter, task with transport or time delays can be paid serious attention in the future.

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