Image Models and Segmentation Algorithms Based on Discrete Doubly Stochastic Autoregressions with Multiple Roots of Characteristic Equations

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Abstract. One of the most important problems of image processing is the problem of mathematical description of objects appearing on image's background. There are many algorithms for current imitation of images, but unfortunately they have some drawbacks. Often, such models are not able to describe images that are inhomogeneous in space, or they can describe only images with slowly changing properties. Therefore, a model is proposed for describing sharp change in the properties of an image. However, due to the use of autoregression with multiple roots, the discrete model will also contain objects with slowly changing brightness characteristics. The paper is devoted to the development of images' models with variable parameters, which can take a limited number of values. We propose methods for discretizing the obtained random parameter fields by transforming the initial fields with a continuous distribution. Furthermore, the probabilistic properties of the proposed model are analyzed. We also describe in details methods for simulating images containing a given number of structures and investigate segmentation algorithms for the proposed model of images.

Keywords: image processing, doubly stochastic models, random fields, image modeling, image analysis, covariation functions, multiple roots, characteristic equations, image segmentation

1 Introduction

Solutions to a number of important problems in various fields of science and technology can be achieved by using the results of processing multidimensional data sets. Among the image processing tasks, a special place is occupied by problems of segmentation, detecting anomalies, reconstruction, and prediction. One of the most important sources of such data arrays are various multidimensional images. Examples of such images are various information signals, digital photographs and video sequences, remote sensing data of the Earth (RS), etc.

The particular urgency of solving these problems is associated with wide dissemination of methods for recording images of various objects including multispectral (up to 10 spectral ranges) and hyperspectral (up to 300 ranges) of recording the Earth areas. As a result of such registration, multidimensional arrays of information are obtained. Thus, there is an increase in the volume of information received, and although significant amounts of processed information potentially improve the quality of satellite material processing, new methods are necessary for qualitative and quantitative analysis of aerospace observations as a single multidimensional aggregate.

In addition, the problem of describing images can not be regarded as solved. Despite the variety of random fields (RF) and random processes models [1–11], most of them do not give a satisfactory description of real images and signals due to a number of reasons. The main problem is presence on an image of several objects having different nature, description of which is difficult for one model. Digital image processing algorithms [11] can be synthesized for some particular cases of mixed image models [7, 8]. In such models, auxiliary RFs are used to implement the basic RF, which are also a set of model parameters.

In this article, we will consider modification of a doubly stochastic model [1, 3, 4, 7–9], which allows one to obtain a smooth change in the brightness properties of an image, as well as the possibility of using a given number of correlation levels. Finally, we show that the use of a countable number of correlation levels allows one to perform effective segmentation of such images.

2 Doubly stochastic model of images based on Habibi and autoregression with multiple roots of characteristic equations models

The main drawbacks of the doubly stochastic model based on the Habibi model or the usual first-order autoregressive (AR) model are its anisotropy, as well as the use of only three neighboring elements to form a new element of the RF. Indeed, it is obvious that the Habibi model is a special case of an AR model with multiple roots of the characteristic equations [2, 6, 10]. The multiplicity of such a model is (1,1). In addition, increasing the multiplicity of the model can expand the number of links between the elements. We now form a doubly stochastic model combining AR models with multiple roots.

The process of such model synthesis is analogous to the process considered in [1], but it has its own peculiarities and is implemented in three stages. First, the basic RFs are created. Secondly, the values of the obtained RFs are converted into a set of correlation parameters $\{\rho_{\boldsymbol{j}}, \boldsymbol{j} \in (j_1, j_2, ..., j_M)\}$, where M is the dimension of the image being formed. These parameters characterize the connection between the current pixel of the simulated image and the neighboring image elements. Finally, the image is formed as a model of a RF with varying correlation parameters $\rho_{\boldsymbol{j}}$.

For simplicity, the basic RFs are simulated using the first order AR model as well as the doubly stochastic RF model based on the Habibi model. After the formation of two basic RFs, the brightness values of one of them should be transformed into a set of correlation parameters $\{\rho_{xij}, i = \mathbf{1, 2, ..., M_1}, j = \mathbf{1, 2, ..., M_2}\}$, and the second RF should be transformed into correlation parameters $\{\rho_{yij}, i = \mathbf{1, 2, ..., M_1},\ j = \mathbf{1, 2, ..., M_2}\}$.

We can write AR with multiple roots of characteristic equations of the second order in the one-dimensional case as follows:

$$x_i = 2\rho x_{i-1} - \rho^2 x_{i-2} + \xi_i. \tag{1}$$

On the basis of formula (1), we can obtain a two-dimensional model

$$\begin{aligned} x_{i,j} &= 2\rho_x x_{i-1,j} + 2\rho_y x_{i,j-1} - 4\rho_x\rho_y x_{i-1,j-1} \\ &-\rho_x^2 x_{i-2,j} - \rho_y^2 x_{i,j-2} + 2\rho_x^2\rho_y x_{i-2,j-1} \\ &+2\rho_y^2\rho_x x_{i-1,j-2} - \rho_x^2\rho_y^2 x_{i-2,j-2} + b\xi_{i,j}, \end{aligned} \tag{2}$$

where b is the coefficient ensuring the equality of the variance of the simulated RF to a given value.

It should be noted that model (2) is an eight-point model, *i.e.* in it we use 8 previous pixels for the formation of the next element of the RF $\{x\}$. Similarly, for the model multiplicity (3,3), we can obtain a 15-point model, for the model multiplicity (4,4), we obtain a model consisting of 24 points from the neighborhood.

Making coefficients of model (2) to be random with the help of the first-order AR model leads to the obtaining of a doubly stochastic RF model based on Habibi and AR with multiple roots of characteristic equations models.

Figure 1 shows examples of images generated by the doubly stochastic model of a RF (Habibi – Multiple roots models) with different mean values of the correlation coefficients.

Thus, using the doubly stochastic AR models, one can generate images and their sequences adequate to real multi-zone images at relatively low computational costs. This allows us to use such models for statistical analysis of the efficiency of image processing algorithms. In this case, it is possible to achieve a smooth change in the brightness properties of the image due to models with multiple roots of the characteristic equations.

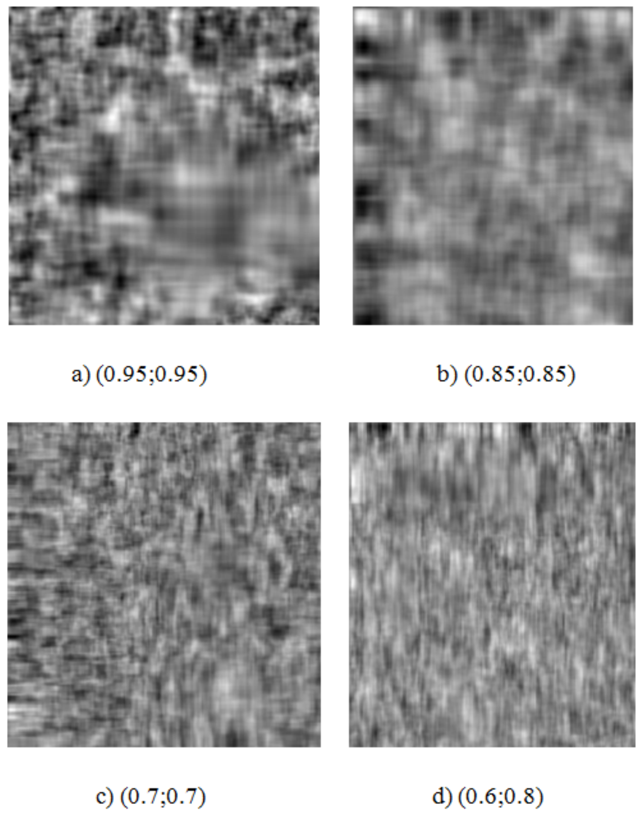

Fig. 1. Images simulated by doubly stochastic RFs based on Habibi and AR with multiple roots of characteristic equations models

3 Discrete doubly stochastic models of images

Consider the doubly stochastic AR model on the example of the RF based on Habibi and AR with multiple roots of characteristic equations models

$$\begin{aligned} x_{i,j} &= 2\rho_{xij}x_{i-1,j} + 2\rho_{yij}x_{i,j-1} - 4\rho_{xij}\rho_{yij}x_{i-1,j-1} \\ &-\rho_{xij}^2 x_{i-2,j} - \rho_{yij}^2 x_{i,j-2} + 2\rho_{xij}^2\rho_{yij}x_{i-2,j-1} \\ &+2\rho_{yij}^2\rho_{xij}x_{i-1,j-2} - \rho_{xij}^2\rho_{yij}^2 x_{i-2,j-2} + b_{ij}\xi_{i,j}. \end{aligned} \tag{3}$$

In model (3), parameters ρ_{xij} and ρ_{yij} are not constant values, but by application of two basic RFs, their brightness values are transformed into a set of correlation parameters in accordance with the following relationships:

$$\begin{aligned} &\tilde{\rho}_{xi,j} = r_{1x}\tilde{\rho}_{xi-1,j} + r_{2x}\tilde{\rho}_{xi,j-1} - r_{1x}r_{2x}\tilde{\rho}_{xi-1,j-1} + \varsigma_{xi,j}, \\ &\tilde{\rho}_{yi,j} = r_{1y}\tilde{\rho}_{yi-1,j} + r_{2y}\tilde{\rho}_{yi,j-1} - r_{1y}r_{2y}\tilde{\rho}_{yi-1,j-1} + \varsigma_{yi,j}, \\ &\rho_{xij} = \tilde{\rho}_{xi,j} + m_{\rho_x}, \rho_{yij} = \tilde{\rho}_{yi,j} + m_{\rho_y}, \end{aligned} \tag{4}$$

where $\varsigma_{xi,j}$ and $\varsigma_{yi,j}$ are two-dimensional RF of independent Gaussian random variables with zero means and variances $M\{\varsigma_{xi,j}^2\} = \sigma_{\varsigma x}^2, \sigma_{\varsigma x}^2 = \sigma_{\rho_x}^2 \left(1 - r_{1x}^2\right)\left(1 - r_{2x}^2\right)$,

$M\{\varsigma_{yi,j}^2\} = \sigma_{\varsigma y}^2 = \sigma_{\rho_y}^2 \left(1 - r_{1y}^2\right)\left(1 - r_{2y}^2\right)$, $\sigma_{\rho_x}^2 = M\{\rho_{xij}^2\}$ and $\sigma_{\rho_y}^2 = M\{\rho_{yij}^2\}$, m_{ρ_x} and m_{ρ_y} define average correlation in row and column.

The correlation coefficients obtained from expression (4) are continuous random variables and usually take different values at each point. To make the correlation coefficients of the basic RF (3) to take a countable number of values, it is necessary to discretize expression (4) as follows:

$$\rho_{xij}^* = round\left(\frac{\rho_{xij} L}{max\{\rho_{xij}\} - min\{\rho_{xij}\}}\right), \tag{5}$$

where $round()$ is the rounding operator, L is the number of sampling levels, max and min are respectively the maximum and minimum values of the RF.

Then it is necessary to convert the obtained discrete RF (5) so that its minimum value corresponds to the minimum value of the continuous RF, and the analogous equality for the maximum values would be satisfied by using expression

$$\rho_{xij}^{**} = min\{\rho_{xij}\} + \frac{(\rho_{xij}^* - min\{\rho_{xij}^*\})(max\{\rho_{xij}\} - min\{\rho_{xij}\})}{max\{\rho_{xij}^*\} - min\{\rho_{xij}^*\}}. \tag{6}$$

Similar actions can be performed for the coefficients by column.

Figure 2 represents realization of model (3) and discrete model (3) taking into account the correlation coefficients (4) and (6) with the following parameters: $\sigma_x^2 = 1$, $m_{\rho_x} = 0.85$, $\sigma_{\rho_x}^2 = 0.0016$, $m_{\rho_y} = 0.9$, $\sigma_{\rho_y}^2 = 0.0016$, $r_{1x} = r_{2x} = 0.95$, $r_{1y} = r_{2y} = 0.8$, $m = n = 320$. Figure 2a corresponds to continuous RF; Figure 2b corresponds to RF with $L = 3$; Figure 2c corresponds to RF with $L = 6$.

Figure 3 shows the cross sections of the covariance functions of the obtained models in a row.

Thus, by varying the number of discretization levels L, it is possible even for identical model parameters to obtain its various implementations. We also can see that on the graphs of Figure 3 the covariance relationships of discrete models decrease more slowly than those in continuous ones.

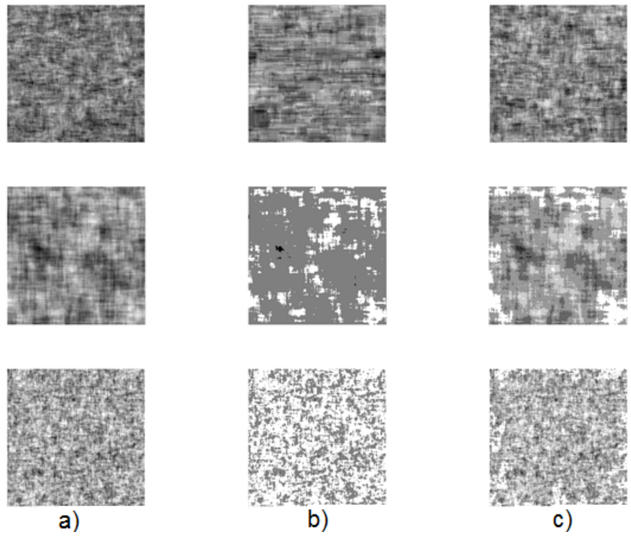

Fig. 2. Images simulated by doubly stochastic model. From top to bottom: basic RF, RF of correlation coefficients in a row, RF of correlation coefficients in a column.

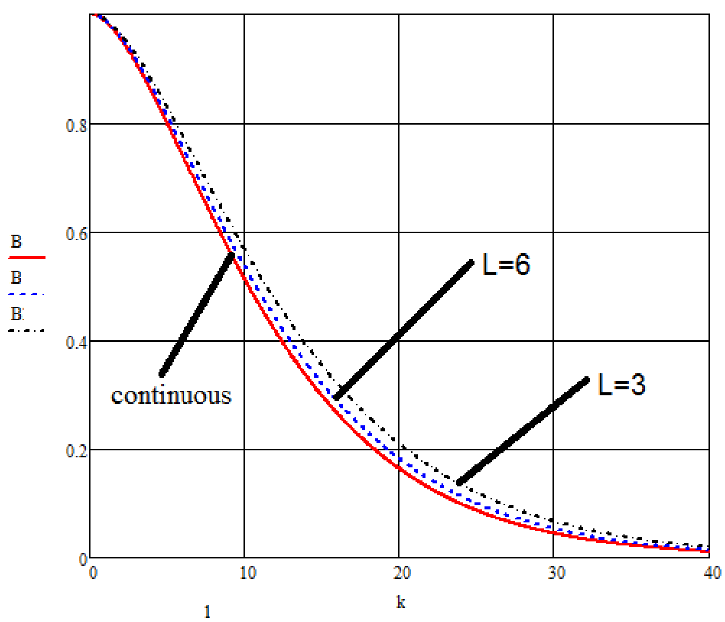

Fig. 3. Covariance functions of doubly stochastic RF based on AR with multiple roots of characteristic equations

4 Segmentation of images with varying correlation properties

Now consider the segmentation algorithm for the generated images in the case where the number of correlation levels is two. To select homogeneous areas, you need to build a histogram of the image. To do this, all the values of the RF $\{\rho_{xij}\}$ are written in the vector of brightness values. It should be noted that all the values have before passed the equalization procedure, and we also taking into account the average additive. So, the result vector is written as follows:

$$L(k) = \tilde{\rho}_{xijeq}, i = 1, 2, ..., M_1, j = 1, 2, ..., M_2, 1, 2, ..., M_1 x M_2. \tag{7}$$

Let us find the number of elements of the vector obtained after the transformation (7), which corresponds to each gradation of brightness. We will use a histogram to find the brightness value between its peaks. Since at this formation the histogram should have two peaks, the threshold value of brightness can be found by the formula

$$L_{thr} = \frac{L_{max1} + L_{max2}}{2}, \tag{8}$$

where L_{max1} and L_{max2} are the brightness values of the first and second maximum of the histogram.

Let us transform the equalized image $\{\tilde{\rho}_{xijeq}\}$ with 256 gradations of brightness into an image $\{R_{xij}\}$ with 2 gradations of brightness. So the image $\{R_{xij}\}$ is binary image, which is obtained by the formula

$$R_{xij} = 0, if\{\tilde{\rho}_{xijeq}\} < L_{thr}, R_{xij} = 255, else. \tag{9}$$

Expression (9) in this case is a segmented image. Furthermore, the proportion of correctly assigned to the first or second area pixels is considered. This proportion is based on the assumption that homogeneous zones are segmented on the binary image of the basic RF. Then the percentage efficiency of segmentation can be found after comparing the resulting image with the segmentation of the basic image.

Figure 4 shows histogram of an image simulated by doubly stochastic model with possible parameter values $\rho_{x1} = \rho_{y1} = 0.8$ and $\rho_{x2} = \rho_{y2} = 0.95$.

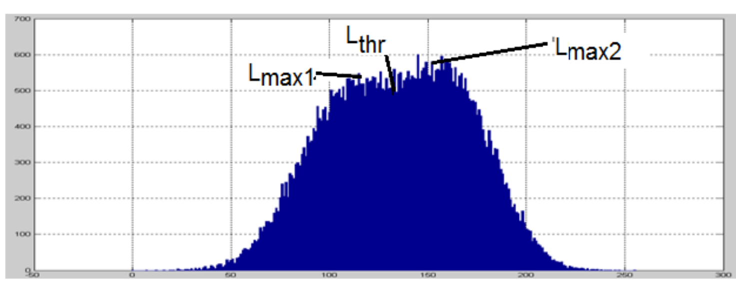

Fig. 4. Histogram of the simulated image

Note that in this case we have the threshold value $L_{thr} = 135$.

It is clear that the best segmentation will be provided by combinational processing of correlation parameters by row and column.

Figure 5 shows the result of the segmentation algorithm using the histogram: Figure 5a corresponds to basic RF; Figure 5b corresponds to nonhomogeneous image with homogeneous regions; Figure 5c corresponds to binarization of the basic RF; Figure 5d corresponds to segmentation of the nonhomogeneous image.

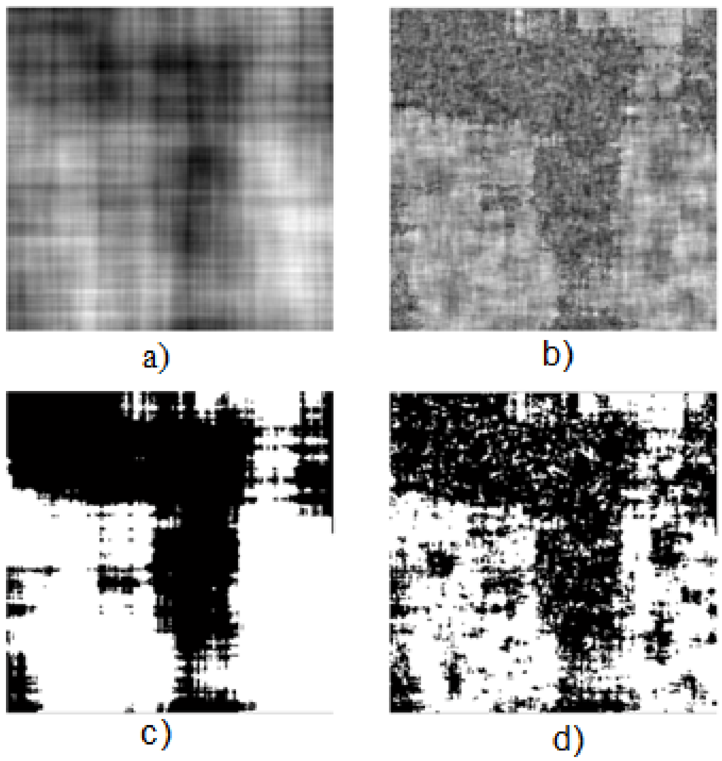

Fig. 5. Segmentation of simulated images

The percentage of the correspondence between Figure 5c and Figure 5d is quite large, namely, 85.7%. Analysis of Figure 5 shows that it is possible to increase segmentation efficiency if small objects are included in a larger neighboring area.

Nevertheless, the result obtained for simulated images allows us to reasonably hope that in the case of applying more complex segmentation procedures in processing the correlation parameter field, the segmentation algorithm found can be applied to real images. Indeed, Figure 6 shows the results of 2-level segmentation (binarization) of a satellite image by applying a combination of the proposed algorithm and the ISODATA algorithm to the field of correlation parameters. In this case, Figure 6a is the original image; Figure 6b shows the correlation parameter fields for the original image; Figure 6c shows the results of segmentation by correlation parameters jointly ISODATA algorithm; and Figure 6d shows the results of applying the ISODATA algorithm to the original image.

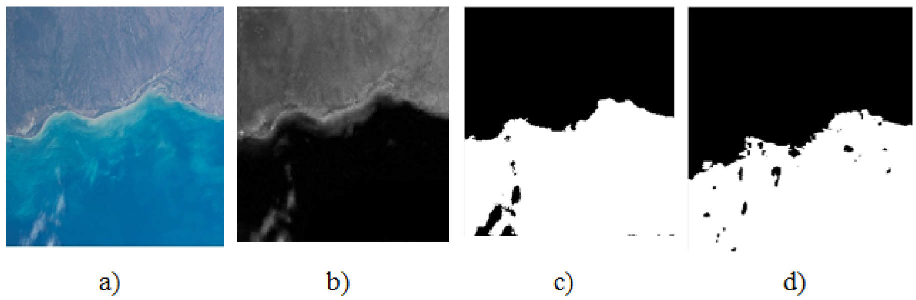

Fig. 6. Segmentation of satellite images

The gain for the presented case on segmentation accuracy was about 6%. However, it is worth to note that the task of assessing the accuracy of segmentation when processing real images is significantly complicated, since there is no predetermined correct version, and most often, we have to use expert estimates.

5 Conclusion

Doubly stochastic models of images based on the AR with multiple roots of characteristic equations are considered. The transition to the discrete case is carried out. Probabilistic properties of the proposed models are analyzed. An algorithm for image segmentation by correlation parameters is described. Gains are obtained when processing images consisting of two structures. The gains are about of 6 –10% in comparison with segmentation by the ISODATA algorithm.

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