Abstract. This paper is devoted to reveal the best way for the reflected signal model implementation for airborne radar systems. The main attention in the paper is paid to the method of the combination of various well-known and enhanced mathematical models of radar signals, terrains, different lengthened objects, etc. for the effective radar scene creation. Also, the key peculiarities of proposed models are discussed in brief. Moreover, questions of the radar echoes computation algorithm of the radar scene are explored. In conclusion, the requirements to the models and suggestions are presented.

Keywords: mathematical model, terrain, reflected signal, airborne radar system, digital signal processing, radar scene simulator

1 Introduction

Technique progress in digital signal processing and computer simulation technologies goes with the rapidly increasing requirements to complex various onboard radar systems and reflected (backscattered) signal simulators for them [1]. This paper concentrated on the radar scene modeling, which is useful for creating multipurpose simulators of radar echoes. It allows researchers to test an influence of the following main parameters: flight parameters (altitude, trajectory, evolutions of the airborne vehicle, etc.), terrain types (forest, ice or water with different salty and wavy), signal distortions (fading, multi-path), type of emitted signal, and radar parameters (antenna pattern, carrier frequency, bandwidth, duration, repetition frequency, etc.).

The proposed digital model is useful in cases of creating the algorithms of analog and digital signal processing and hardware design of radar scene simulators, which are broadly used to check a radar system operation in real-time [6]. The quality of hardware-in-the-loop (HIL) simulators depends on their signal processing capabilities and possibilities to represent all essential aspects of the real flight [6], [13]. So, the develop of models of reflected signals for variety of different flight circumstances is worthwhile.

The first question is what programming environment is best for mathematical model design. It is obvious, that every well-known universal program, which is devoted to a signal computation, can be used for creation of radar signal processing models. It is necessary to decide, which is the best one for the radar scene model.
2 Mathematical Model of Reflected Signals

2.1 Programming Environment

At this point, a brief characteristic of different numerical computing environments will be presented.

The first package is application pure C/C++ environment, but despite the universality of this package and ability of using various libraries freely available through the Internet, it is a very long process to create and debug the model.

So, we turned to the more specialized packages. One of them is the SimInTech environment [2]. This program package contains a lot of libraries for creating various systems from power supply stations to airborne vehicles. This system supplies the visual mode for blocks combinations and signals plotting in the scheme, which is similar to Simulink and Vissim. Moreover, libraries are written in many languages and could be imported in the dll-mode. Other advantage is that many blocks have open-source code and can be modified during the model experiment. So, the producers of this program provide customers support in design and evaluating models for any purposes. On the other hand this program product is the proprietary software.

Another candidate is the GNU Radio [3]. This package is provided only for the Linux OS. So, typical Windows user would feel uncomfortable to use this product. This product provides also the visual mode of design and a lot of libraries for various radar blocks, all libraries are provided with source code and could be modified. But compatibility of libraries causes many questions, because of the conception “as is”. Only enthusiasts use and improve this project.

The next candidate is the SystemVue program [4]. This product contains libraries for radar’s signal multi-path propagation, multiple channels, jammer, interference, etc. It is also compatible with the MATLAB system. The price of this product is equally high.

The next product is the SciLAB, it is an analog of the MATLAB system with free-ware license and open-source code. Most of packages have similar functionality, for example, the Signal Processing and Communication toolbox, Simulink, SAR simulator, import from MATLAB libraries, etc. So, it can be used as the cheap and effective model system for creating the radar scene. This program has a problem with compatibility of different libraries, some of them are unstable and could crash the system. But it is improved every year by European airspace industry.

The last and the most popular program product is the MATLAB, it contains a huge number of libraries, but most of them are unavailable for modification. But the MATLAB has many advantages, some of them are technical support, very effective improvements with each release, ability to build own libraries and import of the third-party modules, and well-designed user interface. Also, it is available in some universities for stuff and students. So, we have chosen the last one.
2.2 Model Structure

It is necessary to divide model in some blocks and describe them separately. Analysis of known sources showed that the most common way is to use the following blocks: underlying surface (terrain) properties, channel of propagation, reflected signal, and evolutions of the airborne vehicle. As it was shown in [1, 5, 6, 7], for radar signals the phenomenological model provides ideas that the superposition of partial signals can be used for radar echoes, the reflection could be presented in terms of geometrical optics, and underlying surface could be split into facets, i.e., tiny pieces of terrain. Because of difficulty and impossibility of implementing other electromagnetic methods for description of big areas of real relief terrain we, have chosen the phenomenological facet model.

So, representation of terrain could be implemented by square or triangle facets, each of them has its own parameters: a square, orientation, backscattering diagram, radar cross section (RCS), and position. All these parameters are sufficient to compute an amplitude and phase of partial signals. This way allows us to model various types of terrains, such as “meadow”, “ground”, “asphalt”, “concrete” and so on.

The rough terrains usually could be presented in the two-scale model, which combines low roughnesses and relief (terrain, such as rocks and hills). The roughnesses could be presented by their mean statistical characteristics. We chose an effective backscattering diagram, which (as it was shown in number of sources [1, 5, 10, 11]) is the most useful statistical characteristic for radar echo signals. The relief could be modeled by position and orientation of the tiny facets [6, 12]. Also, it is possible to model a wavy water surface and forest by this way. But in this paper, other more complicated way is suggested to evaluate reflected signals from these complicated types of terrains.

**Forest Modeling.** Nowadays it is possible to model a reflected signal from each tree and, also, we can change its geometry and its reflection characteristics [1, 7]. For example, we can create the model of pine, aspen, or a birch. For obvious simplification, we have divided the reflected signal in three parts or layers: Canopy, Trunks and Ground. The forest model in Fig. 1 depicts additional (for the Direct beam backscattering) the multi-path signal reflections [7].

![Fig. 1. The multi-path signal reflection from forest layers](image)
For different incidence angles and wavelengths of an emitted signal, the weight of each part would be different. For example, if we talk about the 3 cm-wavelength with about vertical illumination the most weighty component of the reflected signal will be from the crown (if it about leafy trees), a bit less signal will come from a ground, and very few reflected signal returns from trunks. For the millimeters-wavelength, the most strong component is the signal from the canopy, otherwise the meters-wavelength signal is reflected mostly from the ground. So, the reflection depends on wavelength, and we have to get information from real flight experiments to reveal the reflection dependencies.

So it is necessary to design a geometric model for each tree, which will be the base for computing the multi-path signal reflections with precious values of amplitudes and phases of all facets partial addends [6].

After this, it is preferable to model the signal from forest, which can be presented by a number of trees. As soon as we have the geometric model for one tree, we can apply it to many similar trees (clones, which orientation is random) and obtain signal from all the forest. So, we do not need to evaluate signal all the time from each tree, but we can compute it from the geometric model according to proper angles.

There we face to other problem how to evaluate the signal, which is weakened by the leaves of other trees or re-reflected from the lower part of canopy or trunks. There is no common decision, but in some sources [6, 7] it is mentioned that it is possible to neglect the re-reflected signal at all for typical trees because of the weakness their relative magnitudes.

The examples of triangular facet models of the single tree (imported from the 3Ds Max library), woods (reconstructed in the MATLAB system), and the accordingly evaluated received pulse (there about 1.2 million facets in the scene for the radar altitude 50 m with the vertical illumination by a short pulse radar) are presented in Fig. 2.

Fig. 2. Examples of models of the tree, woods and a received pulse

The signal shadowing by canopy can be resolved by implementing the backward raytracing method. The ability of model simplification instead of accurate model of trees is to implement a cloud of randomly spread reflectors. It is mostly useful for the crown model.
But the question is how many reflectors is necessary to use in the model. On the one hand, it can be revealed by the comparison of results of natural experiments and model results. On the other hand, we can take into account the real accuracy of the example of radar system and fill every distance-angle volume or bin (resolved by the common or imaging radar, SAR, etc.) by sufficient number of facets: up to 10–50 facets inside each interesting bin. As the result, we can create an appropriate model for such a complicated terrain type and radar system.

**Wavy Water Modeling.** The next point is the model of a wavy surface. As soon as this topic was highlighted in many researches, we have big amount of carefully debugged models and experimental results of water surface explorations.

At first, it is necessary to mention that the agitation of water surface depends on the wind speed; so, the reflected signal will be different for variety of wind speeds. But not only wind causes surface agitation, it can be caused by ships, water flows, or by the attraction of the moon; and also by a change of water depth, especially, for small depths and steep slope of water bottom. The last one is the most challenging process in modeling the wavy surface. Other feature is that not all emitted signal backscatters on the water surface; it can be reflected from the ground under water, especially, for small depths, unsalted water, and low carrier frequencies.

So, the next thing to deal with is salty of water. The magnitude of the reflected signal depends on salty, as it was shown in [10, 11]. Sequentially, it is necessary to design a wavy water model, which can accurately takes into account the foregoing effects.

Revision of existing models revealed that the most suitable models are the Pierson-Moskowitz (PM) model, Texel, Marson and Arsole (TMA) model, and their enhanced methods [8, 9, 12], which allow us to take into account most of the mentioned effects including the depth dependence of water waving. One approach on the basis of the Joint North Sea Wave Project (JONSWAP) and TMA models can be described by calculating an energy spectrum of waves [9]

\[
E_{\text{TMA}}(f) = E_{\text{JONSWAP}}(f) \cdot \Phi \left( f^*, h \right),
\]

\[
E_{\text{JONSWAP}}(f) = \frac{\alpha \cdot g^2}{(2\pi)^4 \cdot f^5} e^{-\frac{3}{4} \left( \frac{f^*}{f} \right)^4} \cdot \gamma^e \cdot \frac{f^*}{f^p} e^{-\frac{f^*}{f^p}}.
\]

Here, \( f \) is the wave frequency, \( \Phi \left( f^*, h \right) \) is the Kitaigorodskii depth function for the depth of water \( h \); \( \Phi \left( f^*, h \right) = \frac{1}{s \left( f^* \right)} \cdot \left[ 1 + \frac{K}{\sinh(K)} \right] \); \( f^* = f \cdot \sqrt{\frac{h}{g}} \);

\[
K = 2 \left( f^* \right)^2 \cdot s \left( f^* \right); \quad s \left( f^* \right) = \tanh^{-1} \left[ \left( \frac{2\pi \cdot f^*}{f^p} \right)^2 \right];
\]

\( \alpha \) is the scaling coefficient; \( g \) is the gravity constant; \( \gamma \) is the peak enhancement factor; \( \sigma = 0.07 \) for \( f \leq f_p \); \( \sigma = 0.09 \) for \( f > f_p \).
$f_p$ is the maximum spectrum frequency given by $f_p = 3.5 \cdot \left( \frac{g^2 F}{U_{10}} \right)^{-0.33}$, where $F$ is the fetch length; $U_{10}$ is the wind speed at a height of 10 m.

According to this spectrum, the parameters for each elementary wave of water surface are defined (height and wavelength, direction of propagation, wave phase, etc.). These data, together with the aircraft speed vector, current time, and the antenna direction are inserted into the analytical formula [12]

$$\xi(x, y, t) = \sum_{n=0}^{Nf-1} \sigma_n \cdot \sin(K_{0n} \cdot [(x + (V_x - U_n x) \cdot t) \cdot \cos \beta_n + \ldots$$

$$(y + (V_y - U_n y) \cdot t) \cdot \sin \beta_n] - \Omega_n \cdot t + \alpha_n) \quad (3)$$

where $x, y$ is the actual facet location at time $t$;

$n$ is the number of wave trains;

$V_x, V_y$ is the aircraft speed projection;

$U_{nx}, U_{ny}$ is the waves speed projection;

$z + (V U_{nx})t, y + (V_y U_{ny})t$ is the offsets in the $Oxy$ plane;

$\alpha_n$ is the wave phase;

$\beta_n$ is the direction of wave propagation;

$\Omega_n$ is the pulsation;

$K_{0n}$ is the wave number;

$Nf >> 1$ is the number of waves;

$\sigma$ is the standard deviation of sea wave heights.

Examples of model results of wavy water surface according to the model with the wind speed 10 m/s is presented in Fig. 3. The results of model experiments correspond to information from open sources [1, 5, 11]. The salty of water can be taken into account by results of salty measurements. Nowadays we can base on researches provided by many organizations and researcher teams [12].

![Fig. 3. Examples of the wavy surface and computed beat signal of the radio altimeter](image)

**Other complex terrain types and objects.** Another type of terrain, which currently has no implementation in designed model, is an urban development. This is extremely complicated type of terrain, which can be modeled by implementation of 3D models of buildings, bridges, roads, power lines, and huge
amount of other different objects. Some their geometric models are ready and accessible in graphic programs, such as the AutoCAD and 3Ds Max. Their surface forms can be imported, for example, in the MATLAB system, and presented as facets. So, the next step is to accurately set the reflection characteristics for all facet materials. Usually, just a lot of experiments can be helpful for that challenge. Therefore, this type of terrain with some simplifications also can be added into the designed facet model. For many special local objects, such as vans, tanks, or cars the geometric models exist, which are freely accessible (for examples, see [6]). So, we can implement them to fulfill a radar scene.

The next and last point is combination of terrain types in one radar scene. It can be easily presented by brushing (specifying) facets by different reflecting characteristics. So, if one facet presents water, it can be brushed as water-terrain; if other presents the grass, it is brushed with the grass-terrain type. This idea allows us to create lengthened and other usual objects of any form and size.

As the result, we discussed the conceptions for terrain modeling process that allows us to model various terrains and their combinations.

2.3 Signal Representation

The next point is how to represent the signal. It was nothing told about it earlier. For definiteness, we talk about the pulse radar signal, but the very similar operation (as it is written in [6]) can be done for a chirp signal with linear frequency modulation. The emitted signal can be modeled at the carrier frequency. So, it is necessary to have more than two points per a period of signal, but the amount of computations becomes unacceptable. Thus, the usual way is to implement low frequency as the source of information with addition of in-phase and quadrature components, which include the phase information. It is especially necessary in evolution of the evaluate Doppler phase shifts and computation of radar images. The in-phase component could be presented by the sinusoidal signal, the same could be told about the quadrature component, but between components there exists the phase shift of 90 degrees.

The next point is how to sum signals reflected from partial facets. We applied the method, where partial signals can be added to the result signal with their delays, according to the optical geometry theory. For an illuminated spot, we collect the partial signals from all facets in the circle (or ellipse) bounding the half-power level of the antenna pattern. We neglect other partial signals. But the result signal presents only one pulse or repetition interval, so, it is necessary to present the train of pulses. It depends on parameters of pulses: a pause between pulses, duration, envelope, and magnitude. The only envelope should be discussed in detail, others are intuitive parameters. In terms of modeling process, the pulse form could be presented by a number of plots, each of them presents a count of the amplitude (or the power, it is the matter of convenience). For each count with its delay, we accumulate the result signal. Therefore, we have the reflected power (or amplitude). After that it is obvious to evaluate in-phase and quadrature components by multiplication of the sinusoidal signal with counts of
reflected amplitude. At the end, we have the train of reflected pulses, which can be processed, for example, by methods of synthesis the radar image.

3 The Digital Model Implementation

At this moment we described the distinctions of the designed model, and now it is the time to describe exactly the implementation of the designed model.

In Fig. 4, the scheme of the radar scene model is shown, which implements all foregoing ideas in one scheme. Here, following sequence of computation is implemented: constants definition and model parameters; input the track, signal and terrain parameters; track, vehicle, and illuminated spot evaluating; cycles for all antennas and for all points of trajectory where the reflected signal is evaluated; displaying the model results and radar image preparation.

![Fig. 4. The algorithm of the model of radar echo signal](image)

This model allows us to change separately each part of the model without changing others; also, it suits as well for pulse radar as chirp radar. Also, as it was mentioned above, we can add local objects, such as trees or cars, combinations of terrains, change signal, vehicle and terrain parameters. Therefore, this model is flexible and powerful enough to build the simulator of the radar scene.
Therefore, it is possible to extend this model by adding more complex signal forms, extending database of terrains and local objects or connecting this model to the real-time services of vector maps, which are freely available through the Internet, for example, Google-maps.

As soon as we have the model, it is necessary in brief to describe the module structure of the radar scene model. In Fig. 5 the modules are highlighted in bold names. For example, in the MATLAB system, modules are presented in separate files. Also, nearby the names of modules, the brief descriptions are given.

![Module structure of the radar scene model](image)

The module Get_Traject is the clock module, which synchronize all modeling process; so, the parameters of emitted signal and trajectory at first are passed to this block. Also, this model has the following distinctions: the parameters of the transmitter can be passed in the Set_RvParam module; the receiver parameters (if it is necessary) could be implemented afterward.

4 Conclusions

In this paper, the radar scene model is described; it can be implemented and helpful for various explorations from radar image algorithm verification up to simulator design [6, 13]. The designed digital model operates with facets, which can represent difficult shapes and layers of natural surfaces. Additional facet radar properties, such as an orientation, RCS, and backscattering diagram are used to compute the multi-path reflections and overall reflected signal. Also, the radar system carrier motion is taken into account.

Now the model works in the MATLAB environment; so, it allows us to change parameters of signal processing, edit blocks and redesign the model according to specifications of existing and prospective radar systems. Furthermore, the model
is sufficiently flexible, in other words, each block can be improved and transformed separately by a researcher for different radar and navigation systems. The next step is the following: weakening relations between modules, terrain database fulfillment, and addition various algorithms for digital signal processing.

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