Using of Potential Functions Method for Recognition of Two Channels in Additive Noise

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This article represents results of research of potential functions method in case of discrete signal with additive noise. Noise in transmission channel has Gaussian distribution, i.e. it is a white noise. Besides, noises in the channels have a static interrelation. The main aim of the paper is researching of applicability of the method of potential functions for recognition of the binary signal from it mix with additive noise. Number of researches have been conducted to assess the possibility of using the described method. The results were presented in a series of graphics. Tests were made for different numbers of parameters of the model, which had described in this paper. There was done comparative analysis of working of potential functions and k-nearest neighbors algorithm, which had considered in previous author's works. The results of this research can be used as the base for further research and can be applicable in organization of transport channels.

Keywords—signal-to-noise ratio, computer modeling, statistics, potential functions, correlation coefficient, white noise, probability of recognition

I. Introduction

The paper is the logical continuation of [12] and [13], which are devoted to the challenge of recognition of a discrete signal, when there are additive noises in the channels. In that works we showed the results of analytical working on the model of two binary channels. It had suggested the method of linear filtration for noise compensation. It allowed to increase ratio of signal and noise ratio, besides we had made the set of computed experiments to show the influence of different parameters of the transmitted channels [4, 9, 10]. We had used k-nearest neighbors' algorithm for this aim, and compare it results in different conditions [6, 11]. But there are able to use a number of methods for signal recognition, like parametric (including qualifier of a Bayes and working with the conditional probability densities), nonparametric techniques (like density function assessment, assessment by means of parzen's windows, assessment by means of posterior probabilities, the linear discriminant of the Phisher), wavelet transformation, fast Fourier transform, etc. [1-5, 8, 14, 15].

In this paper we propose potential functions for solving the challenge. Signal is transmitted on two channels, noises in the channels are statistically connected. One channel is contained noise only, the second – noise and useful signal. If the second channel doesn't exist, we can introduce it in some cases.

II. Mathematical Representation of the Channels Model

The main idea of using potential function technique consists of finding a decision function, which can determine the class of every object.

There are two classes of objects, w_0 and w_1 ("0" and "1", respectively). Objects of w_0 have a negative potential, and objects of w_1 - a positive one. The value of a potential is computed by potential of the point charge, which is determined as follows:

$$g(x) = \sum q_i K(x, x_i) \qquad (1)$$

where $K(x, x_i)$ – potential function, it is inversely proportional to value $\| x - x_i \|$; x_i – i-th element of the training sample.

q_i – charge of the i-the elements «ones» and «zeros» $q_1 = +1$, $q_0 = -1$.

For every new element is computed it's potential relatively received earlier:

if $g(x) > 0$, then $x \in w_1$;

if $g(x) < 0$, then $x \in w_0$;

case $g(x) = 0$ is impossible from the point of view of the theory of probability, as $K(x, x_i)$ is continuous random value.

There are most commonly used potential function, which has peak at $x=x_i$ and decreases monotonically to null during $\| x - x_i \| \to \propto$. It's convenient to present potential function as function of distance between entry and other elements of the set.

In this paper the potential function $K(x, x_i)$ is:

$$K(x, x_i) = exp(-a \| x - x_i \|^2) \qquad (2)$$

The parameter $a > 0$ – constant.

Evaluation of the successful recognition of the binary signal in the channel looks like:

$$P^* = m / N, \text{ where} \qquad (3)$$

m – an amount of successfully determined classes of the message;

N – the length of the test set.

There are using model of the channels y_1, y_2, which described, as

$y_1 = a * \eta / 2 + \varepsilon + \xi$; $y_2 = b * \eta / 2 + \delta + h * \xi$,

where ε, ξ, η, ξ – random variables,

η takes the values 1 and -1 with probability $P = 1/2$;

$\varepsilon \sim N(0, \sigma_\varepsilon^2)$ - the noise in the first channel; $\delta \sim N(0, \sigma_\delta^2)$ – the noise in the second channel, ε and δ are statistically interrelated, $r_{\varepsilon,\delta}$ it is correlation coefficient of them;

$\xi \sim N(0, \sigma_\xi^2)$ – common noise in the channels, h – constant;

η and ξ – random values, which are independent from ε and δ, and each other;

ε and δ – natural noises, attending in the channels;

ξ – noise, appearing in the channels with some technical conditions;

$a = |m_1^1 - m_{-1}^1|$ – discrete signal in the channel y_1;

$b = |m_1^2 - m_{-1}^2|$ – discrete signal in the channel y_2;

In this work $N(m,\sigma^2)$ – normal distribution law with parameters (m,σ^2)

Conditional probability densities $P(y_1 \mid \eta = 1)$ и $P(y_1 \mid \eta = -1)$ have a normal law with parameters $N(m_1, \sigma_\varepsilon^2+\sigma_\xi^2)$ и $N(m_{-1}, \sigma_\varepsilon^2+\sigma_\xi^2)$. For y_2 that parameters are defined similarly (so, normal law has parameters $N(m_{\pm 1}, \sigma_\delta^2+\sigma_\xi^2)$).

III. EXPERIMENTS

For investigations of method of potential functions there had been conducted the set of experiments, which allowed to valuate recognition probability of the signal in different cases.

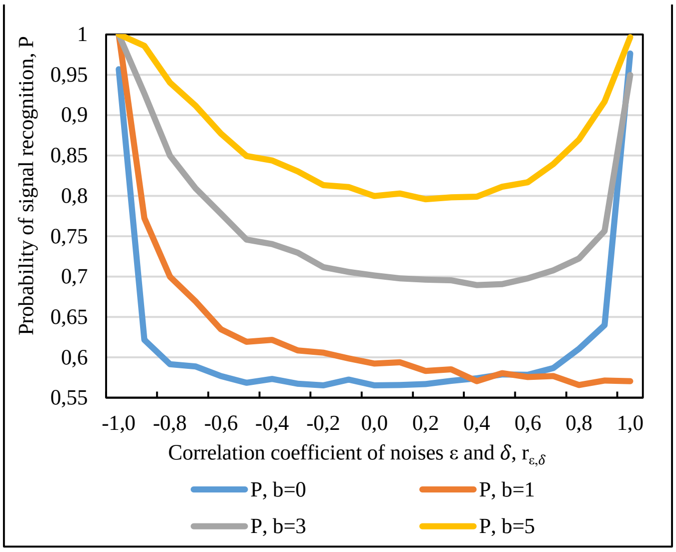

Fig. 1 Dependence of probability P on parameter's value h

Correlation coefficient of the noises ε and δ took values from -1 to 1 in 0.1 steps. Standard deviation of ξ more 10 times than standard deviations of ε and δ. Signal-to-noise ratio [further – ratio] is considerably less, than 1. Matlab 2011a was used for experiments.

The results of the algorithm work in case, where isn't common noise ξ (it's dispersion is equal to zero), are shown if Figure 1. In that case parameter b are changing.

The behavior of the system is predictable in this case, the probability of recognition of the signal grows in the power increasing. Mention must be made of the high level of recognition in case $b = 1$.

As demonstration of the algorithm working are presented results of working for two parameter sets: ($a=1$, $b=1$; $a=2$, $b=1$). Experiments were made for different values of h: $h=0$, $h=|0,5|$, $h=|1|$, $h=|1,5$, $h=|5|$.

Results of case ($a = 1$, $b = 1$) are shown in Fig.2 and Fig.3. It's demonstrates, that the quality of recognition depends on parameter h. Fig.3 shows, that the probability of recognition strives to 1 for correlation coefficient with value -1. This is due to the fact that ratio of the model of the signals is much more, than 1. In some cases in ratio signal grow and noise decreased. When it's happening, you can notice the situation like in the Fig.3, Fig.4, Fig.5.

Results for $a = 1$, $b = 2$ are shown in Fig.4 and Fig.5. Fig.4 shows, that signal recognition significantly improves with $h = 1$. As a whole, we note, that recognition probability with $b=2$ larger, than with $b=1$. You can see, that we found some excludes: when correlation coefficient is going to -1, P→1. It is going because mathematic expression of ratio grows very fast in some point: where fraction signal/noise strives to infinity.

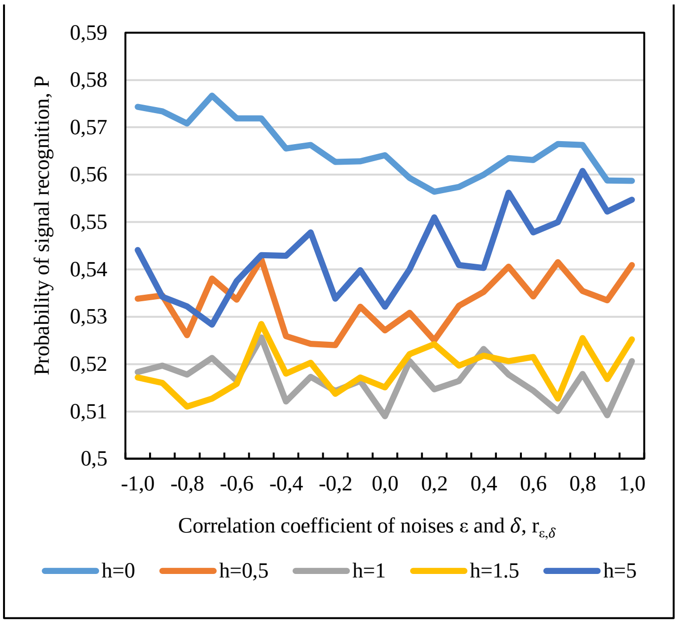

Fig. 2 Signal recognition probability for a=1, b=1, h ≥ 0

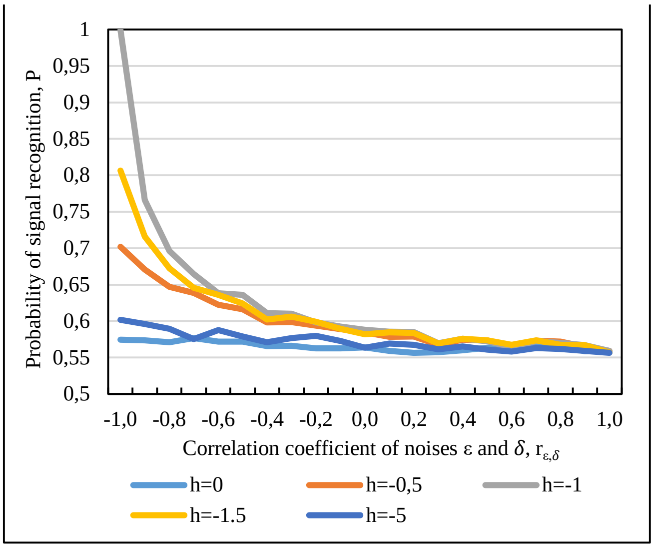

Fig. 3 Signal recognition probability for a=1, b=1, h ≤ 0

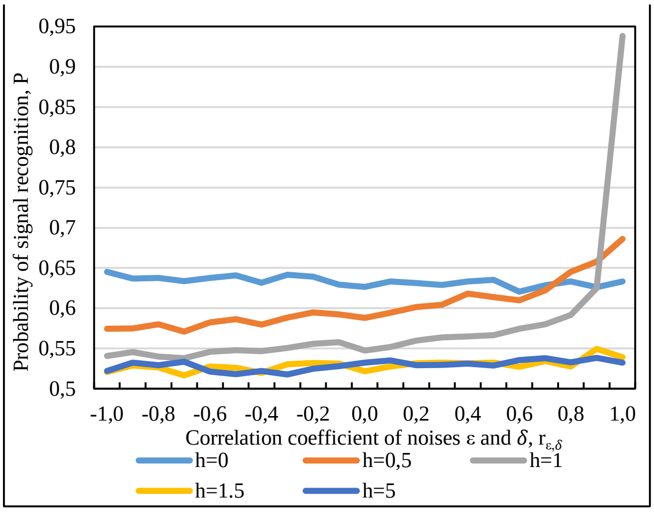

Fig. 4 Signal recognition probability for $a = 1$, $b = 2$, $h \geq 0$

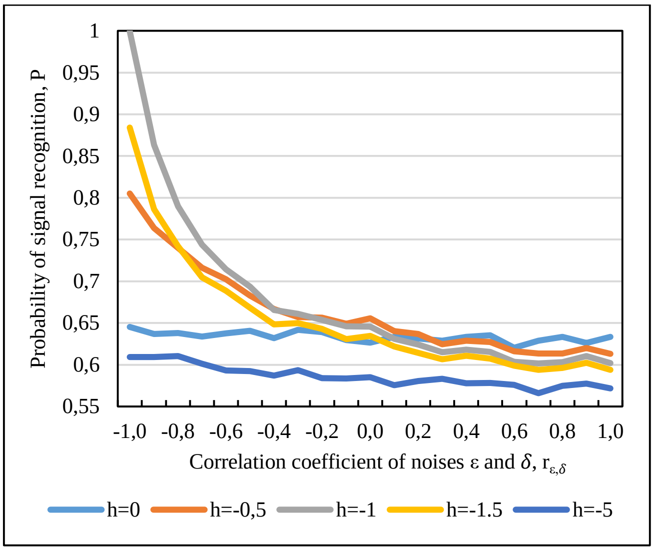

Fig. 5 Signal recognition probability for $a = 1$, $b = 2$, $h \leq 0$

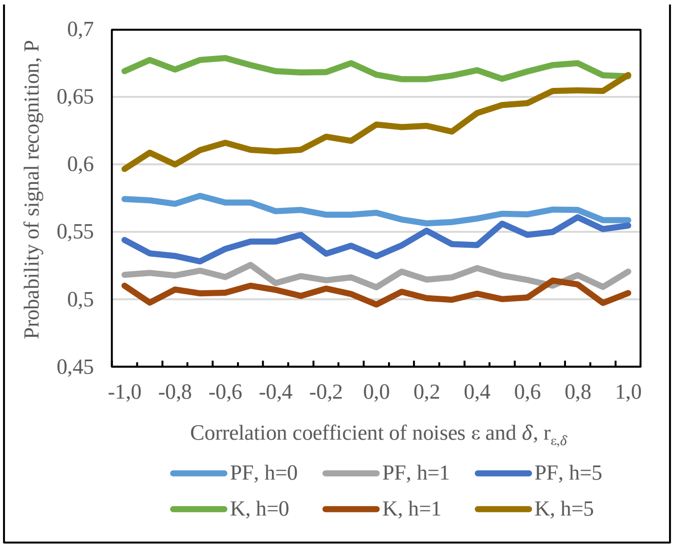

Fig. 6 Comparison of the k-nearest and potential functions in case $a = b$

IV. COMPARISION WITH K-NEAREST NEIGHBORS ALGORITHM

Previous work of authors [12] focused on using of the method of *k*-nearest neighbors algorithm. Compare the results of the computing experiments.

A description of *k*-nearest neighbors is presented further (in the context of the work). There are two classes of elements: w_0 and w_1. Elements with label of the class w_0 are “0”, and with label of w_1 are “1”, respectively. There are generated sample, which size is large enough (in the work the sample’s size is around 10000 [13]). As with potential functions method, we designate x_i as *i*-th element of training set.

A new classified two-dimensional object x comes to the input of the qualifier. The distance $\| x - x_i \|$ between x and elements of the training set is computed. *k* nearest are selected. x will get the label, which had the most of the *k* choosed elements. *k* is always ood in order to avoid collisions.

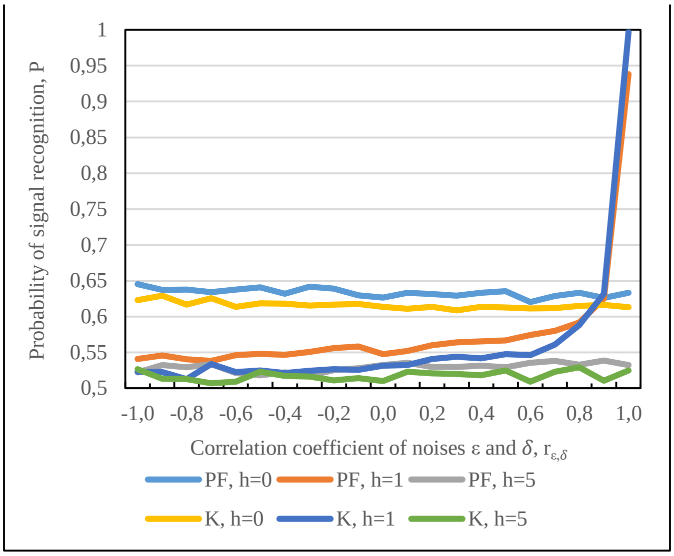

Fig. 7 Comparison of the k-nearest and potential functions in case a=2*b

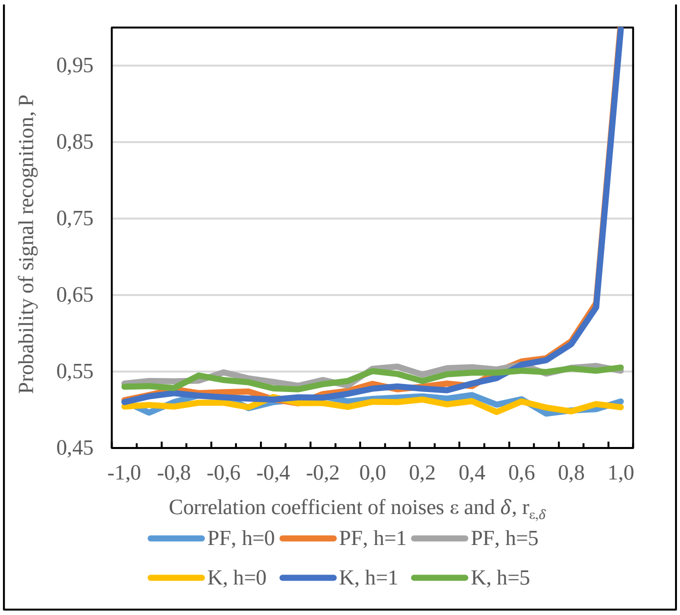

Fig. 8 Comparison of the k-nearest and potential functions in case b=0

So, we compared the method of potential functions and k nearest neighbors' algorithm. As earlier, results are presented as graphs. Size of the training set also are 10 000. On the *x*-axis you have got values of the correlation coefficient $r_{\varepsilon,\delta}$ to within 0.1. On the *y*-axis – the probability of the signal recognition. PF means Potential Functions and K – k nearest neighbors.

We can see, that on the Fig.6, Fig.7, Fig.8 potential function method has a little win in front of the k-nearest method.

V. CONCLUSIONS

In the paper are presented the set of the experiments results, which demonstrate probabilities of using potential function for signal recognition. We considered behavior of the probability of the signal recognition in mathematical model (2) and made the following conclusions. The method can be used in case of two channels. There are opportunity to use it in more channels [7], but it requires additional researches. Besides, we showed there are points, where the recognition probability grows to 1. The reason has been at analytical function of ratio (it had showed in [12] and [13]). In some conditions (it's depends on parameters $a,b,r_{\varepsilon,\delta}$) numerator (signal) in the expression of signal-to-noise grows up and denominator decreases, therefore the fraction goes to infinity. In the second part of the paper we gave a comparison analysis of working potential functions and k nearest neighbors in the context of binary signal recognition. There was investigated three cases: when *a=b=1, b=a*2=2, b=0. h* took values 0, 1,5. It gave us opportunity to see different models of behavior of the recognition. As we can see, potential functions have a little win in compare with k neighbors' algorithm in all presented cases. In whole, potential functions are simple and fast method of classification. It can be used in many various cases. Opportunities of choice of potential functions allow the foundations of the further investigations.

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