Maximum-Profit Domination with Matroid Constraints: A Fixed-Parameter Algorithm and Applications*

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This lecture presents ongoing joint work with Oxana Yu. Tsidulko (Novosibirsk) and Philipp Zschoche (Berlin) on fixed-parameter algorithms for the following generalization of Set Cover and Dominating Set. It models various social network analysis, routing, and facility location problems:

Problem (Max-Profit Domination with Matroid Constraint (MDMC)).
Input: A universe U, for each $u, v \in U$ a profit $p_{uv} \in \mathbb{N} \cup \{0\}$ from dominating u by v, a cost $c_v \in \mathbb{N} \cup \{0, \infty\}$ for using v as dominator, and a matroid (U, I).
Output: Two disjoint sets $D \uplus C \subseteq U$ such that $D \in I$ and that maximize

$$\sum_{u \in C} \max_{v \in D} p_{uv} - \sum_{v \in D} c_v.$$

Informally, we find sets $C, D \subseteq U$ so as to maximize the profit from dominating the elements in C by elements in D minus the cost for the dominators in D.

Example. We obtain the classical problem of covering a maximum number of elements of an universe V using at most k sets of a collection $C \subseteq 2^V$ by choosing $U = V \cup C$, $I = \{C' \subseteq C : |C'| \leq k\}$, and, for each $u, v \in U$,

$$c_v = \begin{cases} 0 & \text{if } v \in C, \\ \infty & \text{if } v \in V, \end{cases} \qquad p_{uv} = \begin{cases} 1 & \text{if } v \in C \text{ such that } u \in v, \\ 0 & \text{otherwise.} \end{cases}$$

We prove the following theorem and present applications to social network analysis, routing, and facility location problems.

Theorem. A maximum-profit solution to MDMC with $|C| = k$ is computable in $2^{O(k)} \cdot \text{poly}(n)$ time, i. e. in poly-time for $k \in O(\log n)$, where n is the input size.

Herein, the universe U, costs c_v and profits p_{uv} for each $u, v \in U$ are given explicitly in the input, whereas the matroid (U, I) is given as an oracle that, in constant time, answers whether a subset of U belongs to I.

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