Design of Complex Products with Regard to Coloristics Based on Discrete Optimization Problems *

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Abstract. In this work, development and study of models of discrete optimization with logical, resource and other constraints to solve the problems of design of complex products are continued. Special attention is paid to the questions of selection of coloristic solutions in the process of complex products design. It allows improving visual variety for consumers without significant production costs. A mathematical model for the distribution of colors to the details of complex products based on the theory of color and the satisfiability problem is proposed. The results of computational experiments which reflect the prospects of further development of the considered approach are presented.

Keywords: Discrete optimization · Mathematical models · Logical constrains · Complex products · Outline design · Coloristics

1 Introduction

In many decision-making problems, related to design, planning and management, logical, resource and other constraints are used. Logical and resource constraints can lead to the partial maximum satisfiability problem. This problem is a generalization of the well known satisfiability problem (SAT), that is one of the central problems in the complexity theory. It is known that these problems are

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NP-hard. The considered problems with the specified constraints can be used to solve a variety of applied problems in many fields. For example, studying and solution of the problem of outline design of clothes that are formed of a set of components [3, 4, 6, 12], design of some technical devices, formulations of rubbers for special purposes [2], the problem of the formation of production groups with regard to interpersonal relations [11], the problem of logical cryptanalysis [14] and others.

Currently, design of complex products has received much attention due to the relevance of this direction. The majority of the problems are solved on the base of different computer systems, for example [15]. However, in most cases these systems do not make much use of the capabilities of the mathematical apparatus. To solve these problems, it is rational to use in particular models and methods of discrete optimization, as well as the development of special algorithms.

In the paper, the development of the described in [3, 4, 10, 12] approach and study of corresponding mathematical models are being continued. The possibilities of the use of the mentioned approach to automation of outline design of complex objects, on the example of some assortment groups of the consumer goods industry, are studied. These groups are distinguished by a great variety and filling. Much attention is paid to the questions of the design of complex products with due regard to coloristic solutions which allow broading the variety of the produced products. During manual design, an expert must search through and compare a large number of variants of color combinations for components and elements. This way, in some cases, not all prospective and interesting variants can be considered, and selected ones are not always optimal. The use of mathematical apparatus and partial automation of the creative process can give the specialist opportunities to meet various demands (economical, resource conditions, the theory of coloristics, fashion trends, etc.) in the optimum way. Therefore, the development of the research in this direction is actual and greatly requested.

2 Problems Formulation and Mathematical Models

We begin with introducing logical variables $x_1, \ldots, x_n$ that can take the values *true* or *false*. Consider the propositional formula $F = C_1 \wedge \ldots \wedge C_m$ where each clause C_i is a disjunction of literals, and each literal is either a variable x_j (that is, a *positive literal*) or its negation $\bar{x}_j$ (that is, a *negative literal*). In the SAT problem, a truth assignment for variables is sought that makes the formula true.

Suppose that every clause C_i has a nonnegative weight c_i. The MAX-SAT is the problem of finding an assignment to the variables that maximizes the weight of the satisfied clauses.

In addition, the practical importance is the problem where a variable assignment is required to satisfy all "hard" clauses and to maximize the total weight of "soft" clauses in a boolean formula (a partial MAX-SAT).

There are models of integer linear programming based on the SAT and the MAX SAT problem. On the basis of these models and the method of regular partition, theoretical research of the problems with logical constraints was conducted [1, 8, 9]. Design and analysis of algorithms of the search of exact and approximate solutions were carried out. The results are used in various applications.

Let us consider the setting of the problem of complex products outline design and the relevant mathematical model with the use of the logical, resource and other constraints. In addition, the partition of elements into the groups of components is taken into account. It can adequately describe the problem situation and can be used for the development of algorithms solving the problem [10, 12]. To formulate the problem, we introduce the following notation:

J – the set of numbers of components of the product, $J = \{1, \ldots, n\}$;

G – the set of groups of components and characteristics;

α – the number of a group of the components and characteristics, $\alpha \in G$;

J_α – the numbers of elements in group α;

v_j^α – the component with number j from group α;

x_j^α – the logical variable that takes the value *true* if v_j^α is included in the product and *false* otherwise;

I – the set of numbers of the logical formulae used in the problem, $I = \{1, \ldots, m\}$;

I_0 – the set of the logical formulae that bind variables from different groups;

C_i – the logical formula corresponding to the i-th logical constraint, $i \in I_0$, which is a disjunction of variables x_j^α and/or their negations. It should be noted that the formulae with numbers from $I'_0 \subseteq I_0$ must be satisfied;

d_i – the weight of formula C_i that defines the significance of satisfying this formula, $i \in I_0 \backslash I'_0$;

$\tilde{C}_{\alpha r}$ – the logical formula corresponding to the r-th logical constraint and bind variables from group α, $r \in I_\alpha$. Equations with numbers from $I'_\alpha \subseteq I_\alpha$ must be satisfied;

d_r^α – the weight of formula $\tilde{C}_{\alpha r}$ that defines the significance of satisfying this formula, $r \in I_\alpha \backslash I'_\alpha$;

The problem is to find the values of the logical variables which satisfy logical formulae C_i with numbers $i \in I'_0$ and logical formulae $\tilde{C}_{\alpha r}$, $r \in I'_\alpha$, $\alpha \in G$. The total number of the satisfied formulae C_i, $i \in I_0 \backslash I'_0$, and formulae $\tilde{C}_{\alpha r}$, $r \in I_\alpha \backslash I'_\alpha$, $\alpha \in G$ is maximized.

Let us assume that the boolean variable y_j^α takes 1 if a corresponding element is included in the product ($x_j^\alpha = true$) and it takes 0 otherwise, $j \in J$, $\alpha \in G$. By the analogy of the ILP model for the partial MAX-SAT problem it is possible to build a model of the considered design problem:

$$f(z) = \sum_{i \in I_0 \backslash I'_0} d_i z_i + \sum_{\alpha \in G} \sum_{r \in I_\alpha \backslash I'_\alpha} d_r^\alpha z_r^\alpha \to \max \qquad (1)$$

$$\sum_{\alpha \in G} \left(\sum_{j \in C^-_{\alpha i}} y^\alpha_j - \sum_{j \in C^+_{\alpha i}} y^\alpha_j \right) \leq |C^-_i| - 1, \; i \in I'_0; \tag{2}$$

$$\sum_{\alpha \in G} \left(\sum_{j \in C^-_{\alpha i}} y^\alpha_j - \sum_{j \in C^+_{\alpha i}} y^\alpha_j \right) + z_i \leq |C^-_i|, \; i \in I_0 \backslash I'_0; \tag{3}$$

$$\sum_{j \in \tilde{C}^-_{\alpha r}} y^\alpha_j - \sum_{j \in \tilde{C}^+_{\alpha r}} y^\alpha_j \leq |\tilde{C}^-_{\alpha r}| - 1, \; r \in I'_\alpha, \; \alpha \in G; \tag{4}$$

$$\sum_{j \in \tilde{C}^-_{\alpha r}} y^\alpha_j - \sum_{j \in \tilde{C}^+_{\alpha r}} y^\alpha_j + z^\alpha_r \leq |\tilde{C}^-_{\alpha r}|, \; r \in I_\alpha \backslash I'_\alpha, \; \alpha \in G; \tag{5}$$

$$0 \leq y^\alpha_j \leq 1, \; y^\alpha_j \in Z, \; j \in J_\alpha; \tag{6}$$

$$0 \leq z_i \leq 1, \; z_i \in Z, \; i \in I_0 \backslash I'_0; \tag{7}$$

$$0 \leq z^\alpha_r \leq 1, \; z^\alpha_r \in Z, \; r \in I_\alpha \backslash I'_\alpha, \; \alpha \in G. \tag{8}$$

Here C^-_i and C^+_i ($C^-_{\alpha i}$ and $C^+_{\alpha i}$ respectively) are the indexes sets of the negative and positive literals in clause C_i ($C_{\alpha i}$). Note that if z_i (z^α_r) is equal to one in a feasible solution of the problem, then clause C_i ($\tilde{C}_{\alpha r}$) is satisfied. The optimal value of the objective function is the total weight of satisfied clauses. In the objective function (1), the sum of weights of the "soft" logical constraints is maximized by the relating variables belonging to different groups of components, and variables of individual groups. Inequalities (2) correspond to the "hard" logical constraints binding the variables among the groups. Inequalities (3) correspond to the "soft" constraints binding the variables among the groups. Constraints (4) and (5) are similar to constraints (2) and (3) but bind variables within one group,(6) – (8) are the conditions on the boolean variables.

A feasible solution of problem (1) – (8) determines a variant of a product satisfying the above-mentioned conditions. Note that the designer is able to correct the previously formulated constraints. There may be several optimal solutions of this problem, so the specialist can choose some of them, taking into account his or her preferences.

One of the effective ways to improve the competitiveness of production is to design not individual products but several models that are connected by a common group of components ("kernel" of series), with the possibility of varying and interchanging other components and elements that is based on the use of models and methods of discrete optimization [4, 12]. Previously, some "kernels" to design series of models of the dress-blouse assortment of women's clothes have been built by the authors. To create them, a number of "hard" logical constraints that define a fixed set of elements and form the "kernel" of the series were distinguished. Other "hard" and "soft" logical constraints create a variety of models. The corresponding computational experiments have been carried out.

Another perspective direction of the development of the approach is the creation of outfits [4], i.e., the sets of clothes that consist of items which belong to two or more various assortment groups interconnected by the unity of style, shape, and the proportional ratio of elements, coherence of articulations, a combination of trimmings and materials, color scheme, etc.

3 On Finding Coloristic Solutions

Coloristic theory plays an important role in the process of design of a visual variety of garments [7, 13]. To find the optimal color decision on the basis of several criteria, it is possible to construct and use various restrictions, including logical ones. As a result, we obtain a technical outline with a selection of color-grades of materials for its production, which satisfy fashion trends, the theory of colors combination and the requests of the designer.

The construction of mathematical models for the problem of finding color solutions is based on the recommendations of the theory of costume design, taking into account preferred color combinations. The selection of harmonious color-grades, that give the impression of color entirety and the relationship between colors of details, is important.

We describe the scheme for finding solutions to the problem of selection of color-grades for sewn products. In the first stage, an optimal solution is found on the basis of the initial mathematical model (without taking color into account). Next, a model of integer linear programming is constructed on the basis of the maximum satisfiability problem. The purpose is to determine the set of colors (palettes) used for colouring technical outlines. The designer can choose a color solution for single or serial production, as well as specify a certain color range, which can also be applied for outfits. In the last stage, a transition to the distribution of selected colors among the details of a garment is made. It takes into account the dimensions of the specific elements to obtain a harmonious combination of proportions, according to the principle of the golden ratio [7]. The corresponding system of inequalities is constructed based on the ratio of areas. Also it is possible to arrange the priority of the use of available remainders of fabrics of specific colors.

To research the problem of finding the optimum coloristic solution from the point of view of several criteria, we use the twelve-part color wheel, the fragment of which is presented in Figure 1. The color wheel is an important foundation for any aesthetic theory of color, since it gives the system of color arrangement and allows us to understand clearly the schemes of harmonious combinations.

The considered scheme of division of a color circle allows classifying colors into groups and harmonious combinations formed [7, 13]:

1) Related colors;
2) Relatively-contrasting colors;
3) Contrast colors.

We consider combinations of related-contrast colors. The main combinations of the colors (see Figure 1) are the pairs of complementary colors (diametrically opposed colors), all combinations of three and four colors in the twelve-colored wheel that are connected to each other through equilateral or isosceles triangles, squares and rectangles.

We now turn to the construction of mathematical models for the problem of finding color solutions for the obtained outlines, based on the problem of aumomation of outline design described, for example, in [12]. We construct a new mathematical model. We introduce the following notations:

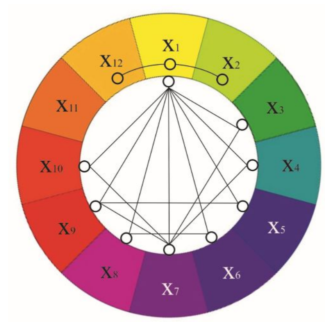

Fig. 1. Fragment of the scheme of harmonious color combinations according to Johannes

J – the set of color numbers, $J = \{1, ..., n\}$;

v_j – the color with number $j \in J$;

x_j – a boolean variable that takes the value of true if v_j is a part of the palette and false otherwise;

s_j – the weight of the color according to $v_j,\ j \in J$;

$p_1,\ p_2$ – the lower and upper bounds for the total number of colors included in the product;

I – the set of logical formula numbers used in the model;

C_i – the logical formula corresponding to the i-th constraint, which is the disjunction of variables and/or their negations;

The task is to find the values of the logical variables that limit the total number of colors included in the product, and formulas $C_i,\ i \in I$ are satisfied, and the weight of the colors included in the gamut will be maximum.

Similar to the previous model we denote by $y_1, ..., y_n$ boolean variables, that y_j corresponds to literal x_j, and $(1 - y_j)$ corresponds to literal $\bar{x}_j$, $j = \{1, ..., n\}$. The problem of integer linear programming for the case under consideration is as follows:

$$g(y) = \sum_{j \in J} s_j y_j \to \max \tag{9}$$

$$\sum_{j \in C_i^-} y_j - \sum_{j \in C_i^+} y_j \leq 1 - |C_i^-|,\ i \in I, \tag{10}$$

$$p_1 \leq \sum_{j \in J} y_j \leq p_2, \tag{11}$$

$$0 \leq y_j \leq 1,\ y_j \in Z,\ j \in J, \tag{12}$$

$$0 \leq z_i \leq 1,\ z_i \in Z,\ i \in I. \tag{13}$$

All the conditions of a harmonious combination of colors (10) considered in the problem are constraints of a hard type. Condition (11) determines the possible number of colors in the found gamma.

In Figure 1 the vertices of the graph correspond to the colors of the circle, and arc (x_1, x_7) and subgraphs (x_1, x_5, x_9), (x_1, x_6, x_8), (x_1, x_3, x_7, x_9), (x_1, x_4, x_7, x_{10}) correspond to the logical constraints that describe harmonious combinations of colors. The part of the system of logical constraints, for example, for color x_1, can be presented the following way:

1) If "yellow" is selected, "purple" must be added:

$$x_1 \rightarrow x_7.$$

2) If "yellow" is selected, "blue" or "red" must be added:

$$x_1 \rightarrow (x_5 \vee x_9).$$

3) If "yellow" is selected, "blue-violet" or "violet-red" must be added:

$$x_1 \rightarrow (x_8 \vee x_8).$$

4) If "yellow" is selected, "green", "purple" or "orange" must be added:

$$x_1 \rightarrow (x_3 \vee x_7 \vee x_9).$$

5) If "yellow" is selected, "blue-green", "purple" or "red-orange" must be added:

$$x_1 \rightarrow (x_4 \vee x_7 \vee x_{10}).$$

6) If "yellow" is selected, "orange-yellow" or "yellow-green" must be added:

$$x_1 \rightarrow (x_2 \vee x_{12}).$$

Similarly, restrictions for all the remaining schemes and colors of the circle are made. As the result of solving these problems, the user will receive a set of color scales that satisfy the set conditions.

After that, the transition to the distribution of the selected colors among details of a garment is carried out. It takes into account their area. The corresponding mathematical model is as follows:

To describe the model of the problem, we introduce the following notation similar to the previous model:

J – the set of numbers of components of the product;

P – the set of numbers of colors;

v_j^p – the component with number j of color p, j, $j \in J$, $p \in P$;

s_j^p – the weight of component v_j^p (the square);

A^p – the volume of the resource p (the square of material of color p), $p \in P$;

Let $p = 1$ is the main color, which is determined by the designer.

$$h(y) = \sum_{j \in J} s_j^1 y_j^1 \to \max \tag{14}$$

$$\sum_{j \in J} s_j^1 y_j^1 \le 2 \sum_{j \in J} \sum_{p \in P \backslash \{1\}} s_j^p y_j^p; \tag{15}$$

$$\sum_{j \in C_i^-} y_j - \sum_{j \in C_i^+} y_j \le 1 - |C_i^-|,\ i \in I,\ p \in P; \tag{16}$$

$$\sum_{j \in J} s_j^p y_j^p \le A^p,\ p \in P; \tag{17}$$

$$\sum_{p \in P} y_j^p = 1,\ j \in J; \tag{18}$$

$$0 \le y_j^p \le 1,\ y_j^p \in Z,\ j \in J,\ p \in P. \tag{19}$$

4 On Development of Algorithms for Searching Exact and Approximate Solutions

Currently, some special algorithms to find exact and approximate solutions of the investigated problems to create series of products based on one "kernel" are being developed.

For the analysis and solution of the problems of ILP, the regular partition method was previously proposed, which was successfully used for various problems of discrete optimization [8, 9]. The L-partition is the most studied among the partitions. On the basis of this approach, algorithms for solving the satisfiability and maximum satisfiability problems were proposed. In this paper, to solve problem (1)-(8) we develop an algorithm based on the search for the L-classes, which makes it possible to find a series of products based on a set of optimal solutions or close to optimal ones. Consider the scheme of this algorithm, based on the combination of the algorithm for the L-class enumeration and the package of applied programs GAMS.

The algorithm for constructing a series of complex products (the problem determined by conditions (1)-(8)):

Step 0. We solve the problem of satisfiability (2), (4) for "hard" constraints with the help of the algorithm for the L-class enumeration of LCE. If the formula is feasible, we get a feasible solution y and go to step 1. Otherwise the algorithm completes the work, the formula is unsatisfiability.

Step 1. We substitute the found admissible solution into constraints (3), (5). We formulate the maximum satisfiability problem. If $z_i = 1$, we exclude the corresponding restriction $C_i,\ i \in I \backslash I'$, from the formula. We solve this problem using the package GAMS. If the executing set y^* is found, it and the value of the objective function are fixed. Go to step 2. Otherwise, immediately move on to the next step.

Step 2. We use the algorithm for of the L-class enumeration to find the following admissible solution in the order of lexicographic descending. The following cases are possible:

a) The requested solution is found. Go back to step 1.

b) There are no admissible solutions or there are no solutions at all. All the received sets y^* are collected, if any, and go to step 3.

Step 3. If there is no admissible solution y^* that satisfy constraints (3), (5), then it is unsolvable. Otherwise, we obtain a set of solutions of the original problem. The algorithm completes the work in two cases: if one or more solutions to the problem are obtained, or the solution to this problem cannot be found. In the future, when constructing a series of products, the designer can use different criteria for selecting the solutions obtained. For example, choose variants for which the values of the objective function (1) are maximal, or deviate from the optimum by no more than a certain predetermined value. In the case of an approximate solution of the problem, the maximum permissible deviation from the optimal solution (by the value of the objective function) is taken into account.

The proposed algorithm is implemented in Visual Studio C++ environment. With the purpose of approbation experimental studies on the class of problems of designing series of products of the dress-blouse assortment were carried out. The task with real initial data was taken. Its solutions resulted in series of sewn products of the dress-bloise assortment [3].

5 Computational Experiments

The computational experiments were divided into the following stages:

1) Finding variants of outlines (optimal solutions of problem (1) - (8)) without taking coloristics into account;

2) Selecting the dominant color (by varying the weights), searching for optimal color combinations using model (9)-(13);

3) For the variants found, model (14) - (19) is used to find the proportional ratio of color spots for one product.

At all stages, the algorithm developed by the authors for finding exact and approximate solutions was also used. For the experiments, the task with real initial data for the design of women's casual dresses was taken. It contained 30 variables, 55 "hard" and 6 "soft" constraints. For the experiments, the weights of the elements and the value of the designer's constraint were varied. The weights of the corresponding variables were changed to select the dominant color. In the third stage, variants of the sets of the colors were obtained for the outlines, some of which are shown in Figure 2.

As it can be seen in Figure 2, in case of applying various color solutions to the products created on the basis of one kernel, specialists can significantly increase visual diversity of finished products at minimal costs. The results of the

Fig. 2. Fragment of computational experiments

experiments confirmed the prospects of applying the developed approach for obtaining a series of various solutions including taking coloristics into account. The authors plan to continue developing this theme in the direction of considering lightness, saturation, temperature, chromatic colors, as well as the description of new combinations schemes, including achromatic colors.

6 Conclusion

In the work, the development and research of the integer linear programming models based on the SAT and the MAX-SAT problems for the complex products design, including consumer goods industry, are continued. The special attention is paid to the creation of complex products on the base of the theory of coloristics. Corresponding mathematical models for automation of outline design of complex products are offered.

The developed algorithm for searching exact and approximate solutions has been proposed. The computational experiments with real input data have been carried out and show the prospects of further development of the appoach.

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