On a Problem of the Utility Network Design ⋆

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Abstract. An adoptable location of utility network elements for processing and distribution of resources between consumers is a complex technical and economic problem. For its solution, it is necessary to take into account both the resource constraints as well as a plan of processing and distributing the resources among consumers, thus providing an efficient scheme for the functioning of a network to be designed. In this study the problem of the utility network design is treated by the hypernet approach according to the compatibility of different types of resources with allowance for their laying in the same track. Also, we study the reliability aspect of the designed utility network for obtaining the optimal laying of a reliable enough utility network on a given area with minimal costs. The performed numerical experiments show how the method proposed works.

Keywords: Multilayer network · Utility network · Deployment area · Track · Graph · Hypergraph · Hypernet · Connectivity · Network reliability

1 Introduction

Utility network is a composite geographically distributed system that performs such vital functions as providing consumers with energy and water resources, communication facilities, information, route network, traffic, and other services. It includes local authorities, economic objects and consumers, and, also, considers the nature of the relationship between them.

Currently, there is a significant number of published works related to design and construction of networks for various purposes [8, 2, 14, 16, 10, 9, 26, 28, 3, 13, 11, 22, 7]. In particular, in [14, 16] the authors consider the search for the optimal routes for laying of the utility networks. The task of locating the construction objects on a given territory is solved in [10, 9]. In [26, 28] the possibility

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of GIS-technologies applying in tracing and placing of linear objects is studied. An application of splines in the optimization of the car roads tracing is treated in [3]. Moreover, problems of the utility networks optimization also arise in other adjacent areas, such as the placement of logistics facilities [13] and power supply systems [11], design and reconstruction of transport networks [22], and the routing of service networks [7].

In all the above mentioned papers, the utility network is considered to be a two-dimensional object located on a plane. However, such a two-dimensional representation of networks does not completely correspond to reality, since as the basis of the design and construction of the utility network underlies the interaction of, at least, two interrelated objects. Unlike its predecessors, in this work a utility network is treated as a hierarchical system, in which resources (gas, oil, water, etc.) are transferred from producers to consumers through the points of processing via communication (transport channels). Such systems are characterized by the existence of the so-called fixed costs, which must be done regardless of the volume of production and processing of products.

For the simulation of various network objects, graphs and hypergraphs are commonly used. As was mentioned above, in many areas of applications there is a bunch of interconnected networks, forming a hierarchical structure. In such cases it is completely inconvenient to use graphs and hypergraphs. Instead of them, other network models are used [8, 2, 21, 12, 19, 29, 1]. For instance, there are such mathematical objects as nested graphs [21], layered complex networks [12], hypernets [19, 29, 1].

For the problems of a utility network design we use the hypernet approach, which makes it possible to operate with hierarchical, multilevel, and heterogeneous networks. More specifically, we study a simple case of hypernet — the two-level network. It consists of a primary network, which represents a physical network, and a secondary network, which represents a logical network, embedded into a primary network. Our previous results on the hypernet using for the utility network design are presented in [30, 31]. A of nodes of a secondary network is a subset of nodes of a primary network, whereas each secondary network edge is a chain in a primary network, i.e. a sequence of the adjacent edges. For the sake of definiteness, we call edges of a primary network as branches. Note that each branch can be included into several edges.

Based on the hypernet approach, we solve the task of providing the connectivity of given consumers with given sources on a given area with minimization of laying and maintenance costs and within given constraints. As a result, we propose a new method for the selection of routes for the laying of the utility networks taking into account the compatibility of different types of resources for placing them in the same track. For example, in one collector it is possible to lay in one direction the heat communications, pressure water pipes and sewerage systems, over ten communication cables and power cables with voltage up to 10 kV. However, the joint laying of gas pipelines and pipelines with combustible substances is not permitted due to the fact that the distance between communications of different purposes is normalized according to building codes and regulations [4].

Also, we introduce an algorithm for obtaining not only the cheapest solution, but reliable enough as well. Note that reliability analysis of hypernets is not a new task, see, for example [25, 24]. As a reliability measure, the hypernet analogue of the 2-terminal reliability is considered, under assumption that failures occur in a primary network, and nodes in a secondary network should be connected. In this case we design a utility network, in which each consumer should be connected with the necessary suppliers with probability no less than a given reliability threshold $0 < R_0 \leq 1$. For solving this problem, we introduce the ant colony algorithm. Unlike the existing optimization techniques, the procedure proposed makes it possible to obtain the variant of the utility network placement in a given area, which would be appropriate in terms of reliability and minimum total costs.

2 Definitions and Notations

We use the classical definitions from the hypernet theory [19]:

Definition: hypernet is represented by the four sets: $HN = (X, V, R, F)$, where:
$X = \{x_1, x_2, ..., x_n\}$ – a set of nodes;
$V = \{v_1, v_2, ..., v_g\} \subseteq X \times X$ – a set of branches;
$R = \{r_1, r_2, ..., r_m\}$ – a set of edges;
$F : R \to 2^V$ – a mapping associating each item $r \in R$ with a path $F(r) \subseteq V$.

These four sets determine the two graphs:
$PN = (X, V)$ – a graph of a primary network;
$WN = (X, R)$ – a graph of a secondary network.

It is assumed that for the utility network laying, the following objects are given:
D – s two-dimensional placement area;
$Y_{source} \subseteq D$ – a set of point objects, which represents sources;
$Y_{consumer} \subseteq D$ – a set of point objects, which represents consumers;
$Y = Y_{source} \cup Y_{consumer}$ – a set of point objects to be connected by linear facilities of various assignment.

Let us describe the utility network in terms of the hypernet theory.

In a primary network $PN = (X, V)$ we assume that:
$\rho(v)$ – the length of $v \in V$;
$a(v)$ – the land cost (rent, taxes, etc.) of $v \in V$;
$b(v)$ – the cost of constructing (excavation) of $v \in V$;
γ_1 – a discount factor of building costs (this factor is for bringing economic indicators of different years to comparable values).

In a secondary network $WN = (Y, R)$ we assume the following:
$\rho(r) = \sum_{v \in F(r)} \rho(v)$ – the length of $r \in R$;

$c(r)$ – the cost of $r \in R$, including its installation and laying between the corresponding elements of D;
γ_2 – a discount factor of equipment costs;
$d_v(r)$ – maintenance costs of linear facilities on plots $v \in F(r)$ for a chosen edge (route) r;
T – a set of all types of utility resources. Thus, each edge r has the type $type(r) \in T$.

As was said above, different resource types may be incompatible with each other for their laying in the same track. For the description of the compatibility of different types of resources, we introduce the following notation:

A binary relation $CT \in T \times T$ is defined by the rule: if $(t_1, t_2) \in CT$, then these types of resources can be placed in the same track, i.e. both of them can include the same branch.

Let $MinCT(t_1, \ldots, t_h)$ be the minimum number of disjoint subsets into which a subset of types $\{t_1, \ldots, t_h\}$ can be divided.

For example, if there are types $\{t_1, t_2, t_3\}$ such that $(t_1, t_2), (t_2, t_3) \in CT$, but $(t_1, t_3) \notin CT$, then $MinCT(t_1, t_2, t_3) = 2$, since these types can be divided into the subsets $\{t_1, t_2\}$ and $\{t_3\}$.

3 Problem Statement

In the general case, the task of a utility network design can be formulated as a problem of providing the connectivity of given objects from $Y = Y_{source} \cup Y_{consumer}$ with minimization of the laying and maintenance costs. The obtained configuration determines the topology of the designed utility network.

As we said above, we use the hypernet approach, so the network structure is separated into a primary and a secondary networks. As a primary network, we consider a discrete analogue of the placement area D for the utility network. As a secondary network, we consider routes through branches for the laying of various utility systems.

The problem is to find the hypernet HN, i.e. embed each path $r \in R$ in WN into the edges of the graph PN in such a way that all conditions and restrictions imposed on the utility network be met, and the functional below would take the minimum value:

$$Q(HN) = \sum_{v \in V'} \Big(a(v) + b(v) \cdot \gamma_1\Big)\rho(v) \cdot MinCT(v) + \\ + \sum_{r \in R} \Big(c(r) + \sum_{v \in F(r)} d_v(r) \cdot \gamma_2\Big)\rho(r), \quad (1)$$

where $v \in V' \subseteq V$ if $\exists r \in R$ such that $v \in F(r)$. Let $v \in V'$ and $v \in F(r_i)$, $i = \overline{1, h}$ $(r_1, \ldots, r_h \in R)$, then $MinCT(v) = MinCT(type(r_1), \ldots, type(r_h))$.

4 The Two-Stage Algorithm

This Section proposes an algorithm for obtaining an approximate solution of the problem stated. The basic idea of our algorithm is to find an initial estimation of HN, its total cost (1), and to improve it further. The initial estimation is found by the "greedy" algorithm or the Floyd algorithm, which is commonly used for finding the shortest paths.

The cost of $v \in V$ is $(a(v) + b(v) * \gamma 1 + c(r) * \gamma 2 + d_v(r)) * \rho(v)$ (for the sake of simplicity $c(r) * \gamma 2 + d_v(r) = const$, $\forall r$) for the Floyd and the Dijkstra algorithms in the primary network PN.

Below we outline the steps of the algorithm:

Algorithm Stage 1 (Greedy)

Divide a set of types into incompatible subsets.
repeat
  Choose a subset of types T' with the maximum number of edges (return the values $a(v)$, $b(v)$ for all $v \in F(r)$ in the graph PN).
  repeat
    Find all the shortest paths (x_i, x_j) $i, j = \overline{1, n}$, $i \neq j$ in the graph $PS = (X, V)$ by the Floyd algorithm ($type(x_i, x_j) \in T'$).
    Choose the minimal value from a set of the shortest paths, where $(x_i, x_j) \in R$ in the graph WN (the path (x_i, x_j) is not assigned for any $r \in R$).
    Assign the edge $r \in R$ to the shortest path (x_i, x_j) in the graph PN.
    $a(v) := 0$, $b(v) := 0$ for all $v \in F(r)$ in the graph PN (the costs of branches are zero for assigning a path).
  until $\forall r \in R$ in the graph WN, there is an assigned path (x_i, x_j) in the graph PN.
until $T' \neq \emptyset$.

The greedy algorithm is approximate, therefore the solutions obtained are not always optimal. We can improve it if for an edge $r \in R$ we reassign new paths $F(r)$. For example, a new path $F(r)$ is cheaper if it includes a greater number of branches $v \in F(r)$ with zero cost (branches that have already been used for other edges $r \in R$).

We can order the edges by a certain technique. Earlier we have considered a few techniques [31], for example, ordering by the inclusion of the most rarely used branches. However, on the average, the results obtained did not strongly depend on the ordering method used.

We can use some iterations of the Stage 2 algorithm for finding better results.

5 Results of Numerical Experiments

For numerical experiments we have chosen the 10x10 grid as a primary network PN. The number of edges in WN for laying in PN varied from 10 up to 4000

Algorithm Stage 2 (Improvement)

Order $r_i \in R$ by decreasing the costs (renumber them) and obtain the new list $\{r_i\}$.
repeat
"Delete" r_i from the graph HS according to the list (i.e. $\forall v_k \in F(r)$, restore for $a(v_k)$ and $b(v_k)$ the initial values if v_k is not included into other edges).
Find a new shortest path for r_i in the graph PN by the Dijkstra algorithm. Assign it for r_i.
$a(v) := 0$, $b(v) := 0$ for all $v \in F(r_i)$ in the graph PN.
until All edges $\{r_i\}$ from the list are updated.

(the abscissae axis). For a chosen number of edges n_{edges}, we randomly place in the grid n_{edges} sources and n_{edges} consumers. As a result, a grid node can contain more than one object: source(s) and consumer(s). The random placement is carried out 10 times for a given n_{edges} value. As a laying cost for n_{edges} we take the average among the 10 values found.

We consider the set of resource types which has been mentioned as the example in the Section 2. Thus, we assume that there three different types of edges.

In Figure 1 the normalized costs are shown (i.e. we find minimal cost and divide all the obtained values by it). The results show that the Greedy algorithm is more convenient for a small number of edges; otherwise the Floyd algorithm gives a better solution.

In Figure 2 we show the results of the previous numerical experiments for the case of compatibility of all resources type for laying in same track [31]. Performing Improvement stage with different orders of branches leads us to techniques Improve1 and Improve2. The results are very similar with the numerical experiment 1.

6 Utility Network Laying with Reliability Constraint

In this Section, we improve the algorithm proposed in order to obtain not only the cheapest, but also reliable enough solution. Most of the results presented coincide with those from our previous publication [30].

Random graphs are commonly used for modeling the networks whose elements are subject to random failures [5, 15, 17, 18, 27, 23, 20]. As a rule, the network reliability is defined as some connectivity measure. The most common reliability measure of such networks is the probability that all terminal nodes in a network can keep being connected together, given the reliability of each network node and edge. The problem of calculation of the network probabilistic connectivity is known to be NP-hard [5]. Nevertheless, it is possible to do the exact calculation of reliability for networks with a dimension of practical interest by taking into consideration some special features of real network structures and based on modern supercomputers [15, 17, 18].

Another well-accepted network reliability measure is the 2-terminal probabilistic connectivity [27]. The average pairwise connectivity [23, 20] is the aver-

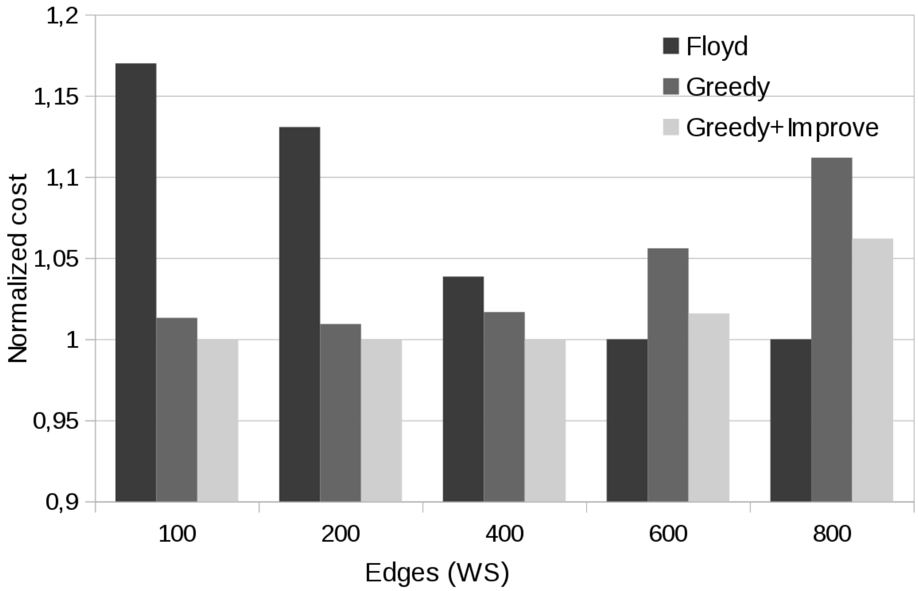

Fig. 1. Numerical results: the normalized costs for the 10x10 grid, the number of edges from 10 up to 800.

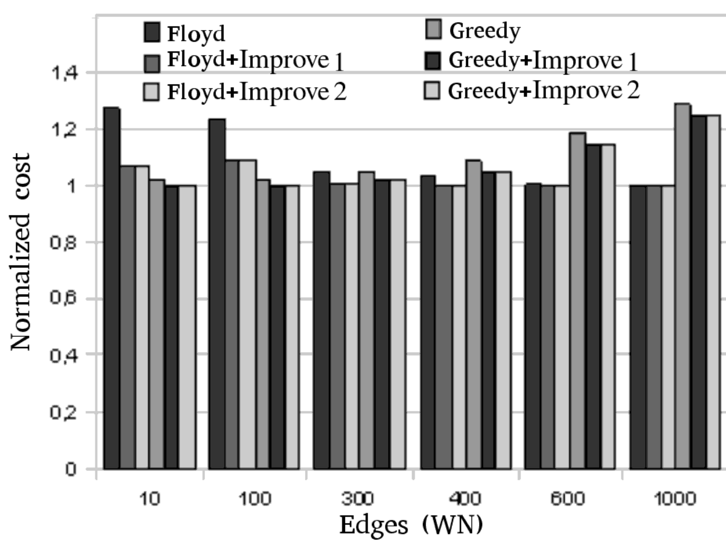

Fig. 2. Numerical results: the normalized costs for the 10x10 grid, the number of edges from 10 up to 1000.

age between all 2-terminal probabilistic connectivities of the network. The later parameter demonstrates the network reliability from the point of connectivity between a pair of nodes even if a network is disconnected.

Let us assume that branches (edges of the primary network) are subject to random failures that occur statistically independently with known probabilities $p_i, \ 1 \leq i \leq g$.

The reliability of an edge $r \in R$ we define as

$$R_r(HN) = \prod_{v \in F(r)} p(v). \tag{2}$$

If for an edge $r \in R$ the path $F(r)$ has the endpoints a and b, we use the notation $R_{ab}(HN)$ instead $R_r(HN)$. If for the nodes a and b there are more than one of such edges, we choose one with a maximum reliability value.

We consider the following reliability measure $R(HN)$, taking into account the fact that failures occur in the primary network, and all consumers should be connected with the corresponding sources:

$$R(HN) = min\{R_{ab}(HN)\}, a \in Y_{source}, b \in Y_{consumer}. \tag{3}$$

Therefore, $R(HN)$ is the minimum among all 2-terminal probabilistic connectivities $R_{ab}(HN)$, where b is a consumer and a is a corresponding source.

Based on the two-stage algorithm, we propose a new algorithm with a reliability constraint. It is assumed that we are given a reliability threshold $0 < R_0 \leq 1$. The objective is to find a sufficiently reliable solution of problem (1). In other words, we find a solution HS of problem (1) such that $R(HS) \geq R_0$.

7 Ant Colony Algorithm for the Reliable Utility Network Design

In the case of the reliability constraint, we use the Ant Colony Algorithm [6] instead of the Floyd Algorithm in the two-stage algorithm. For each edge, we create l ants acting according to the following rules:

- The probability of an ant moving to the next vertex at the iteration of t is defined as:
$$P_{ij}(t) = \frac{\tau_{ij}^{\alpha} \nu_{ij}^{\beta}}{\sum \tau_{ij}^{\alpha} \nu_{ij}^{\beta}}$$
- Every ant l_i $(0 < i \leq l)$ has TimeToLive label $(TTL(l_i))$
$$TTL(l_i) = \prod_{v \in Path(l_i)} p(v),$$
where $Path(l_i)$ is the path of the ant l_i. If $TTL(l_i) < R_0$, then ant l_i dies.
- After finding an edge, each ant lays the pheromone on a branch. $\Delta\tau_{ij}(t) = Q/length(t)$
- In a set of the most frequent routes we find the minimum and fix it, so we decrease the cost of branches of the primary network.

8 Numerical Results for the Reliable Utility Network Design

Below in Figure 3 we present the numerical results of the algorithm proposed for $p_i = 0.99, 1 \le i \le g$, $R_0 = 0.9$. For the primary network PN we consider the 10x10 grid ($|X| = 100$), $\alpha := 1$; $\beta := 3$; $\tau_0 := 1$; $Q := 50$.

The number of edges was from 10 up to 100 in WN (the abscissae axis). Axis of ordinate is cost $Q(HN)$. The first and the second algorithms are the Floyd and the Greedy respectively, without constraint (2). Therefore, the found value $Q(HN)$ is less than the result of FloydProb and AntColony (FloydProb is the Floyd algorithm with constraint (2)). Figure 2 shows that the AntColony results are better than the FloydProb results only for a small number of edges.

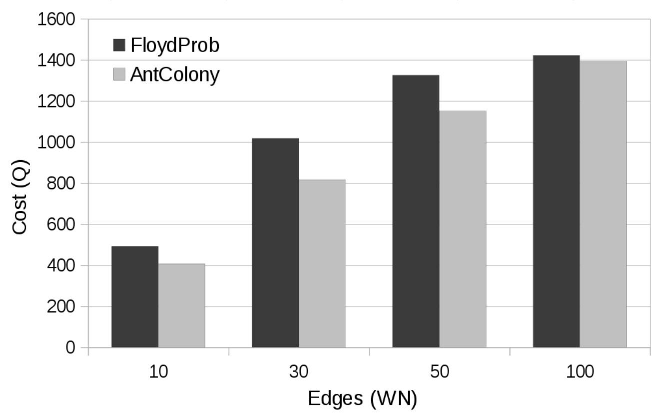

Fig. 3. Numerical results: costs for 10x10 grid, the number of edges from 10 up to 100

9 Conclusion

The new approach to the utility network design has been studied. Based on the hypernet theory, we consider a natural-technical system of a land plot and a utility network within the framework of the unified mathematical model. As a result, we propose the new method to provide the connectivity between given consumers and corresponding sources with minimization of laying and maintenance costs and within given constraints. This method gives an appropriate solution with allowance for the compatibility of different types of resources for placing them in the same track.

For ensuring the reliability of a designed utility network, we, also, introduce the method for obtaining not only a cheap solution, but a reliable enough one as well assuming that failures occur in a primary (physical) network. As a reliability measure we consider the minimum among all 2-terminal probabilistic connectivities between each consumer and corresponding sources.

The conducted numerical experiments which show the applicability of the methods proposed are presented.

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