Inverse analysis of time-resolved LII data

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This study reviews techniques for recovering aerosol particle size distributions from time-resolved laser-induced incandescence data in the context of solving a mathematically ill-posed inverse problem.

Introduction

Time-resolved LII has recently evolved into a tool for evaluating particle size distributions in aerosols. Since larger particles cool slower than smaller particles the size distribution can be recovered from the observed monochromatic incandescence or effective temperature decays.

This involves solving a mathematically ill-posed inverse problem, which is complicated by the fact that ill-posed problems may not have a solution, have multiple solutions, or have a solution that is sensitive to perturbations to the problem. Although solution existence is guaranteed in this case, uniqueness and stability are not, which is of particular concern in LII experiments since physical parameters are not known with a high-degree of certainty and experimental data often contains substantial shot-noise. Because of these difficulties, special explicit and implicit techniques must be used to solve ill-posed inverse problems.

Explicit Methods

Explicit methods solve the mathematically ill-posed governing equations directly. In this problem, the monochromatic incandescence at any instant, $J_\lambda(t)$, is governed by a Volterra integral equation of the first-kind,

$$J_\lambda(t) = C_\lambda \int_0^\infty f(d_p) K_\lambda(t, d_p)\, d\, d_p, \qquad (1)$$

where C_λ is a constant, $f(d_p)$ is the unknown particle size distribution, and $K_\lambda(t, d_p)$ is the radiation emitted by particles of diameter d_p at time t and wavelength λ. Explicit methods transform ill-posed integral equations into ill-conditioned matrix equations, $\mathbf{Ax} = \mathbf{b}$, which are then solved using regularization methods. Roth and Filippov [1] used iterative regularization to solve Eq. (1) for $f(d_p)$.

Implicit Methods

Implicit methods work on the well-posed forward problem, which in this case is to determine the $J_\lambda(t)$ or the effective temperature, $T_e(t)$ that corresponds to a particular $f(d_p)$. Different size distributions are then substituted into the governing equations until the modeled $J_\lambda(t)$ or $T_e(t)$ matches the experimentally-observed values. This is most efficiently done by casting the problem as a least-squares minimization problem,

$$\min_x [F(\mathbf{x})] = \min_x \left[\left\| \mathbf{f}^{exp} - \mathbf{f}^{mod}(\mathbf{x}) \right\|_2^2 \right], \qquad (2)$$

where $\mathbf{x}$ specifies $f(d_p)$ (which is usually log-normal) and $\mathbf{f}^{exp}$ and $\mathbf{f}^{mod}(\mathbf{x})$ contain experimentally-observed and modeled data, respectively. $F(\mathbf{x})$ is then minimized using nonlinear programming; the minimizer $\mathbf{x^*}$ specifies the particle size distribution that best describes the experimentally-observed results. This approach has been used [2-5] to find $f(d_p)$ using $J_\lambda(t)$ or $T_e(t)$. Liu et al. [5] transform this problem into an easier-to-solve univariate minimization problem.

Although implicit methods work on the well-posed forward problem, the ill-posedness of the inverse problem is manifested in the topography of $F(\mathbf{x})$, shown in Fig. 1. The valley surrounding $\mathbf{x}^*$ corresponds to particle size distributions that nearly produce the experimentally-observed data. Diamonds show solutions found using data contaminated with shot noise. Accordingly, care must be taken to select a method that is insensitive to uncertainties in the parameters and experimental error.

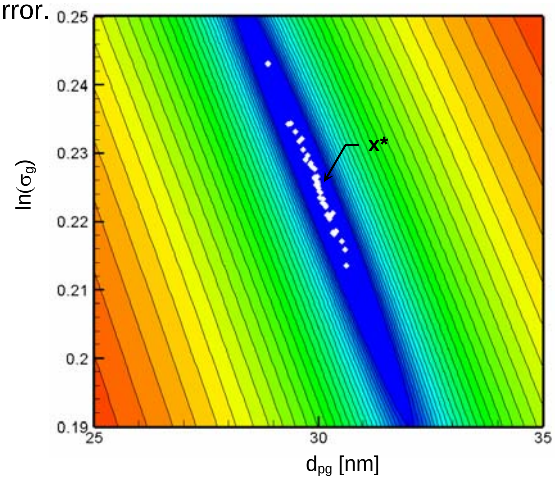

Fig. 1: Plot of F(x), showing solutions obtained using perturbed incandescence data.

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