Visual reasoning in the Marlo diagram

Marcos Bautista López Aznar[/[0000-0001-6931-660X]]

University of Huelva, SPAIN
C/ Fuenteheridos, 25. 2d Huelva, 21002, Spain
pensamosdistintorazonamosigual@gmail.com

Abstract. The Marlo diagram can represent and solve, at least in theory, logic problems with an unlimited number of terms. In López Aznar (2014), (2018b), it was confirmed that the diagram far exceeded the requirements which, according to Moktefi & Shin (2013), are expected from any diagrammatic system. Diagrams are created when trying to add the ability of expressing implicit possibilities to the Venn diagram. This idea was developed in 2013 when studying Johnson Laird's theory of mental models, in González Labra (1998). Diagrams prioritise the concept of "being associated to" instead of the concept of belonging and inclusion. This has allowed the recombining of their DNA with the connective network models, described in Crespo (2006). In López Aznar (2017), (2018a), the Marlo diagram is translated into tree diagrams that show the logical connectives as the simple elimination of different possibilities. This idea is already present in the classic truth tables, but logical trees make it possible to simulate how truth and falsity propagate through the networks of a cognitive system.
The diagram has been in development since 2013, in high school classes, with the help of students and other teachers who are experts in mathematics.

Keywords: diagrammatic reasoning, visualization, cognitive science

1 Basic building rules

> Thought is surrounded by a halo. —Its essence, logic, presents an order, in fact the a priori order of the world: that is, the order of possibilities, which must be common to both world and thought. (Wittgenstein, 1958, p.26)

1.1 An overview of the diagram

The Marlo diagram uses propositional models to communicate the associations that a variable maintains with others in a mental space S. As we already know, in truth tables the number of elements of S is determined with the formula 2^n . In this work, for reasons of simplicity, we will use, like the tables of truth, dichotomous systems in which A means *to be* and ¬A means *not to be*. For example, if *"b"* means being *beautiful*, then *"¬b"* means *not beautiful*.

To construct the models of the simplest propositions, we must place the subject in the middle of what we will call the propositional model. For its part, the predicate must also be placed within the model, but on one of the sides or regions thereof (see figure 1). **All the variables within the same region of a model are necessarily associated and form a unique and distinct type of object in which the contradiction is not possible**. A very important characteristic of our diagram is that we will also write the predicate on the external margin of the model whenever we can conceive that it is possible to have that predicate not associated with the subject of the proposition. For example, if we declare that *all numbers that finish in zero are divisible by two*, we are explicitly communicating the association between *ending in zero* (subject) and *being divisible by two* "t" (predicate). That is why in the first model of figure 1 we can see that *"z"* and *"t"* form a type of object *"zt"*, although now we see that relation from the perspective of the set of *"z"*. However, it is implicit in our proposition that the possibility of being divisible by two without ending in zero is left open, and that is why we write a *"t"* apart from the "z" model. The letter *"t?"* stands for the possibility *"t¬z"*. We have put a question mark in "t?" because, if we eliminate prior knowledge, we cannot know based only on the proposition, if in fact that type of numbers exists or does not exist**. Variables outside any model A should be read as associated with ¬A. Uncertainty is indicated by a question mark.**

Suppose we now add the premise *all the numbers ending in zero are divisible by five* "*f*". We represent the model of this proposition also in figure 1, with the same rules as before, and we realize that the two models have the common denominator "z".

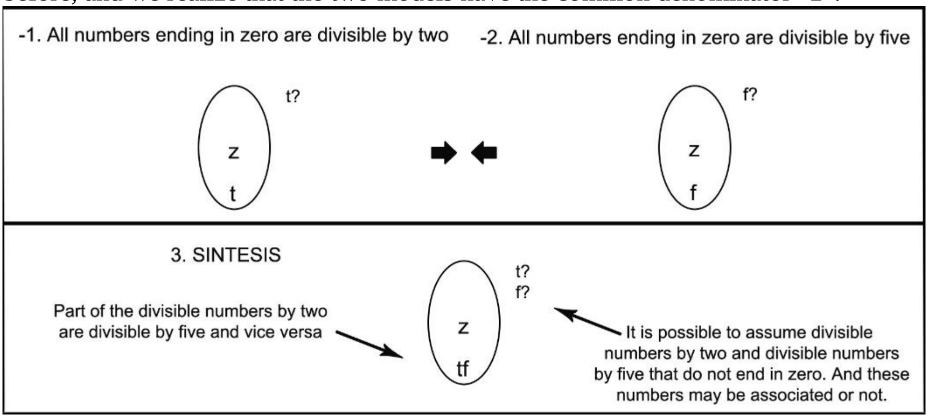

Fig.1. Universal synthesis example

When different models share the same subject, we can synthesize them into one according to the law of identity. It is in this way that we obtain the synthesis -or model 3- of figure 1. We can see that the *"z"* model associates *"t"* and *"f"* necessarily within it in the same region, providing the object type *"ztf"*. We can already affirm that, at least in theory, there are such objects which end in zero and are divisible by two and by five in the space of S. However, as we have also compiled the implicit possibilities in the synthesis, we see that, apart from the numbers that end in zero, there may be numbers divisible by two and others divisible by five, of which we know nothing. On these marginal terms, we can only state with certainty that these are possibilities that must be taken into account when we draw our conclusions. To draw solid conclusions from model 3, we must make the correct questions in an orderly manner about all the associations shown in it. First question: How many types of Z are associated with F and T? All or part? Answer: we see in the image that all Z, the only one that appears, is F and T. Second question: Are all types of T necessarily associated with F or only a part of them? Answer: we see that only a part of the types of T is necessarily associated with F, so we can conclude that *at least part of T is F*. We cannot say that all T is F because there is a possibility, outside Z, of T not being associated to F. Although we cannot say, based solely on the premises, that there is a part of T which is not F. Third question: Can we say that all F is T? or can we only affirm with certainty that some numbers divisible by five are divisible by two? Answer: we observe model 3 and we see that only a part of F is necessarily associated with T; the other part of F is outside Z, neither united nor separate from T.

When drawing conclusions, the model functions as an external memory device that reminds us of all the possibilities that we must consider, which helps to avoid the most common errors when reasoning. This turns the diagram into a powerful didactic tool of logic. In fact, the simplest version of the diagram was tried as a didactic tool with seventy high school students of sixteen years of age. The students received both theoretical (seven hours) and practical instruction with a booklet of exercises (3 hours). The test consisted of solving six syllogisms using the diagram: three by synthesis (universal, particular and probable) and three by exclusion (universal, particular and null). 88.3% of the students passed the test. And 36.6% of them without making a single mistake. See López Aznar (2014).

1.2 Logical rendering of the propositions into diagrams.

> Every simple proposition may be reduced to the expression of a relation between two ideas: the two ideas are represented by the two terms (subject and predicate) […] These terms may be either both of them individual, or one of them individual and the other collective, or both of them collective. (Bentham, 1827, p. 129)

The first decision we must make when representing a proposition is whether to divide the model of the term which functions as the subject. **In particular propositions, the model must be divided, but not in universal propositions.** When we later put the diagrams into formal language, we will indicate that a term is universal with the subscript “x”. The formula $a_x b$ means whenever a appears, we will have b (the subscripts x does not appear in the diagrams). By contrast, if a term - or variable - is not written in the formal language with the subscript “x”, this term should be considered particular. In a proposition in which the subject is particular, it is only associated with the predicate within a region of the model. In figure 2 we can see that: $e_x t_x;\ mh_x\ h_x m;\ po.$

The second decision we must make to represent sentences in the diagram is whether or not we will write the predicate P outside the model of the subject, that is, outside the circle. But we must forget our previous knowledge! When a child first hears that *all vertebrates have a trunk*, he can still assume that there are other beings with a trunk but that they are not vertebrates. But if the child hears the propositions *only vertebrates are* *mammals* or hears the proposition *donkey is the same as burro*, then he has to consider the predicates burro and mammal universally. And the child should think that the terms are universal because the propositions forbid him to think of mammals that are not vertebrates or burros that are not donkeys. **The main idea is that when the proposition does not allow the possibility of conceiving the predicate outside the subject, the predicate is universal. And when the predicates are universal, they are not represented separately from the subject.**

I acknowledge that there is a certain amount of difficulty in getting used to considering whether the predicate of a proposition is taken universally or not, because there are no explicit quantifiers for the predicate of propositions. However, I have verified with my students that it is almost always possible to reach a complete agreement in this sense. And besides, this way of proceeding has several advantages: Hamilton lamented that classical logic had forgotten the extensional dimension of the predicate and he claimed that syllogistic logic could be improved if not only subjects were quantified, but also the predicates in premises, see Lombraña (1989). Figure 2 shows with examples the logical structure of the four elementary propositions. This option differs from the Aristotelian orthodoxy followed by Oriol (2015) and from the Frege's tradition described by Diez Martínez (2005), but it facilitates the calculus.

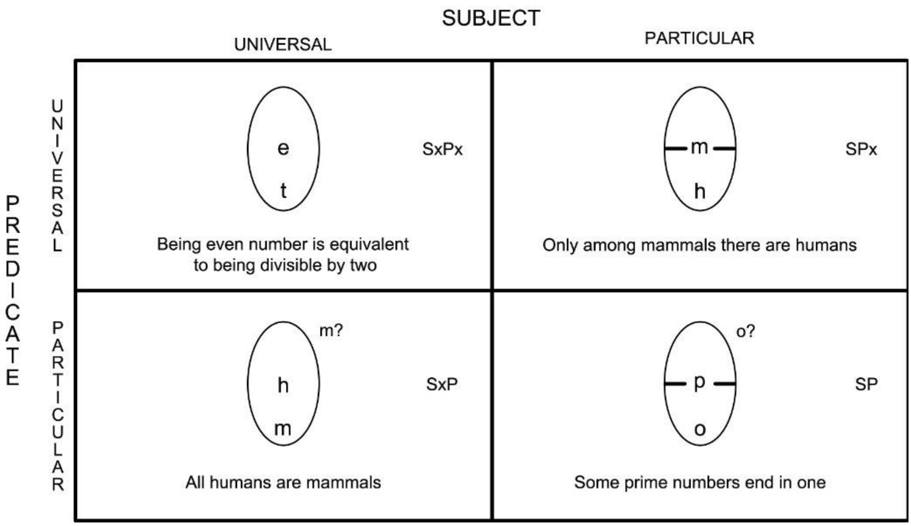

Fig.2. Basic structure of elementary propositions

We should note that the fact that the propositions are affirmative or negative does not affect their basic structure. For example, the proposition *no human flies*, conforms to the form SxP, and is formalized as *"*$h_x\neg f$*"*. In the same way, the proposition *some non-smokers are not drinkers* conforms to the structure SP and is formalized in the diagram as *"*$\neg s \neg d$*"*. We can also learn from Figure 2 that the blank spaces within a particular model should be interpreted as possibilities neither confirmed nor eliminated, which are also contradictory with respect to those explicitly shown in the model. For example, in the "m" model, the upper space should be interpreted as *"*$\neg h$*?"* because in the context of "m", *"*$\neg h$*"* is the only thing that could be different from *"*h*"*. In the same way, in the *"*p*"* model, the upper part could be completed with *"*$\neg o?$*"*.

During the research processes our models maintain spaces of uncertainty, but all the possibilities do not have the same weight. For instance, suppose you meet my family for the first time and ask all my brothers if they are hunters. Everyone answers *yes*. You can now affirm with certainty that all my brothers hunt "$b_x h$", and you can also establish with certainty that at least part of those who hunt in my family are my brothers *"*hb_x*"*. Nevertheless, can you say there are hunters in my family other than my brothers? Is there *"*$h\neg b$*"*? It is possible, but only at the level of conjecture. And if I told you now that I'm going hunting with a member of my family, what option would you choose if your life was at stake? That hunter of my family, is he or is he not my brother? Is the so-called fallacy of the affirmation of the consequent always irrational?

Figures 5, 8, 9, 10 and 11 can facilitate understanding of the diagrams because they contain lots of examples of propositions that combine particular and universal subjects and predicates.

1.3 Complex models

> We have observ'd (sic), that whatever objects are different are distinguishable, and that whatever objects are distinguishable are separable by the thought and imagination. And we may here add, that these propositions are equally true in the inverse, and that whatever objects are separable are also distinguishable, and that whatever objects are distinguishable are also different. (Hume, 1896, p. 16)

Figure 3 shows first, in its upper part, four models of increasing complexity about schizophrenia. Then, in its lower part, it exemplifies what the result of integrating in a single model the information coming from different sources would be: Suppose that a psychiatrist has recently started working in a new hospital and he has received some information about the patients that he must integrate applying some rules, which will be explained later on (fig.9). When integrating the models that the assistants have notified him, the psychiatrist comes to a single model in which he can deduce that some of his patients are schizophrenics with catatonia and no anxiety. **Each particular premise requires opening a new extension in the models in order to temporarily separate different types of things which, however, could eventually become the same thing.** (In the end, we may discover that there is only one type of schizophrenic patient in the hospital). Note that, what is represented in each region of the diagram are abstract entities, types of objects, not concrete entities. In the model in the lower part of figure 3 we can see that three components of schizophrenia are confirmed in the hospital: hearing imperative voices, suffering delusions and suffering from catatonia without anxiety. We also see it is possible to assume that there are patients with schizophrenia but without any of those symptoms (this is shown on the side of the square where we have placed a question mark). In addition, it is possible to assume patients who do not suffer from anxiety, delusions, catatonia, or schizophrenia (all these latter possibilities are expressed outside the schizophrenia model with a question mark, because they have not been confirmed nor eliminated by the sources).

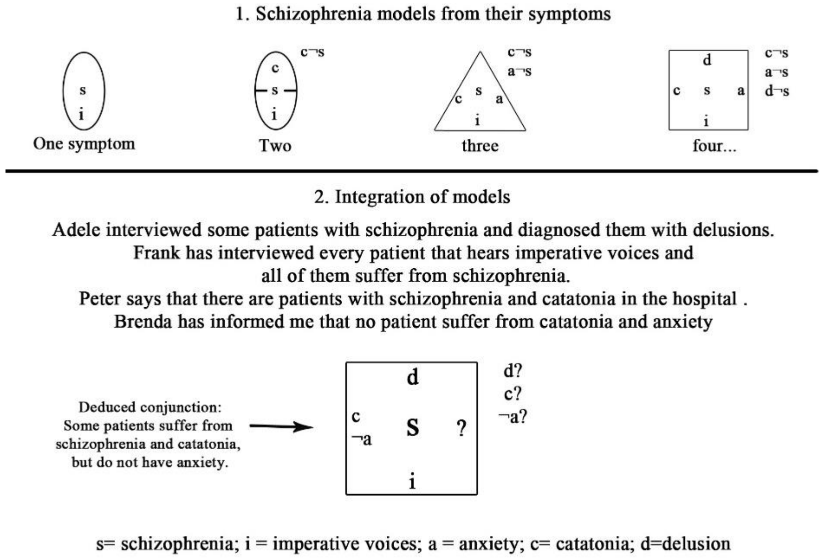

Fig.3. Integrating complex models during research processes

We must bear in mind that the **variables included in the region of a model are hypothetically combinable with the variables of other regions, provided that contradictory pairs are not established.** The main idea is that each region of a propositional model contains a brick or element with which it is possible to create different types of objects by combining them. This idea has repeatedly appeared in the history of logic. See Ramón (1990). For example, *"sdic¬a"* is a probable conjunction of types of patients, however, others are possible (but not probable). In models that contain many terms, it would be impossible to explicitly express all the possibilities at stake. However, we can establish as a rule that **all variables not explicitly confirmed in the models, as well as all their possible combinations, are acceptable at the level of conjecture**. That is, variables that do not appear explicitly in the diagram, or those that appear only with a question mark, should be considered as acceptable possibilities at the level of conjecture. In our example, eighteen of the thirty-two combinations we can make with the dichotomous criteria *a*, *s*, *i*, *d*, *c* are still possible. And this is because we only discard the fourteen combinations that contain "ca", "ac", "i¬s" or "¬si". But remember that those fourteen possibilities that have not yet been discarded do not have the same evidence in their favour. In López Aznar (2016), (2018a) we have given indications to calculate the weight of the evidence for and against each of the possibilities.

1.4 Connectives in the diagram: Inferring by elimination

Classical connectives can be represented in the Marlo diagram based on the definitions that can be found in Frege (1879). Each connective is defined by the possibilities it eliminates. The following figure compares the models of exclusive disjunction ⊻ and inclusive disjunction ∨. First, they are shown in a tree diagram and then in the Marlo diagram. If we first observe the exclusive disjunction model in the tree diagram we will see that starting from the criterion $\underline{p}$ and situating ourselves in the affirmation of P it is only possible to arrive at ¬Q. We also see that placing ourselves in ¬P we can only reach Q. If we now observe the same diagram, but starting from the perspective of the criterion $\underline{q}$, we will first see that starting from ¬Q it is only possible to arrive at P, while starting from Q it is only possible to arrive at ¬P.

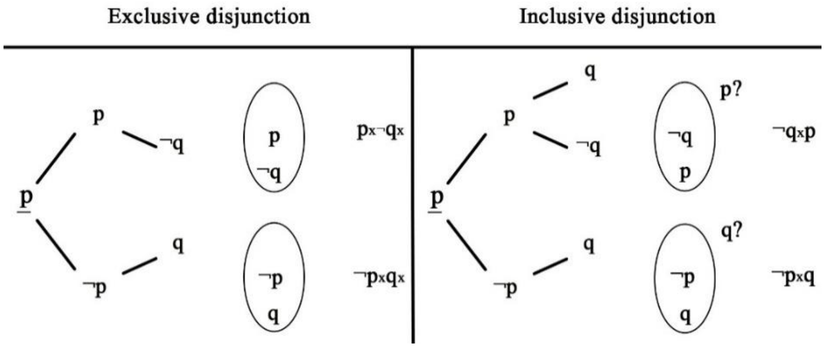

Fig.4. Disjunctions in a tree diagram and in the Marlo diagram.

We should note that this information is the same as that included in the truth table of $\veebar$ when we affirm that 0110. Therefore, we see that the whole extension of P is associated with ¬Q, being impossible to find ¬Q outside of P in the Marlo diagram. Written in formal language: $\boldsymbol{p \veebar q = p \leftrightarrow \neg q = p_x \neg q_x}$. On the other hand, the tree diagram of the inclusive disjunction $p \vee q$ tells us that starting from P it is both possible to reach Q and ¬Q. Thus, it would be a bad option to select that route to communicate information because it is a path that does not discard anything. However, we see that starting from ¬P we necessarily arrive at Q, so: $\neg p_x q$. Likewise, when reading the inclusive tree diagram from the perspective of criterion $\underline{q}$, we find that the relevant information is on the line of ¬Q. Starting from ¬Q we are sure to obtain P, so: $\neg q_x p$. However, starting from Q, it is possible to reach P following the upper route and it is possible to reach ¬P through the lower route. So this does not inform us of anything. The same information is offered by the Marlo diagram, the tree diagrams and the truth tables of $p \vee q$, where by affirming 1110 we are communicating that we eliminate the possibility ¬P¬Q. The zero in the truth tables equals a removed path in the diagrams. All the logical connectors can be defined as conditionals, Oriol (2015). When we communicate with conditionals, we focus our attention on a single path, which does not contain bifurcation. That is, to verify the truth of a conditional the only thing we need to see is that we can only reach the consequent from the antecedent. For example, in the diagram of the inclusive disjunction of Figure 4 the evidence of $\neg p \rightarrow q$ is easily reviewed in a single line. However, when we communicate our expectations about the set of possibilities using the $p \vee q$ disjunction, we must pay attention to the entire network to verify it. That is, we must verify all the routes one by one to see if it is true that at least one of the variables P or Q always appears. This process is visually more complex and requires connecting several intuitions, if we interpret "intuition" with the meaning given to this concept by Descartes (1988), see also Johnson-Laird (1983). Figure 5 shows that every model can be converted and transformed into four equivalent expressions. **During conversion processes, there is no change in the quality of the variables in play. During transformation processes, subscripts "x" must be switched and the quality of the expressed variables is changed**.

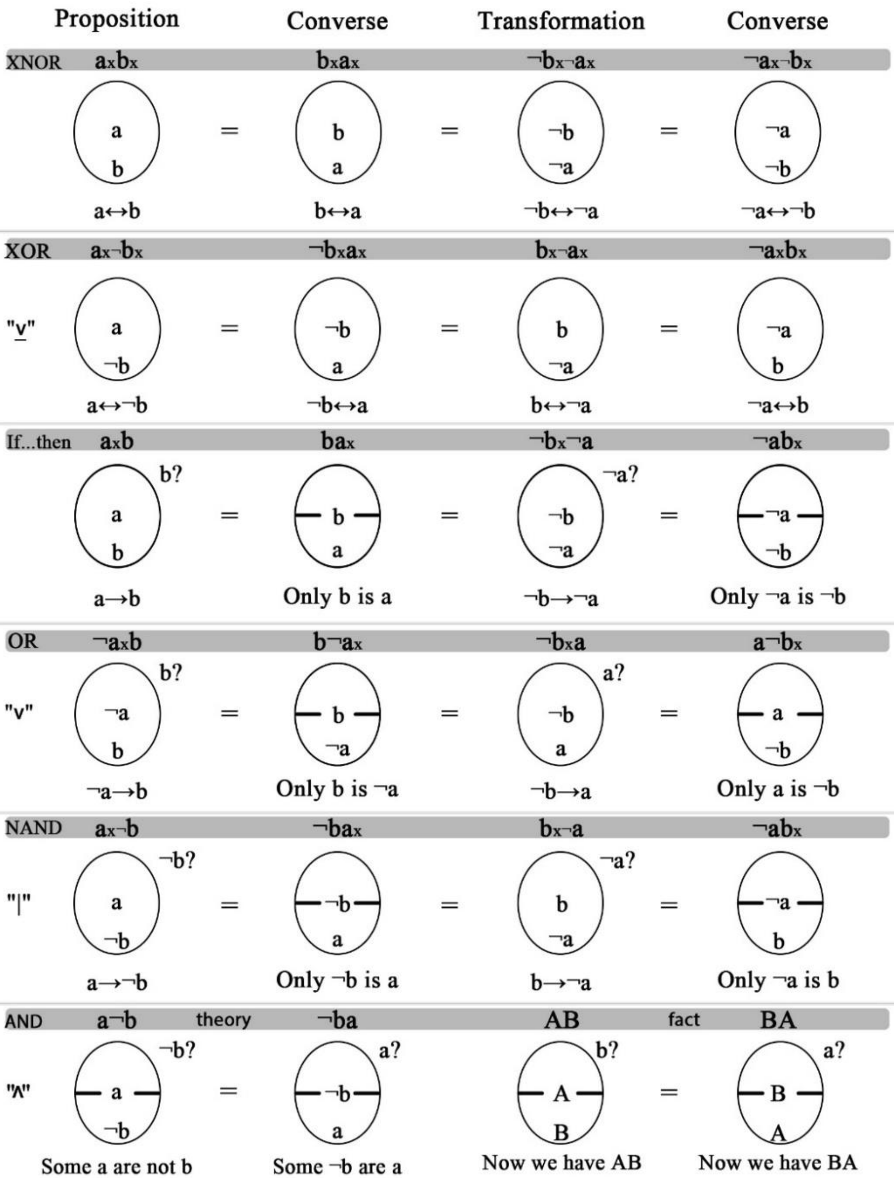

Fig. 5. Logical connectives in the Marlo diagram. Although the truth table of the conjunction states that 1000, it is possible to have ab, a¬b, ¬ab, ¬a¬b in a set at the same time. López Aznar (2016).

2 Certainty degrees and the hierarchy in elimination:

Belief models do not only inform about what things are or are not. Cognitive systems also communicate information about the subjective certainty of their beliefs. When eliminating possibilities, actual facts prevail over theories and theories over suppositions. In this way, the belief models expressed by the Marlo diagram have the capacity to adapt to the continuous evolution of the environment, following the spirit of the Pragmatist Philosophy of Life in Ortega y Gasset, see Ortega (1942). In our classes we distinguish the following degrees of knowledge:

- Conjecture (1): any combination of variables is possible a priori. They will be expressed with a question mark:

$$a\neg b? \tag{1}$$

- Theory (2): combination of variables based on facts. They determine reasonable expectations, this is, beliefs rationally justified. They may present more or less evidence in their favour. Theories are expressed in lowercase letters, without question mark:

$$a\neg b \tag{2}$$

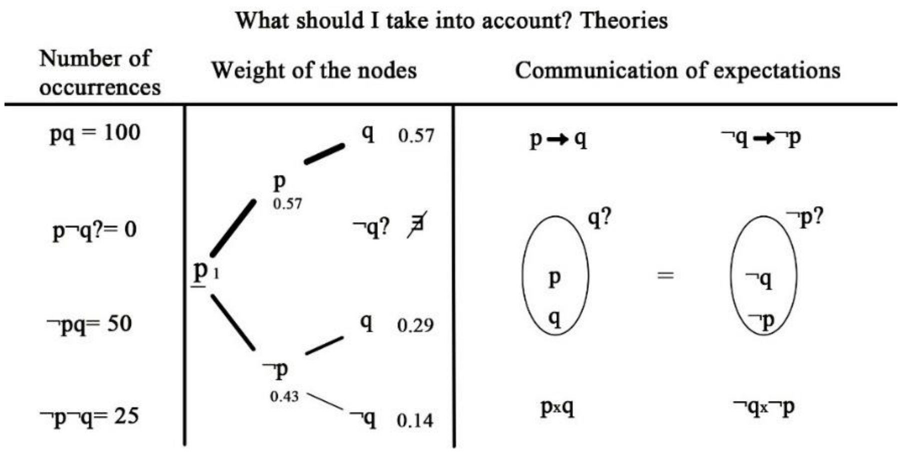

Fig.6. The influence of experience in the models. Uncertainty linked to communication processes

- Theoretical implications: they postulate what is impossible on the basis of accepted theories. In the Marlo diagram, the eliminated possibilities disappear from the space of mental representation. Expressions that contain the denial of existence communicate, in a compressed way, the rest of the remaining possibilities which can be decoded by a decompress or unzip process (3). This process consists in denying the first, denying the second and denying both:

$$\text{zip } \neg(ab) \leftrightarrow (\neg ab,\ a\neg b,\ \neg a\neg b) \text{ unzip} \tag{3}$$

- Probable facts (4): Possibilities linked to a confirmed event. For example, if it is certain that *one part of 'a' is 'b'* and that *one part of 'a' is 'c'*, then:

$$\%abc \tag{4}$$

- Evidence-based hypothesis: When solving complex problems such as the one with schizophrenia, we notice that it is possible to combine an affirmed variable with another possible one. Those combinations are conjunctions which are more possible than those based only on conjectures. This is, *ab?* starting from *a and b?* is more likely than *ab?* starting from *a?* and *b?*.
- On-going facts: they determine the presences and absences (5) which must be taken into account now, and are capable of changing emotions, behaviour, perception or thought in a possible world defined as m_0 in the universe of discourse. These are expressed in capital letters:

$$\text{A¬B} = \text{presence a¬b} \qquad \overline{A\neg B} = \text{absence a¬b} \tag{5}$$

> A VERB, is that which, besides something else, signifies time [..] But I say that it signifies time, besides something else, as for instance, "health" is a noun, but "is well" is a verb; for it signifies besides being well, that such is the case now. (Aristotle, 1889)

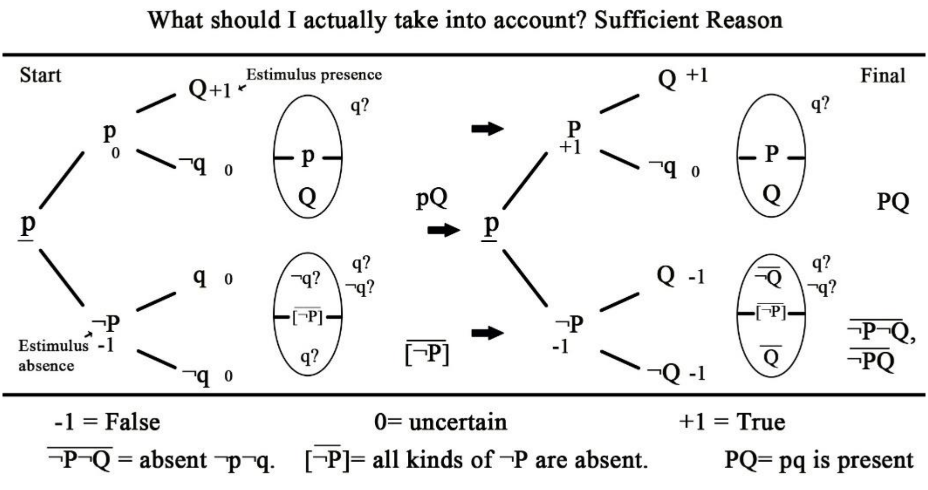

Fig.7. Inferences based on actual evidence

If *a→b*, it can be supposed that, theoretically, if there is A, there will be B, but B will not be an on-going fact unless there is a stimulus, here and now, that can be codified as "*a*". Only such a stimulus can be considered a sufficient reason to affirm the presence of B. Figure 8 shows an example of inference based on the propagation of activation in a network which takes into account a conditional (*d→m* or $d_x m$). The example is from Carlson (1977)

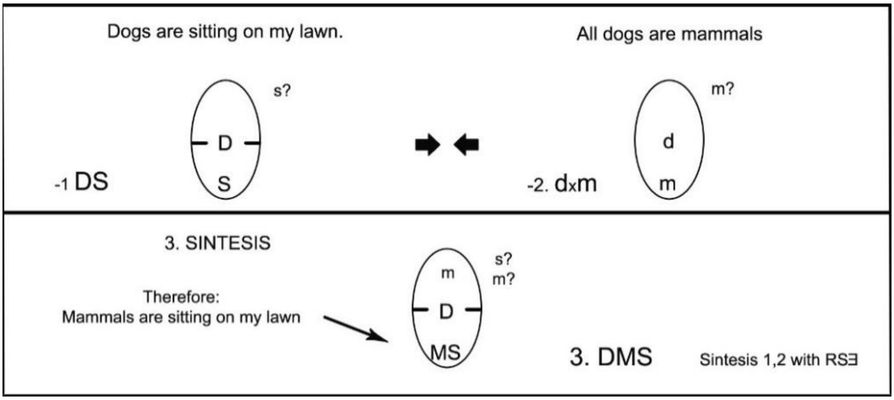

Fig.8. Partial synthesis with sufficient reason to affirm existence.

3 Inferring by synthesis and exclusion.

> […] first, if two terms agree with one and the same third, they agree with each other: secondly, if one term agrees and another disagrees with one and the same third, these two disagree with each other (Whately, 1853, p. 108).

Inferring by synthesis and generalisation. To avoid committing fallacies during synthesis, the basic laws of identity, uncertainty and distinction must be applied. They are the basis of the logic inferences expressed in figure 9. The principle of uncertainty states that what is uncertain in premises must remain uncertain in conclusions.

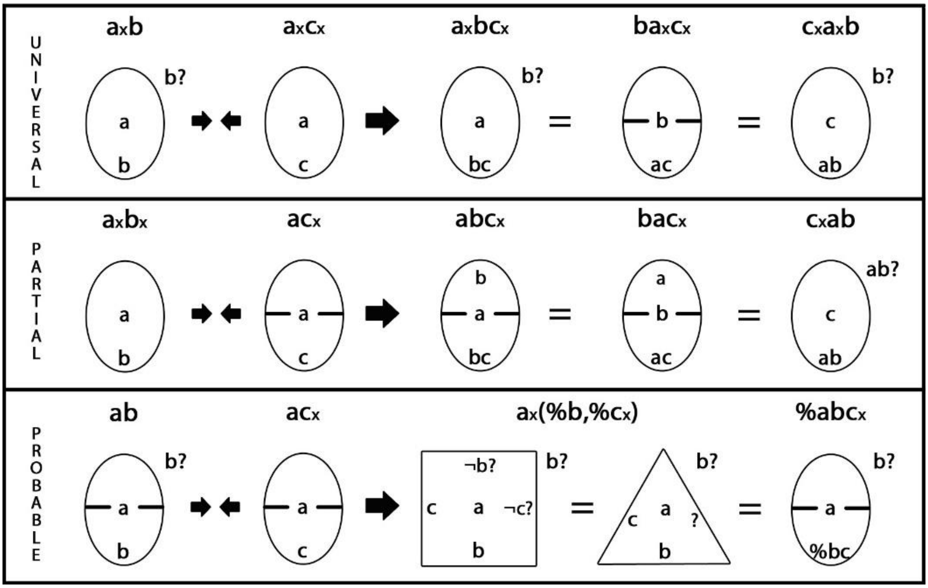

Fig. 9. The laws of synthesis. *%bc* means *abc* is probable and more than possible, because *ab* and *bc* are confirmed. López Aznar (2016)

The principle of distinction forces to provisionally separate the variables when there is not sufficient reason to associate them as a unit in the same region of the model (see probable synthesis in figure 9). Consequently, those variables will fill different sides in the model. However, as long as it is not explicitly expressed that those variables represent incompatible objects, their combination is probable. For example, if we have been informed that a primate species hunts small monkeys in Borneo, and that a primate species cleans fruit in Borneo; therefore, it is likely that the species which hunts and cleans fruit is the same. It is a probable belief, but it is not necessary.

Inferences by exclusion. If two variables are respectively related to incompatible variables (between themselves), the mutual relation is impossible. Therefore, if A_1 is B and C_1 is ¬B, then, it is impossible that A_1 could be associated to C_1. Nevertheless, in this case, the relations between other types of A and other types of C remain uncertain. When confronting two models defined with any excluding variable, three types of inferences are obtained. See figure 9. We have highlighted in grey colour the incompatible regions of the models.

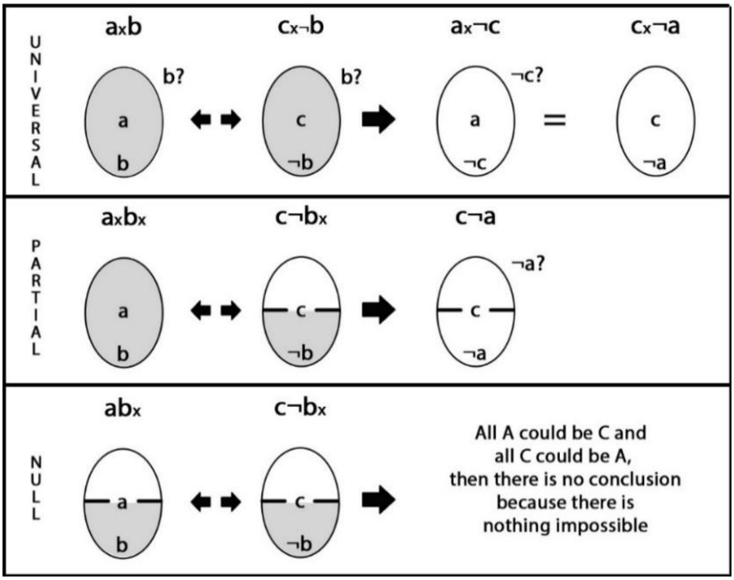

Fig. 10. The laws of exclusion. López Aznar (2016)

1. Universal: No part of A is C and no part of C is A. We reach this conclusion when there are no parts of A or C that could be combined without incurring in the contradiction B¬B. For example: *Minerals are electrical conductors. Carbon is not an electrical conductor.*
2. Partial: One part of C is not A, although every A may be related to the other supposed part of C. We reach this conclusion when there is a part of C that could be combined with A. This is the most difficult type of logical inference for my students. We can see an example in Figure 11.

3. Null: in this case we cannot say anything with certainty because there are no impossible combinations. We cannot affirm that two variables are excluded when a part of both remains uncertain. We can only say that a certain part of A cannot be associated with a certain part of C, or vice versa.

Exclusion is a difficult process and can be avoided by transforming the models in order to operate by synthesis. However, it is important to understand its principles to get a more complete picture of what it means to reason. In Synthesis, the logical inference is based on understanding the necessary relationships. In Exclusion, the logical inference is based on the understanding of impossible associations, when two variables are placed on different paths. Sometimes the relations between two variables are not symmetrical and it is possible, as we have seen in Figure 11, that although every A are C, a part of C is not A. In such cases, it is very difficult for students to consider possible and impossible relationships in an orderly manner. Teaching logical reasoning is simply teaching to look in the correct order and show that a shift of focus is always necessary to review all conclusions.

All hominids are bipedal. Some primates are not bipedal

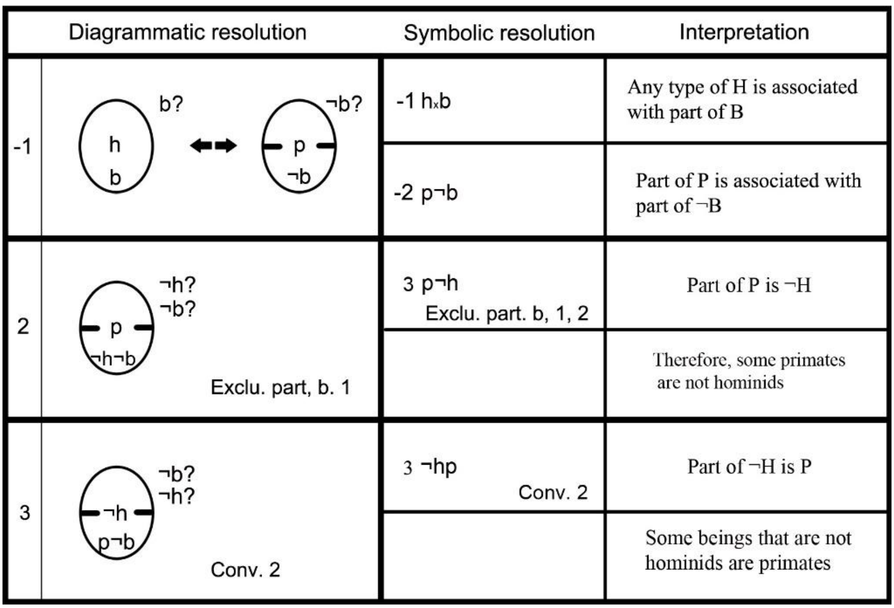

Fig. 11. Example of partial exclusion in BAROCO

In Figure 11 we get that it is only true that, based on the premises, some primates cannot be hominids, although every hominid could be a primate. It is evident that primates that are not bipedal cannot be bipedal hominids. However, considering only the premises and forgetting our previous knowledge, it would be possible, although not necessary, to associate every "*h*" with the indeterminate "*p*" part. Figure 12 shows a problem in which the models have been transformed to be able to operate by synthesis.

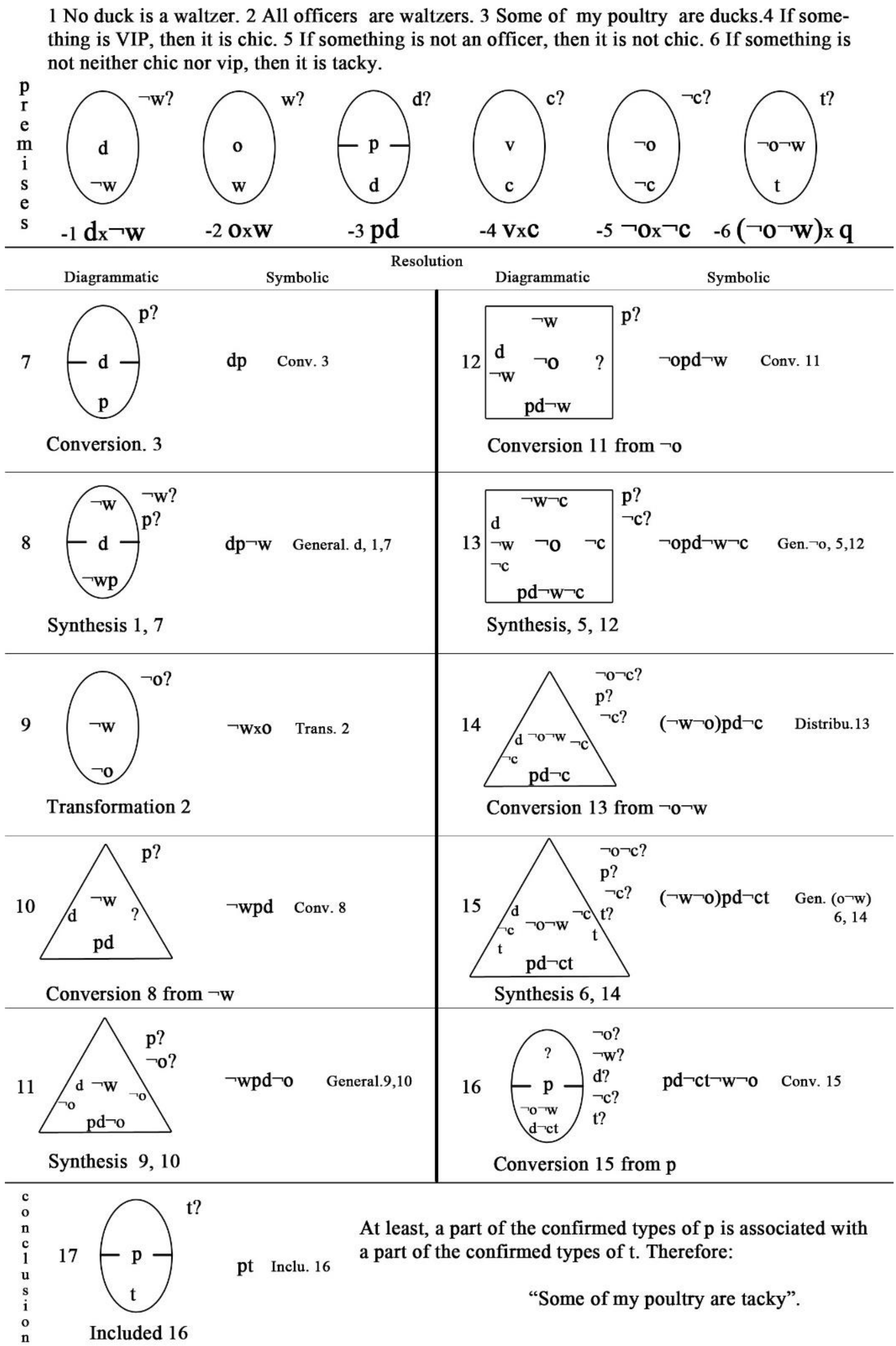

Fig. 12. Problem with seven terms

4 Conclusion

In theory, the Marlo diagram makes it possible to work with an unlimited number of terms, although in practice it must fit the available space. Furthermore, the understanding of the differences between the visual processes necessary to solve inferences by synthesis and by exclusion, allows us to explain, for example, why the EI syllogism of the fourth figure, which is solved in the diagram by exclusion, is more difficult than the syllogism EI of the first figure, which is solved by synthesis, although both have a particular negative conclusion, García Madrugá (1982). The main idea underlying the operation of the diagram is to specify which possibilities are eliminated and which are maintained, an idea that we already found in the explanation of the Carroll diagrams that appears in Moktefi (2013). However, the fact of differentiating between synthesis and exclusion could become a source of hypotheses for cognitive science. By reducing the elementary principles of inference to the law of identity and the Law of non-contradiction, we come close to two fundamental concepts of the psychology of learning: the capture of similarities and differences, see Tarpy (2000).

It is also worth remembering that the Marlo diagram is a powerful tool for the didactics of logic. I have been able to see for myself repeatedly that the students fully understand their principles at the beginning of the high school period. But understanding a language is one thing and knowing how to produce messages with it is another. In any case, the diagram facilitates, in my classroom, the coordination and integration of language and eye movements while troubleshooting a problem. And a student, who looks at where he points and points out what he names, is a student who makes himself understood and because of that convinces me that he understands the problem himself.

Explaining the logic with the Marlo diagram has allowed me to obtain another point of view about what reasoning means. The tree diagrams show us all the mental space simultaneously, while the Marlo diagrams are like the pieces of a jigsaw which fit together.

> [...] we must, I say, observe two sorts of propositions that we are capable of making: -First, mental, wherein the ideas in our understandings are without the use of words put together, or separated, by the mind perceiving or judging of their agreement or disagreement. Secondly, Verbal propositions, which are words, the signs of our ideas, put together or separated in affirmative or negative sentences. (Locke, 1825, pág. 441)

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