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On the Simulations of the Limited Resources Queueing Systems

Eduard S. Sopin[*†], **Konstantin E. Samouylov**[*†], **Sergey Ya. Shorgin**[†]

[*] *Department of Applied Probability and Informatics*
Peoples' Friendship University of Russia (RUDN University)
6 Miklukho-Maklaya st., Moscow, 117198, Russian Federation
[†] *Institute of Informatics Problems, Federal Research Center*
"Computer Science and Control" of the Russian Academy of Sciences
44-2 Vavilova st., Moscow, 119333, Russian Federation

Email: sopin_es@rudn.university, samuylov_ke@rudn.university, sshorgin@ipiran.ru

Queuing systems with limited resources are widely applicable in the modelling and analysis of modern infocommunication systems. The main feature of them is that customers occupy not only a server, but also a volume of multiple resources, for the whole serving time. Processor time, memory, disc space may serve as examples of resources in a computing system, and frequency range and signal power are examples of resources in modern wireless networks. However, even under simplest assumptions on the arrival flows and service times distributions, computational algorithms still have very high complexity, since deduced formulas for the main stationary characteristics include multiple convolutions of the resource requirements distribution function. Therefore, there is absolute necessity in simulation tools for limited resources queuing systems. In the paper, we describe the general queuing system with multiple customer types and multiple limited resources, present the developed simulator and provide some simulation results for various types of arrival and serving processes.

Key words and phrases: queuing system, limited resources, random requirements, simulation.

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1. Introduction

In the queuing systems with limited resources, each arriving customer requires not only a server, but also a random volume of resources. This concept represents the evolution of Kelly networks [7], in which customers occupy fixed volume of resources. The key advantage of the queuing systems with limited resources in the communication systems analysis is possibility to capture characteristic properties of radio resources allocation schemes in modern wireless networks [2,5,9] by definition of specific cumulative distribution function (CDF) of resource requirements.

There are plenty of analytical results of resource queuing systems analysis. However, they were obtained under assumption of Poisson [10, 13, 17–20] or state-dependent Poisson arrivals [11]. Moreover, exact algorithms for evaluation of the main probabilistic characteristics obatined from the analytical formulas remain too complex due to multiple convolutions of the resource requirements CDF.

In case of discrete resources, the recurrent algorithm was developed for evaluation of stationary measures [16]. In [16], the sampling approach was also proposed for the case of continuous resources, but the algorithm still have high complexity.

Since arrivals of customers in modern networks do not have Poisson distribution in most cases, a simulation tool is required to evaluate stationary measures of queuing systems with limited resources. In the paper, we start with brief description of the mathematical model, then provide the design of the developed simulation tool. We provide the detailed numerical analysis for various distributions of arrival and serving processes and finally give a brief summary in the conclusion.

2. Queuing system with limited resources

We consider a multiserver queueing system with N servers and M types of resources, in which arriving customer occupies a server and a vector of resources. The total volume of resources in the system is $\mathbf{R} = (R_1, R_2, \ldots, R_M)$ and L is the number of customer types. Interarrival and serving times of l-type customers are independent identically distributed random variables with CDFs $A_l(x)$ and $B_l(x)$ respectively. Volumes of l-type customers' resource requirements are also independent identically distributed random variables, independent of arrival and serving processes and have CDFs $F_l(\mathbf{x})$. Figure 1 shows the scheme of the queuing system.

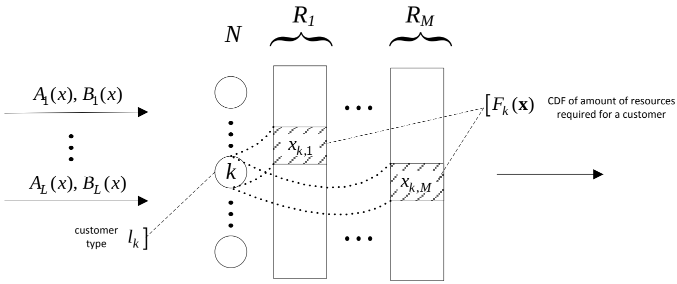

Figure 1. Queuing system with limited resources

Let $a_i^{(1)}$ and $b_i^{(1)}$ be the average of interarrival and serving times respectively. Then the offered load is

$$\rho = \sum_{l=0}^{L} \frac{b_i^{(1)}}{a_i^{(1)}}.$$

The stochastic process $X(t) = (\xi(t), \alpha(t), \beta(t), \theta(t), \gamma(t))$ describes the system behaviour in time. Here $\xi(t)$ is the number of customers in the system at moment $t > 0$, $\alpha(t) = (\alpha_1(t), \dots, \alpha_L(t))$ is the vector of the remaining times before next arrival, $\beta(t) = (\beta_1(t), \dots, \beta_{\xi(t)}(t))$ is the vector of residual service times, $\theta(t) = (\theta_1(t), \dots, \theta_{\xi(t)}(t))$ is the vector of customer types and $\gamma(t)$ is the matrix of occupied resources, where $\gamma_{i,j}(t)$ denotes the volume of i-type resource occupied by j-th customer, $1 \leqslant i \leqslant M$, $1 \leqslant j \leqslant \xi(t)$. Note that $\alpha(t)$ and $\beta(t)$ are decreasing with unit speed, while other components of $X(t)$ change only at the moments of arrival or departure.

Let $\delta(t) = (\delta_1(t), \dots, \delta_M(t))$ be the vector of total volume of occupied resources, where $\delta_m(t) = \sum\limits_{k=1}^{\xi(t)} \gamma_{m,k}(t)$. Denote the moment of arrival of the i-th customer and its resource requirements as t_i and $\mathbf{r}_i$, $i \geqslant 1$, respectively. If there is no free server ($\xi(t_i - 0) = N$) or not enough unoccupied resources ($\delta(t_i - 0) + \mathbf{r}_i \geqslant \mathbf{R}$), then the customer is lost. On the contrary, if $\xi(t_i - 0) < N$ and there are enough resources, then the customer is accepted and it occupies $\mathbf{r}_i$ resources. Thus, if the customer is accepted, the system state changes from $(k, (\alpha_1, \dots, \alpha_{i-1}, 0, \alpha_{i+1}, \dots, \alpha_L), (\beta_1, \dots, \beta_k), (\theta_1, \dots, \theta_k), (\gamma_1, \dots, \gamma_k))$ to $(k + 1, (\alpha_1, \dots, \alpha_{i-1}, \alpha_i, \alpha_{i+1}, \dots, \alpha_L), (\beta_1, \dots, \beta_k, \beta_{k+1}), (\theta_1, \dots, \theta_k, i), (\gamma_1, \dots, \gamma_k, \gamma_{k+1}))$.

The volume of resources occupied by a customer remain constant until departure. On the departure of a customer, it releases the server and resources, i.e. the system moves from the state $(k, (\alpha_1, \dots, \alpha_L), (\beta_1, \dots, \beta_{i-1}, 0, \beta_{i+1}, \dots, \beta_k), (\theta_1, \dots, \theta_k), (\gamma_1, \dots, \gamma_k))$ to $(k - 1, (\alpha_1, \dots, \alpha_L), (\beta_1, \dots, \beta_{i-1}, \beta_{i+1}, \dots, \beta_k), (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_k), (\gamma_1, \dots, \gamma_{i-1}, \gamma_{i+1}, \dots, \gamma_k))$.

In [10], the described system in case of Poisson arrivals and exponential service times was analyzed. In [14], it was proved that stationary behavior of the limited resource queuing systems under Poisson arrivals are insensitive to the service time distribution, analogously to [15], so formulas (1) and (2) hold true for any distribution of the service times. Stationary probabilities $p^k_{l_1,\dots,l_k}(\mathbf{x}_1, \dots, \mathbf{x_k})$ that there are k customers of types $l_1, \dots, l_k$ and i-th customer occupies no more than $\mathbf{x}_i$ resources is given by

$$p^k_{l_1,\dots,l_k}(\mathbf{x}_1, \dots, \mathbf{x_k}) = p_0 F(\mathbf{x}_1) \cdots F(\mathbf{x}_k) \prod_{n=1}^{k} \frac{\lambda_{l_n}}{\sum\limits_{i=1}^{k} \mu_{l_i}}, \tag{1}$$

where

$$p_0 = \left(1 + \sum_{r=1}^{N} \sum_{k_1 + \ldots + k_L = r} F_1^{k_1} * \ldots * F_L^{k_L}(\mathbf{R}) \frac{\rho_1^{k_1}}{k_1!} \cdot \ldots \cdot \frac{\rho_r^{k_r}}{k_r!} \right)^{-1}. \tag{2}$$

Here $F^{(k)}(\mathbf{x})$ is k-fold convolution of the CDF $F(\mathbf{x})$ and sign $*$ denotes convolution operation.

3. Simulation tool description

In this section, we describe the simulation tool. The event-based approach was utilized in the simulator, which is widly used for the analysis of event-based dynamic systems [4, 8, 12]. Figure 2 shows block-diagram of the simulation algorithm. Let us describe it briefly.

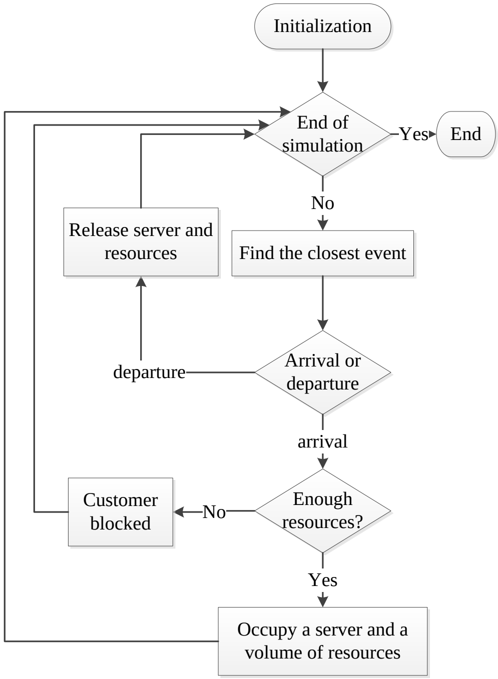

Figure 2. Block-diagram of the simulation tool

1. Initial parameter setup. Set *current_time*, *arrivals*, all components of M-dimensional vector *total_occupied_resources* and all components of $N \times M$ matrix *resources* to 0. There are two event types: arrival and departure of customers. Define components of L-dimensional vector *arrival_time* according to CDF $A_l(x)$, $1 \leqslant l \leqslant L$ and set all elements of N-dimensional vector *departure_time* to infinity. Define *total_number_customers* in the simulation session.
2. Start simulation cycle. The simulations continue until *arrivals* become equal to *total_number_customers*. If simulation session continues, find the closest event (corresponding to the smallest value in vectors *arrival_time* and *departure_time*), set *current_time*=*event_time*.
   (a) If the closest event is arrival, then go to 3.
   (b) If the closest event is departure, then go to 5.

3. Set *resource_requirements* of the customer according to CDF $F_l(\mathbf{x})$, where l corresponds to the customer type, increment *arrivals* by one, check whether the customer is accepted or not.
   (a) If $\mathbf{R} - total\_occupied\_resources \geqslant resource\_requirements$ and one of the components of *departure_time* is equal to infinity, then customer is accepted, go to 4.
   (b) If $\mathbf{R} - total\_occupied\_resources < resource\_requirements$ or all components of *departure_time* are less than infinity, then the customer is blocked. Set new *arrival_time*, update statistics and go to 2.
4. Define a server to serve the customer, set the corresponding element of the vector *departure_time* according to the the CDF $B_l(\mathbf{x})$ and set corresponding row of matrix *resources* to *resource_requirements*. Increase *occupied_resources* by *resource_requirements* and set new *arrival_time*. Update statistics and go to 2.
5. Decrease *total_occupied_resources* by corresponding row of matrix *resources* and set the row to zero. Then set the corresponding element of *departure_time* to infinity, update statistics and go to 2.

4. Simulation results

In this section, we present and discuss the simulation results. For the simplicity, we consider the system with only one type of customers ($L = 1$) and one type of resources ($M = 1$). Assume that $N = 100$ and $R = 1$.

We used gamma distribution for the interarrival times with the following probability density function:

$$a(x) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)}, x \geqslant 0,$$

where $\Gamma(k)$ is the gamma-function. Three set of parameters were used with the same average interarrival time $a^{(1)} = 10s$: $(k = 1, \theta = 10)$, $(k = 5, \theta = 2)$, $(k = 10, \theta = 1)$. The resource requirements CDF $F(x)$ was derived in [16], based on works [3, 6] for machine-to-machine communications in a LTE cell:

$$F(x) = \begin{cases} 0, & x \leqslant 0, \\ C\left(e^{\frac{D}{x}} - 1\right)^E, & 0 < x \leqslant \phi, \\ 1, & x > \phi, \end{cases} \tag{3}$$

where $C = 0.144$, $D = 0.01$, $E = -0.4$ and $\phi = 0.1318$. The values of constants in formula (3) were calculated according to 3GPP standards [1].

Figures 3 and 4 show the behaviour of blocking probability and average volume of occupied resources in case of equiprobability distribution of service times. The serving intensities were chosen so that the offered load ρ varies from 20 to 100.

5. Conclusions

In the paper, we developed the tool for simulation of queuing systems with limited resources and random resource requirements. The simulation algorithm was described briefly and some numerical examples were shown. The tool may be used for not only evaluations of performance measures of contemporary wireless networks, but also for our future research in developing efficient approximate recurrent algorithms for estimation of stationary characteristics of limited resource queuing systems.

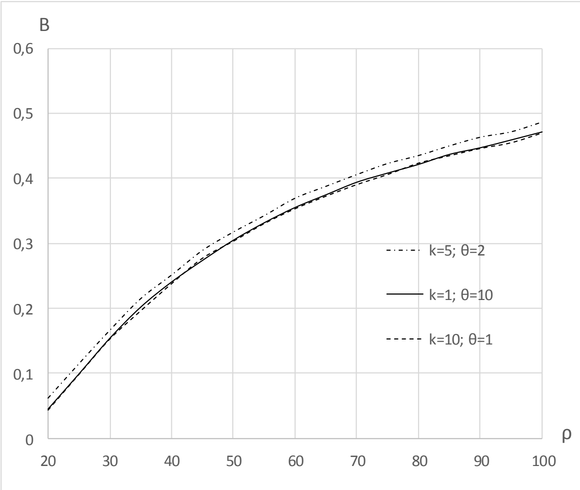

Figure 3. Blocking probabilities in case of exponential service times

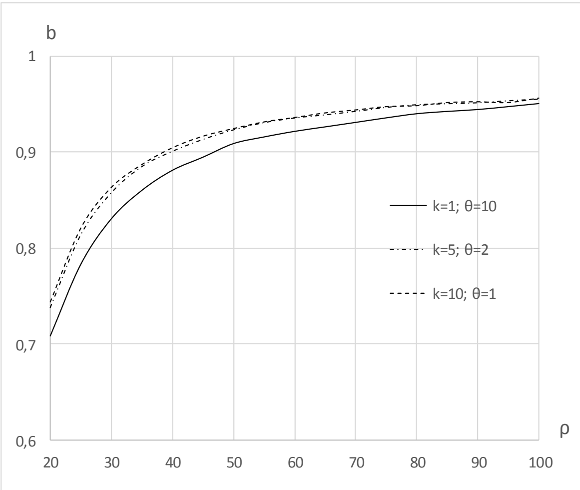

Figure 4. Average volume of occupied resources in case of exponential service times

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