# On an approach to assessing the inter-channel phase synchronization of electroencephalogram signals

**Y V Obukhov[1], R A Tolmacheva[1], A F Polupanov[1] and L A Zhavoronkova[2]**

[1]Kotel'nikov Institute of Radioengineering and Electronics of RAS, Mokhovaya 11-7, Moscow, Russia, 125009
[2]Institute of Higher Nervous Activity and Neurophysiologyof RAS, Butlerova 5a, Moscow, Russia, 117485

**Abstract.** A new approach to evaluation of the phase coherence of electroencephalogram (EEG) signals in different channels based on the calculation and comparison of definite phase characteristics of signals at the points of the ridges of their wavelet spectrograms is considered. The approach is applied to evaluation of inter-channel phase coherence of EEG signals at cognitive tests performed for a healthy subject and for a patient after a traumatic brain injury. The method makes possible to efficiently detect phase-coupled pairs of channels of EEG and distinguish them from phase-uncoupled ones.

## 1. Introduction

The study of EEG inter-channel coherence is a conventional method for diagnosing brain pathologies. Phase connectivity or phase synchronization of signals in two EEG channels is evaluated with the help of the coherence. Typically [1-3], the coherence of two signals is estimated using their normalized complex cross-correlation calculated by multiplying the normalized Fourier components of the signals. The coherence between the two channels of the EEG is defined as the linear dependence of two signals at a certain frequency [2]. Let $x_i(f)$ and $x_j(f)$ are the complex Fourier transformations of time series $x_i(t)$ and $x_j(t)$ of channels *i* and *j*, respectively. Then the cross-spectrum is defined as:

$$S_{ij}(f) = \langle x_i(f) x_j^*(f) \rangle \qquad (1)$$

where * is the complex conjugation and <> is the mathematical expectation.

Coherence is defined as a normalized cross-spectrum [2]:

$$C_{ij}(f) = \frac{S_{ij}(f)}{(S_{ii}(f) S_{jj}(f))^{1/2}} \qquad (2)$$

and the connectivity is defined as the absolute value of the coherence:

$$Coh_{ij}(f) = |C_{ij}(f)| \qquad (3)$$

The phase connectivity is calculated using the phases of the signals *i* and *j*. If $x_i = r_i \exp(i\Phi_i)$ and $x_j = r_j \exp(i\Phi_j)$ are Fourier transforms of the signals, then the cross-spectrum is calculated as [2]:

$$S_{ij}(f) = \langle r_i r_j \exp(i\Delta\Phi) \rangle \qquad (4)$$

where $\Delta\Phi = \Phi_i - \Phi_j$ is the phase difference of signals in channels *i* and *j* at a certain frequency.

In order to calculate the phase connectivity, the cross-spectrum is normalized to the 'global' amplitudes $\langle r_i^2 \rangle^{1/2}$ and $\langle r_j^2 \rangle^{1/2}$. If the signals in the two channels are independent, then $\Delta\Phi$ is a random number and the connectivity is equal zero. Phase connectivity or phase synchronization is defined as an unweighted average:

$$P = \langle \exp(i\Delta\Phi) \rangle \qquad (5)$$

Further, the phase difference is averaged over a certain frequency range predetermined from the neurophysiological considerations. Usually these are the ranges corresponding to the EEG rhythms, such as delta, theta, alpha, etc. rhythms. Phase-coupled pairs of brain regions are received by calculating the averaged phase difference between all pairs of signals and selecting the cut-off threshold.

The averaging of the coherence or the phase difference for different time periods and in the frequency range predetermined on the basis of neurophysiological experience is done in the coherent analysis. Shortcomings of this approach are considered in [4]. In this regard, the actual task is the development of the method for determining the phase-coupled pairs of signals from single trial data and for a more sustainable choice of the phase coherency threshold.

We consider a new approach to the evaluation of phase synchrony of non-stationary EEG signals in cognitive tests. As a criterion of phase synchronization of two signals the following condition [5] is considered:

$$\left|\varphi_{i,\,j}(t)\right| \le const \tag{6}$$

where $\varphi_{i,\,j}(t) = n\phi_i(t) - m\phi j(t)$, $\phi$ is the phase of the signal, $n, m$ are integers.

We consider the case $n = m = 1$, which can easily be generalized to the case of any $n \neq m$.

## 2. Method description

A method of evaluation of the inter-channel phase coherence of EEG signals is based on the calculation and comparison of definite phase characteristics of signals in different channels at the points of the ridges of their wavelet spectrograms. In case the signal satisfies asymptotic properties, the wavelet transform can be approximated in the stationary phase approximation, the points of ridges are the points of the stationary phase where the instantaneous frequency of the signal is equal to the wavelet frequency [6, 7].

At first we find a ridge with the maximum value of $|W|$ at each reference point $\tau_i$ of the Morlet wavelet spectrogram:

$$W(\tau, T) = \frac{1}{\sqrt{T}} \int x(t)\psi\left(\frac{t-\tau}{T}\right)dt, \tag{7}$$

$$\psi(\eta) = \frac{1}{\sqrt{\pi F_b}} e^{2i\pi F_c \eta} e^{-\frac{\eta^2}{F_b}}, \tag{8}$$

where we accept $Fb = Fc = 1$ [8].

Further, at the points of the ridge $R_i = \max_{\{f(t_i)\}} W(f(t_i), t_i)$ we calculatethe phase characteristic of the signal which is defined as a product of the instantaneous frequency of the signal at the time $t_i$ and time: $\varphi_i = 2\pi f(t_i) t_i$ and the difference $\Delta\varphi_{i,j} = \varphi_i - \varphi_j$ for two signals.

In this case points beyond the ridge of the wavelet spectrogram are not taken into account in the evaluation of the phase coherency. Some points of the ridges may not satisfy the asymptotic conditions, which will lead to errors in the calculation of the phase. However, this error seems to be substantially less than errors associated with averaging the phase difference in a wide frequency range.

By removing the frequency range of the processed ridge in the wavelet-spectrogram, we can further apply the described algorithm to distinguish another ridge (in another frequency range, etc.).

## 3. Results

Histograms of the values of portions $\rho_{i,j} = n_{i,j}/N$, where $n_{i,j}$ is number of reference points of ridges with $|\Delta\varphi_{i,j}| < 0.05\pi$, and $N$ is a summary number of EEG signal reference points during the test, are represented in Figure 1. The first pair of leads (Figure 1a) can be referred to a phase-coupled pair. Another pair (Figure 1b) can be referred to a phase-unconnected pair. Figure 1a shows, that $\rho_{i,j}$, less than 0.1, can be considered as a background. We consider the threshold $\rho_{i,j}^{thr}$ equal to 0.15 and we will assume that above this value of the points portions the ridge correspond to the phase-coupled pairs of leads.

EEG of healthy subjects were analyzed, which performed cognitive tasks in isolation. Below, for example, the results of the phase connectivity analysis are presented for two cognitive tests. Some items that belong to the category "clothes" or "food" were randomly listed to the subject during the cognitive test (CT1). During the test, he counts in his mind the quantity of items belonging to one of these categories, and at the end of the test declares the result. When performing a cognitive test (CT2), the doctor randomly called the time.The test subject must imagine in his mind the dial of the clock and the position of the clock hands on it in accordance with the time mentioned. If both clock hands are in the same half of the dial, he says "yes," and if they are in different halves, he keeps silent. All tests were performed for 60 seconds.

Distribution graphs of the portions of the reference points for pairs of EEG leads, based on the histograms obtained, were built and they were sorted in order of increasing $\rho_{i,j}$ with EEG records without tests, with cognitive tests. These distributions are shown in Figure 2 for a healthy subject and for a patient after a traumatic brain injury.

The distribution of the portions of the reference points by pairs of EEG leads, sorted in ascending order $\rho_{i,j}$ for certain tests correspond to lines of a certain color (blue line: recording EEG without test; red line: record EEG with cognitive test CT1; lilac line: recording of EEG with cognitive test CT2). The abscissa axis shows the number of pairs of EEG leads. In the article records of 19-channel EEG were analyzed, therefore the number of pairs of leads is 171. Figure 2 represent that for a certain value on the abscissa axis for each test corresponds, possibly, a different pair of EEG leads. Based on the obtained pairs of EEG leads, it is calculate the number of pairs of leads for each test above the threshold $\rho_{i,j}^{thr}$ and it can be concluded that for a healthy subject the number of such pairs is higher with cognitive tests than when record is without a test (CT1: $n_{i,j}$= 131, number of coupled pairs of EEG leads: 171-131=40; CT2: $n_{i,j}$=130, number of coupled pairs of EEG leads: 171-130=41; record is without test: $n_{i,j}$=150, number of coupled pairs of EEG leads: 171-150=21 (figure 4). As well as, according to figure 3, it is possible to $n_{i,j}$=144, number of coupled pairs of EEG leads: 171-144=27).

Pairs of EEG leads in a healthy subject with a cognitive test CT1 are given as an example on the figure 3a. These pairs of leads are absent when EEG record is without test. Pairs of EEG leads in a healthy subject with a cognitive test CT2 are given as an example on the figure 3b. These pairs of leads are absent when EEG record is without test. Pairs of EEG leads in a patient with craniocerebral trauma with a cognitive test CT1 are given as an example on the figure 3c.

a) b)

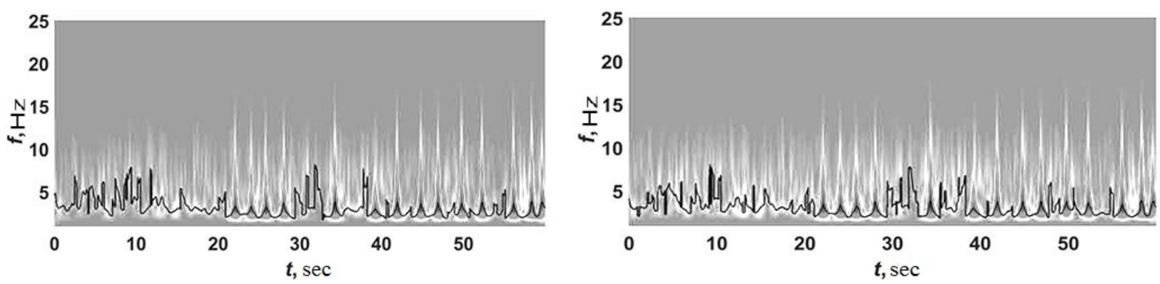


**Figure 1.** Wavelet spectrogram of the Morlet signal in a pair of EEG leads with a cognitive test with aselected ridge: a)For one lead. b) For the second lead. The abscissa axis is the time in seconds, the ordinate axis is the frequency in Hz, the color isproportinal to the power spectral density (red - max).

a) b)

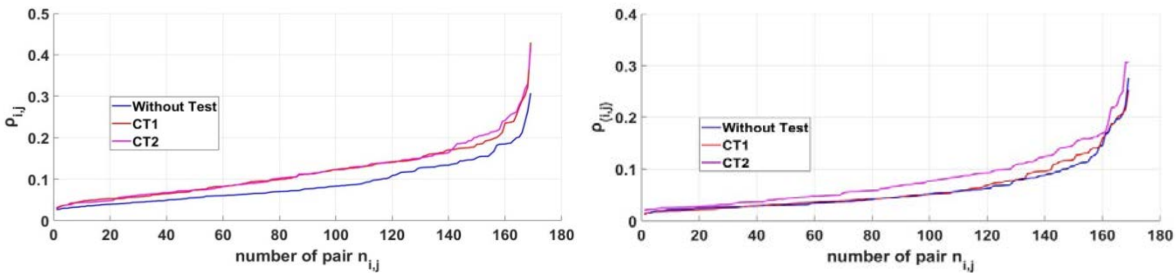


**Figure 2.** The distribution of the portions of the reference points for pairs of EEG leads, sorted in ascending order $\rho_{i,j}$ . The abscissa is the number of the pair of EEG leads that correspond to different pairs of EEG leads. a) healthy subject; б) patient with craniocerebral trauma.

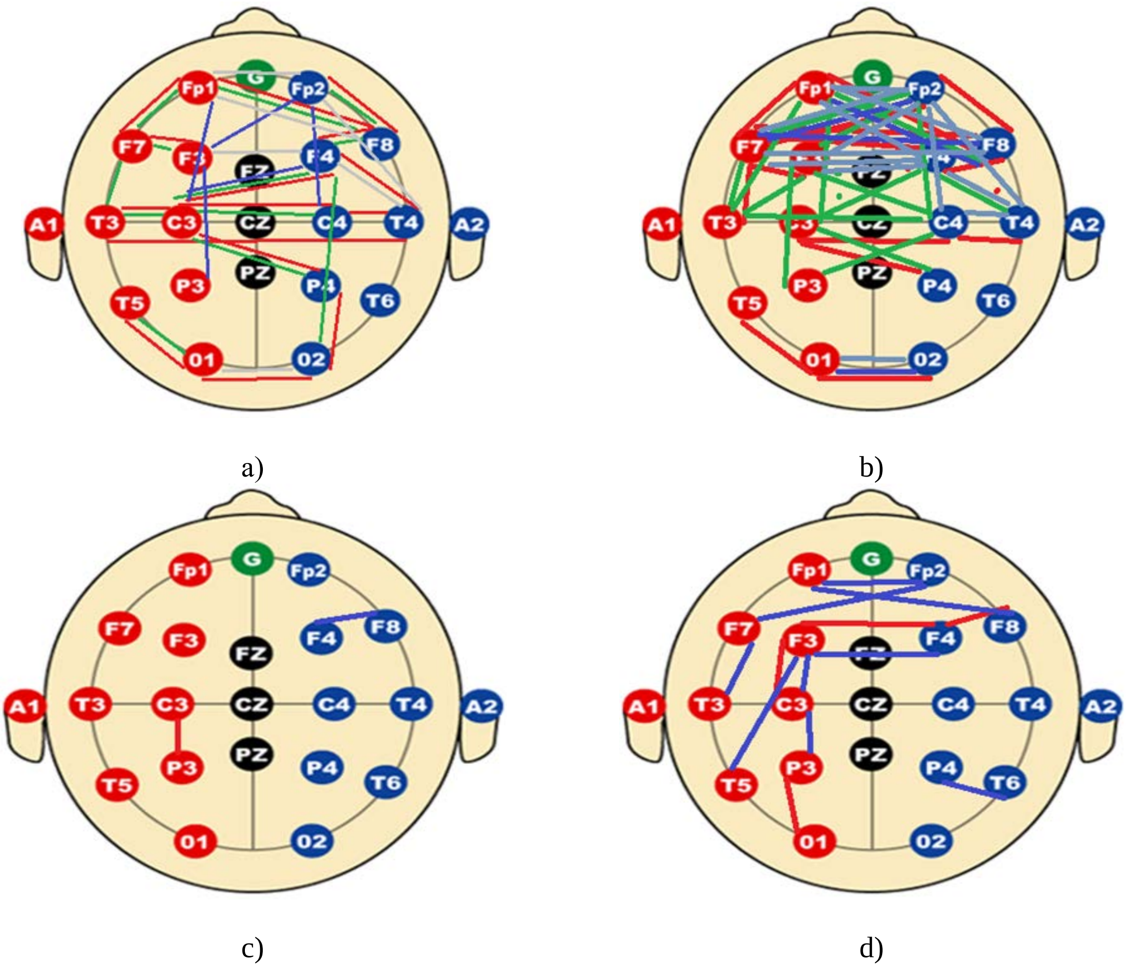


**Figure 3.** Distinctive pairs of EEG leads in cognitive tests CT1 and CT2 in a 4 healthy subjects: (a) – CT1, (b) – CT2; 2 patients with craniocerebral trauma: (c) – CT1, (d) – CT2.

These pairs of leads are absent when EEG record is without test. Pairs of EEG leads in a patients with craniocerebral trauma with a cognitive test CT2 are given as an example on the figure 3d. These pairs of leads are absent when EEG record is without test.

According to [9] the test CT1 is accompanied by a predominant activation of the prefrontal areas of the left hemisphere and the test CT2 is accompanied by a predominant activation of the prefrontal areas of the right hemisphere.

The distinctive phase-coupled pairs of leads demonstrate this (figure 3a and figure 3b).

## 4. Conclusion

The proposed approach improves the calculation of phase synchronization of EEG signals in order to get rid of the shortcomings of the existing estimates of coherence, related to the necessity of averaging the estimates over a wide range of frequencies. The method is tested in the problem of analysis of inter-channel phase synchronization in cognitive tests by healthy subjects and patients after craniocerebral injuries. The number of phase-coupled of the EEG leads is about the same as with cognitive tests for patients after craniocerebral injuries than when record is without a test.

## 5. References

[1] Wending F, Ansari-Asl K, Bartolomei F and Senhadji L 2009 From EEG signals to brain connectivity: a model-based evaluation of interdependence measures *J of Neuroscience Methods* **183(1)** 9-18

[2] Nolte G, Bai O, Wheaton L, Mari Z, Vorbach S and Hallet M 2004 Identifying true brain interaction from EEG data using the imaginary part of coherency *Clin. Neurophysiol.* **115** 2292-2307
[3] Zhana Y, Halliday D, Jiang P, Liu X and Feng J 2006 Detecting time-dependent coherence between non-stationary electrophysiological signals. A combined statistical and time-frequency approach *Journal of Neuroscience Methods* **156** 322-332
[4] Kulaichev A P 2009 On the informative value of coherence analysis in EEG studies *Journal of Higher Nervous Activity* **59(6)** 757-775
[5] Le Van Quyen M, Foucher J, Lachaux J P, Rodriguez E, Lutz A, Martinerie J and Varela F J 2001 Comparison of Hilbert transform and wavelet methods for the analysis of neuronal synchrony *Journal of Neuroscience Methods* **111** 83-98
[6] Mallat S A 2009 *Wavelet Tour of Signal Processing* (Elsevier Inc.)
[7] Guilleemain P and Kronland-Martinet R 1996 Characterization of Acoustic Signals Through Continuous Linear Time-Frequency Representations *Proceedings of the IEEE* **84(4)** 561-585
[8] Daubechies I 1992 *Ten lectures on Wavelets* (SIAM) p 194
[9] Zhavoronkova L, Shevtsova T and Maksakova O 2017 *How the human brain simultaneously solves two problems* (Saarbrücken: LAP LAMBERT Academic Publishing) p 68 (in Russian)

**Acknowledgments**
The reported study was funded by RFBR according to the research project № 18-07-00609.