Application of time series analysis for structural and parametric identification of fuzzy cognitive models

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Abstract. The article deals with problems of structural and parametric identification of fuzzy cognitive models on the basis of statistical data analysis. The feasibility of application of time series analysis for solving these problems is justified. The Granger causality test is proposed for structural identification. An approach for parametric identification based on distributed-lag time series model is also proposed. The results of experimental verification of the described approaches are presented.

1. Introduction

A cognitive approach is one of approaches to the study of semi-structured systems, which is widely used at the present time. According to the definition given in [1], this approach focuses on the development of formal models and methods supporting the intelligent problem-solving process as they include human cognitive capabilities (perception, conception, cognition, understanding, explanation) in solving management problems. Structure and target modeling and simulation modeling methods based on cognitive approach are commonly subsumed under the umbrella term “cognitive modeling”. In general terms, cognitive modeling refers to the study of structure, functioning and development of a system by analyzing its cognitive model. The cognitive model is based on a cognitive map, which reflects researcher's subjective notion (individual or collective) of the system as a number of semantic categories (known as factors or concepts) and a set of cause-and-effect relationships between them.

A cognitive model is an effective tool for exploratory and estimative analysis of the situation. It does not give an opportunity to obtain accurate quantitative characteristics of the system under study, but it allows to assess trends related to its functioning and development, and to identify the key factors influencing these processes. Thus, we can search, generate and develop effective solutions for system management, as well as identify risks and develop strategies to reduce them.

Cognitive modeling starts with creating a cognitive map of the system under study on the basis of information received from experts. The next step includes direct simulation. Its main objectives are forming and testing hypotheses for the structure of the system under study, which can explain its behavior, also developing strategies for various situations in order to reach the specified target states.

Tasks solved by means of cognitive modeling can be divided into two groups:

1. Tasks of structure and target analysis:
   - finding the key factors influencing the targets;
   - identification of contradictions between the targets;
   - identification of feedback loops.
2. Tasks of dynamic analysis (scenario simulation):
   - self-development (“what if we do nothing”);

- managed development:
  - direct task (“what if”);
  - inverse task (“how to”).

Thus, the scenario simulation allows prediction of the simulated system states under different control actions and search for alternative control solutions bringing the system to the target state.

Mathematical apparatus most commonly used to represent cognitive models and underlying methods for their analysis is fuzzy logic. As a result, there appeared a whole class of cognitive models based on different types of fuzzy cognitive maps (FCM). A detailed overview of such models can be found, for instance, in monograph [2]. One of FCM varieties, well-proven in practical analyzing and modeling of semi-structured organizational, social and economic systems are Sylov’s FCMs. They were firstly proposed in [3] and represent the development of signed cognitive maps [4].

2. Formal definition and structure of Sylov’s fuzzy cognitive map

As previously mentioned, a cognitive model is based on formalization of cause-and-effect relationships which occur between factors characterizing the system under study. The result of the formalization represents the system in the form of a cause-and-effect network, termed a cognitive map and having the following form:

$$G = < E, W >,$$

where $E = \{e_1, e_2, \ldots, e_K\}$ is a set of factors (also called concepts), W is a binary relation on the set E, which specifies a set of cause-and-effect relationships between its elements.

Concepts can specify both relative (qualitative) characteristics of the system under study, such as popularity, social tension, and absolute, measurable values: population size, cost, etc. Moreover, every concept e_i is connected with a state variable v_i, which specifies the value of the corresponding index at a particular instant. State variables can possess values expressed on a certain scale, within the established limits. Value $v_i(t)$ of state variable at instant t is called the state of concept e_i at the given instant. Thus, the state of the simulated system at any given instant is described by the state of all concepts included in its cognitive map.

Concepts e_i and e_j are considered to be connected by relation W (designated as $(e_i , e_j) \in W$ or e_iWe_j) if changing the state of concept e_i (cause) results in changing the state of concept e_j (effect). In this case, we say that concept e_i influences concept e_j. Besides, if the value increase of the concept-cause state variable leads to the value increase of the concept-effect state variable, then the influence is considered positive (“strengthening”); if to the decrease – then negative (“inhibition”). Therefore, the relation W can be represented as a union of two disjoint subsets $W = W^+ \cup W^-$, where W^+ is a set of positive relationships and W^- is a set of negative relationships.

Fuzzy cognitive model is based on the assumption that the influence between concepts may vary in intensity; whereas, intensity may be constant or variable in time. Taking into account this assumption, W is set as a fuzzy relation, however, its setting depends on the adopted approach to formalization of cause-and-effect relationships. A cognitive map with fuzzy relation W is termed a fuzzy cognitive map.

Sylov’s fuzzy cognitive map represents FCM, characterized by the following features:

1. State variables of concepts can possess values on the interval [0, 1].
2. Influence intensity is considered constant, so relation W is specified as a set of numbers w_{ij}, characterizing the direction and degree of influence intensity (weight) between concepts e_i and e_j:

$$w_{ij} = w(e_i , e_j),$$

where w is a normalized index of influence intensity (characteristic function of the relation W) with the following properties:

a) $-1 \le w_{ij} \le 1$;
b) $w_{ij} = 0$, if e_j does not depend on e_i (no influence);
c) $w_{ij} = 1$ if positive influence of e_i on e_j is maximum, i.e. when any changes in the system related to concept e_j are univocally determined by the actions associated with concept e_i;

d) $w_{ij} = -1$ if negative influence is maximum, i.e. when any changes related to concept e_j are uniquely constrained by the actions associated with concept e_i;
e) w_{ij} possesses the value from the interval (–1, 1), when there is an intermediate degree of positive or negative influence.

Clearly, FCM of this structure can be graphically represented as a weighted directed graph, which points correspond to elements of set E (concepts) and arcs correspond to nonzero elements of relation W (cause-and-effect relationships). Each arc has a weight which is specified by the corresponding value w_{ij}. In this case, relation W can be represented as a matrix of dimension $n\times n$ (where n is the number of concepts in the system), which can be considered as the graph adjacency matrix and is termed a cognitive matrix.

3. The present state of research in the field of fuzzy cognitive models identification

In the course of building a FCM we can distinguish two stages:

- structural identification, which implies determining a set of concepts E and a crisp relation W over this set, i.e. verification of connections between concepts;
- parametric identification, which implies transition from the crisp relation W to a fuzzy one, i.e. determination of connection weights (influence intensity) between concepts.

Experts are the key source of information at both stages of building a map. In particular, at the structural identification stage, a list of concepts is formed by an expert (or a group of experts). Then, connections between concepts are added to the cognitive model on the basis of the expert notion of the simulated situation.

Expert methods are also most commonly used at the parametric identification stage. They can be direct and indirect. The direct methods imply immediate (explicit) weighing by an expert. The indirect methods are used to minimize the impact of subjectivity in the process of weighing, and they are based on breaking the general task of determining weights into a number of simpler sub-tasks. Saati's pairwise comparison method, Yager's level set method and Churchman-Ackoff method are examples of indirect methods. Description of these methods, as applicable to defining FCM weights, can be found in monograph [5] (section 3.2).

As previously noted, some concepts can set quantitative parameters of the system under study and consequently have numerical state variables. Provided that there is statistical information about the values of these variables, it can be used to identify connection weights between such concepts instead of expert assessment. Thus, statistical methods can be used to identify FCM parameters alongside with the expert methods. The possibility of using this or that method is determined by the nature of the available statistical information [6]. For instance, if statistical data about concepts are represented in the form of spatial sampling, a linear regression model can be applied to identify the sign and intensity of influence between the concepts.

Method based on a pair linear regression model was proposed in monograph [7] (sections 4.2-4.3). With its help, it's possible to identify the sign and intensity of influence between two concepts. Generalization variations of this method based on multiple regression analysis are of special interest and allow identifying parameters of influence of several concepts on a concept.

Attempts to apply correlation and multiple regression analysis to build fuzzy cognitive models of social and economic systems were undertaken in [8, 9]. Nevertheless, the results given in these papers can't be regarded satisfactory for several reasons. First, the authors use the correlation and regression analysis to reveal the very existence of cause-and-effect relationships between concepts and to ascertain the direction of these relationships. However, it is well known that high value of a coefficient of correlation between factors as well as reliability of the regression model built on their basis are not sufficient to draw a conclusion of a cause-and-effect relationship between these factors. Moreover, it is impossible to define accurately the direction of this relationship through the specified methods. Second, the authors propose to use regression equation coefficient values as connection weights between concepts. Yet, the weights obtained can acquire values outside the range [-1, 1], which contradicts the formal definition of Sylov's FCMs. Finally, these papers don't examine the multicollinearity problem, i.e. a high degree of intercorrelation among explanatory variables in regression models. This inevitably leads to abundance of redundant connections in the FCMs obtained.

The approach described in [10] is also based on the multiple regression analysis but is free of the enumerated drawbacks. Nevertheless, there are still a number of current problems connected with the identification of fuzzy cognitive models on the basis of statistical data.

First of all, it should be noted that since the modeled systems are dynamic (i.e. their state changes in the course of time), statistical information about them is likely to be represented in the form of time series in most cases. In this context, the regression analysis is not viable because one of its conditions is representation of data in the form of spatial sampling. Considering this, methods should be developed to identify connection weights between concepts on the basis of time series analysis. This issue was partially addressed in [11], but no approach to identification based directly on time series analysis was proposed in it – the primary focus was on correlation analysis.

Development of FCM structural identification methods based on statistical data is another advanced problem. As has been mentioned above, methods based on spatial sampling analysis do not enable us to establish cause-and-effect relationships between concepts, while time series analysis methods provide us with such an opportunity.

Further, we describe approaches to solving the specified problems.

4. Application of Granger causality test to structural identification

As noted above, the decision of adding a connection between two concepts to a cognitive model is made on the basis of expert notion of the system modeled. Even if there are statistical data about the concepts in the form of spatial sampling, it is impossible to establish either a cause-and-effect relationship between concepts or its direction. In this case, statistical information is used only for the sign and influence intensity identification if there is such influence in the expert's opinion.

If there are statistical data about some concepts X and Y in the form of time series, then Granger causality test can be used for verification of feasibility and viability of adding a connection between them [12].

The idea of the test is as follows: if X influences Y, then a change in X must precede a change in Y, but not vice versa. Moreover, the following two conditions must be met:

- X must contribute significantly to the prediction of Y;
- Y must not contribute significantly to the prediction of X.

If every variable contributes significantly to the prediction of the other one, there are two options possible:

- there is a two-way causality between them;
- there is a third variable influencing both.

Two null hypotheses are sequentially checked in Granger test:

- "X does not Granger-cause Y";
- "Y does not Granger-cause X".

To test these hypotheses two regressions are built; in each of them, the regressand is one of the variables tested for causality, and the regressors are the lags of both variables:

$$y_t = a_0 + a_1 y_{t-1} + \ldots + a_p y_{t-p} + b_1 x_{t-1} + \ldots + b_p x_{t-p} + \varepsilon_t; \tag{1}$$

$$x_t = c_0 + c_1 x_{t-1} + \ldots + c_p x_{t-p} + d_1 y_{t-1} + \ldots + d_p y_{t-p} + u_t. \tag{2}$$

For each regression (1) and (2) the null hypothesis is that the coefficients of the lagged values of the second variable simultaneously equal zero:

$$H_0^1 : b_1 = \ldots = b_p = 0; \tag{3}$$

$$H_0^2 : d_1 = \ldots = d_p = 0. \tag{4}$$

To test hypotheses (3) and (4), an F-test should be performed. To arrive at the conclusion that X influences Y, it is necessary that the first hypothesis be rejected and the second one be accepted (generally, at significance value 0.05).

The number of lagged variables included in the regressions influences the result of the test. Therefore the test is recommended to be performed at a variety of p.

Granger causality between variables doesn't guarantee a cause-and-effect relationship between them but implies the *possibility* of such relationship. Meanwhile, no Granger causality guarantees absence of such relationship. In other words, Granger causality between time series is an *essential but not sufficient* condition for a cause-and-effect relationship between the corresponding concepts. Thus, the final decision whether to add a connection to the cognitive model remains with the expert.

Suppose a cognitive model includes concepts X_1, K, X_n and there are data about them in the form of time series $x_t^1,\ldots,x_t^n$. Then, at the structural identification stage, it is required to establish between which pairs of concepts connections should be added. For this purpose, the described test should be conducted between the series corresponding to each pair of concepts.

Moreover, it is important to consider the fact that influence between concepts can be realized not only directly but also transitively. Granger causality will be also detected in the latter case but with a longer lag (i.e. at larger p) than under the direct influence. Since existence of a relationship between concepts in a cognitive model means that a change of cause concept state leads to a change of effect concept state in one step, the question of adding a connection should be raised only if causality is detected between time series at minimum value of p.

5. Parametric identification based on distributed-lag time series model

At the parametric identification stage, it is necessary to determine signs and weights of all connections between concepts added to the model following the results of the structural identification stage.

Choosing a time series model for weighting FCM connections, it is required to correlate it with the impulse process model which is supposed to be used for the dynamic analysis of the map under study. Research on various impulse process models can be found in one of the authors' previous papers [13].

Further description of the proposed approach is given by the example of the most common impulse process model, namely an additive model with absolute changes. Within this model, a change of concept Y state in a given step t is supposed to be determined (except control and external actions) by absolute changes of influencing concept states in a previous step (t – 1). Meanwhile, previous state changes of concept Y itself are not taken into account. Considering this, for the simplest case (such as when influence on concept Y is realized from one concept X) we obtain the expression:

$$\Delta y_t = a_1 \Delta x_{t-1} + \varepsilon_t, \tag{5}$$

where $\Delta y_t = y_t - y_{t-1}$; $\Delta x_{t-1} = x_{t-1} - x_{t-2}$; a_1 is a coefficient, determining the intensity of influence transmission from X to Y; ε_t is an error.

It is easily seen that model (5) is equivalent to the following:

$$y_t = a_0 + a_1 x_{t-1} + u_t, \tag{6}$$

where a_0 is an absolute term; u_t is an error.

The described model (6) is a special case of a distributed-lag time series model (*DL*), which, in its turn, can be represented as a special case of autoregressive distributed lag model (*ADL*) [12].

This model can be assessed by the least squares method (in fact it is a model of concept Y values regression on previous values of the influencing concept X), so that we can obtain the desired value of a_1 (regression coefficient).

The described model is naturally generalized in case of several influencing concepts. In this case, we receive a multiple regression model. Application of this model to the parametric identification was viewed in detail by the authors in [10].

The same principle of transition from regression coefficients to connection weights should be applied as in the regression analysis. The principle was also described in detail in [10].

6. Experimental validation of the proposed approaches to structural and parametric identification

Suppose concepts *X* and *Y* are added to a fuzzy cognitive model, and there is statistical information about them in the form of time series. Time series corresponding to the concepts are illustrated by graphs in figure 1.

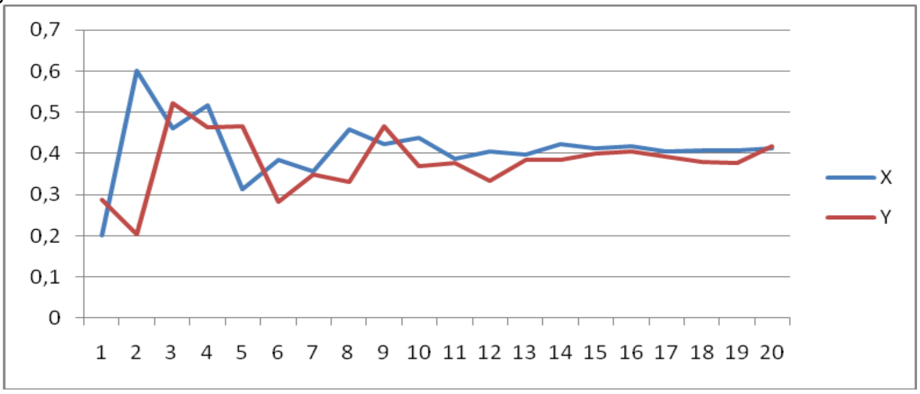

Figure 1. Time series corresponding to concepts *X* and *Y*.

According to the described approach to the structural identification and taking $p = 1$, let us plot the following regressions:

$$y_t = a_0 + a_1 y_{t-1} + b_1 x_{t-1} + \varepsilon_t; \tag{7}$$

$$x_t = c_0 + c_1 x_{t-1} + d_1 y_{t-1} + u_t. \tag{8}$$

By assessing models (7) and (8) by the least squares method, we obtain:

$$y_t = 0.011 + 0.053 y_{t-1} + 0.858 x_{t-1}; \tag{9}$$

$$x_t = 0.61 - 0.31 x_{t-1} - 0.159 y_{t-1}. \tag{10}$$

Next, for each regression (9) and (10), it is necessary to test the hypothesis of the coefficient equal to zero with the second variable lagged, that is $H_0^1 : b_1 = 0$ and $H_0^2 : d_1 = 0$. F-test reveals that the first hypothesis is rejected at significance value 0.05 and the second one is accepted. Thus, *X* Granger causes *Y* if $p = 1$.

Detection of Granger causality between these concepts at minimum value of *p* is a reason to question the expert whether to add a connection directed from concept *X* to concept *Y* to the fuzzy cognitive model.

Suppose the expert decides to add such connection to the model. In this case, the same statistical data about the concepts used at the previous stage can be applied to identify the sign and influence intensity between the concepts. For this, let us develop a model using the existing time series

$$y_t = a_0 + a_1 x_{t-1} + u_t, \tag{11}$$

Having assessed the model by the least squares method, we obtain $a_1 = 0.857$. Determination coefficient R^2 of model (11) equals 0.9, which indicates its acceptable quality. Besides, the obtained value a_1 is significant according to Student’s t-test. With the help of transformations described in [10], let us pass from the regression coefficient obtained to the influence intensity of concept *X* on concept *Y*. As a result, we obtain $w_{XY} = 0.88$ (with normalizing function parameter $b = 3$).

7. Conclusion

The paper deals with problems of structural and parametric identification of fuzzy cognitive models and the existing problem-solving techniques: expert and statistical. The viability of new approaches to solving these problems on the basis of time series analysis is substantiated. An approach to solving the problem of structural identification is proposed, based on Granger causality test. Also a possible approach to parametric identification on the basis of distributed-lag time series model is studied. The results of experimental validation of the proposed approaches are presented.

Let us consider possible directions for further research which are of major interest.

First, one of the features of data analysis represented in the form of time series is that, besides the measurements themselves (levels of a series), there is information about real time moments at which these measurements were obtained. Knowing the difference in time between two successive levels of time series (and consequently knowing the time of influence spreading between directly connected concepts), we can approximately correlate model time steps with the real time of the simulated system and thus improve accuracy and concreteness of prediction resulting from the cognitive model dynamic (scenario) analysis.

Second, it is worthwhile developing the existing impulse process models towards taking account of different rates of influences: between different pairs of concepts, influences can spread at a variable speed (at a varying number of simulation steps). Meanwhile, rates of influence spread between pairs of concepts are determined on the basis of time series pairs corresponding to them.

Finally, use of statistical data for fuzzy cognitive model identification in the form of time series provides the model verification with new opportunities. In case of comprehensive statistical data, model identification can be performed using only a part of them; the rest can be used for its verification. Degree of the model adequacy will be determined by the accuracy of recalling in the process of dynamic simulation the data by which it was trained and by the efficiency of predicting data, which were not accounted for while training.

8. References

[1] Avdeeva Z K, Kovriga S V and Makarenko D I 2007 Cognitive modeling approach to control of semi-structured systems (situations) *Managing Large Systems* **16** 26-39

[2] Borisov V V, Kruglov V V and Fedulov A S 2012 *Fuzzy Models and Network*s (Moscow: “Goryachaya Liniya – Telekom” Publisher)

[3] Sylov V B 1995 *Strategic Decision Making in Fuzzy Environment* (Moscow: “INPRO-RES” Publisher)

[4] Roberts F S 1976 *Discrete Mathematical Models with Application to Social, Biological and Environmental Problems* (Englewood Cliffs, N.J.: Prentice-Hall)

[5] Erokhin D V, Lagerev D G, Laricheva E A and Podvesovskii A G 2010 *Strategic Enterprise Innovation Managemnet: Monograph* (Bryansk: Bryansk State Technical University Press)

[6] Denisova A Y and Sergeev V V 2015 Impulse response identification for remote sensing images using gis data *Computer Optics* **39(4)** 557-563 DOI: 10.18287/0134-2452-2015-39-4-557-563

[7] Averchenkov V I, Kozhukhar V M, Podvesovskii A G and Sazonova A S 2010 *Monitoring and Prediction of Regional Demand for Highest Scientific Degree Specialists: Monograph* (Bryansk: Bryansk State Technical University Press)

[8] Makarova E A, Gabdullina E R, Zakieva E Sh and Valiullina K M 2016 Algorithms for intelligent analysis of life quality in the domain of public health on a regional level *Proc. of the 6th Int. Conf. on Information Technologies for Intelligent Decision Making Support* **2** 222-228

[9] Makarova E A, Zakieva E Sh, Gabdullina E R and Makhmutova A E 2016 Knowledge generation algorithms for construction of cognitive model of life quality in the domain of high education on a regional level *Proc. of the 6th Int. Conf. on Information Technologies for Intelligent Decision Making Support* **2** 54-59

[10] Podvesovskii A G and Isaev R A 2016 Application of multiple regression analysis for parametric identification of fuzzy cognitive models *Proc. of the 6th Int. Conf. on Information Technologies for Intelligent Decision Making Support* **2** 28-33

[11] Rogachyov A F and Melikhova E V 2014 Problems of statistical estimation of cognitive map characteristics on the basis of correlation analysis *Proc. of the Int. Conf. “Physico-Mathematical Sciences: Theory and Practice”* 55-62

[12] Magnus Ya R, Katyshev P K and Persetskii A A 2004 *Econometrics: Basic Course* (Moscow: “Delo” Publisher)

[13] Isaev R A and Podvesovskii A G 2017 Generalized model of pulse process for dynamic analysis of Sylov’s fuzzy cognitive maps *CEUR Workshop Proceedings* **1904** 57-63