

Optimal Strategies for Bidding on Perfect Substitutes in Simultaneous Vickrey Auctions

Enrico H. Gerding Rajdeep K. Dash David C. K. Yuen
Nicholas R. Jennings

University of Southampton, Southampton, SO17 1BJ, UK.
{*eg,rkd,dy,nrj*}@ecs.soton.ac.uk

Abstract

We derive optimal bidding strategies for a global bidder who participates in multiple, simultaneous second-price auctions with perfect substitutes. We first consider a model where all other bidders are local and participate in a single auction. For this case, we prove, somewhat surprisingly, that the global bidder should always place non-zero bids in all available auctions, irrespective of the local bidders' valuation distribution. The optimal strategy is then to bid high in one auction and equal or lower in all other auctions. In addition, by combining analytical and simulation results, we demonstrate that similar results hold in the case of several global bidders, provided that the market consists of both global and local bidders. In the case of global bidders only, there is no stable solution.

1 Introduction

The recent surge of interest in online auctions has resulted in an increasing number of auctions offering very similar or even identical goods and services [8, 9]. In eBay alone, for example, there are often hundreds or sometimes even thousands of concurrent auctions running worldwide selling such substitutable items¹. Against this background, it is important to develop bidding strategies that agents can use to operate effectively across a wide number of auctions. To this end, in this paper we devise and analyse optimal bidding strategies for one such setting — namely, a bidder that participates in multiple, simultaneous second-price auctions for goods that are perfect substitutes.

To date, much of the existing literature on multiple auctions focuses either on sequential auctions [5] or on simultaneous auctions with complementarities, where the value of items together is greater than the sum of the individual items (see Section 5 for related research on simultaneous auctions). In contrast, here we consider bidding strategies for the case of simultaneous auctions and perfect substitutes. In particular, our focus is on Vickrey or second-price sealed bid auctions. We choose these because they are communication efficient and well known for their capacity to induce truthful bidding, which makes them suitable for many multi-agent system settings. Within this setting, we are able to characterise, for the first time, a bidder's utility-maximising strategy for bidding in any number of such auctions and for any type of bidder valuation distribution. In more detail, we first consider a market where a single bidder, called the *global bidder*, can bid in any number of auctions, whereas the other bidders, called the *local bidders*, are assumed to bid only in a single auction. For this case, we find the following results:

- Whereas in the case of a single second-price auction a bidder's best strategy is to bid its true value, the best strategy for a global bidder is to bid below its true value.
- We are able to prove that, even if a global bidder requires only one item and assuming free disposal, the expected utility is maximised by participating in all the auctions that are selling the desired item.

¹To illustrate, at the time of writing, over one thousand eBay auctions were selling the iPod mini 4GB.

- Empirically, we find that a bidder’s expected utility is maximised by bidding relatively high in one of the auctions, and equal or lower in all other auctions.

We then go on to consider markets with more than one global bidder. Due to the complexity of the problem, we combine analytical results with a discrete simulation in order to numerically derive the optimal bidding strategy. By so doing, we find that, in a market with only global bidders, the dynamics of the best response do not converge to a pure strategy. In fact it fluctuates between two states. If the market consists of both local and global bidders, however, the global bidders’ strategy quickly reaches a stable solution and we approximate a symmetric Nash equilibrium outcome.

The remainder of the paper is structured as follows. In Section 2 we describe the bidders and the auctions in more detail. In Section 3 we investigate the case with a single global bidder and characterise the optimal bidding behaviour for it. Section 4 considers the case with multiple global bidders. Finally, Section 5 discusses related work and Section 6 concludes.

2 Bidding in Multiple Vickrey Auctions

The model consists of M sellers, each of whom acts as an auctioneer. Each seller auctions one item; these items are complete substitutes (i.e., they are equal in terms of value and a bidder obtains no additional benefit from winning more than one item). The M auctions are executed simultaneously; that is, they end simultaneously and no information about the outcome of any of the auctions becomes available until the bids are placed². We also assume that all the auctions are symmetric (i.e., a bidder does not prefer one auction over another). Finally, we assume free disposal and risk neutral bidders.

2.1 The Auctions

The seller’s auction is implemented as a Vickrey auction, where the highest bidder wins but pays the second-highest price. This format has several advantages for an agent-based setting. Firstly, it is communication efficient. Secondly, for the single-auction case (i.e., where a bidder places a bid in at most one auction), the optimal strategy is to bid the true value and thus requires no computation (once the valuation of the item is known). This strategy is also weakly dominant (i.e., it is independent of the other bidders’ decisions), and therefore it requires no information about the preferences of other agents (such as the distribution of their valuations).

2.2 Global and Local Bidders

We distinguish between global and local bidders. The former can bid in any number of auctions, whereas the latter only bid in a single one.

Local bidders are assumed to bid according to the weakly dominant strategy and bid their true valuation³. We consider two ways of modelling local bidders: *static* and *dynamic*. In the first model, the number of local bidders is assumed to be known and equal to N for each auction. In the latter model, on the other hand, the average number of bidders is equal to N , but the exact number is unknown and may vary for each auction. This uncertainty is modelled using a Poisson distribution (more details are provided in Section 3.1).

As we will later show, a global bidder who bids optimally has a higher expected utility compared to a local bidder, even though the items are complete substitutes and a bidder only requires one of them. However, we can identify a number of compelling reasons why not all bidders would choose to bid globally. Firstly, participation costs such as entry fees and time to set up an account may encourage occasional users to participate in auctions that they are already familiar with. Secondly, bidders may simply not be aware of other auctions selling the same type of item. Even if this is known, however, additional information such as the distribution of the valuations of other bidders and the number of participating bidders is required for bidding optimally across multiple auctions. This lack of expert information often drives a novice to bid locally. Thirdly, an optimal global

²Although this paper focuses on sealed-bid auctions, where this is the case, the conditions are similar for last-minute bidding in iterative auctions such as eBay [9].

³Note that, since bidding the true value is optimal for local bidders irrespective of what others are bidding, their strategy is not affected by the presence of global bidders.

strategy is harder to compute than a local one. An agent with bounded rationality may therefore not have the resources to compute such a strategy. Lastly, even though a global bidder profits on *average*, such a bidder may incur a loss when inadvertently winning multiple auctions. This deters bidders who are either risk averse or have budget constraints from participating in multiple auction. As a result, *in most market places we expect a combination of global and local bidders*.

In view of the above considerations, human buyers are more likely to bid locally. The global strategy, however, can be effectively executed by autonomous agents since they can gather data from many auctions and perform the required calculations within the desired time frame.

3 A Single Global Bidder

In this section, we provide a theoretical analysis of the optimal bidding strategy for a global bidder, given that all other bidders are local and simply bid their true valuation. After we describe the global bidder's expected utility in Section 3.1, we show in Section 3.2 that it is always optimal for a global bidder to participate in the maximum number of auctions available. In Section 3.3 we further characterise the optimal global bid by considering specific settings.

3.1 The Global Bidder's Expected Utility

In what follows, the number of sellers (auctions) is $M \geq 2$ and the number of *local* bidders is $N \geq 1$. A bidder's valuation $v \in [0, v_{max}]$ is randomly drawn from a cumulative distribution F with probability density f , where f is continuous, strictly positive and has support $[0, v_{max}]$. F is assumed to be equal and common knowledge for all bidders. A global bid \mathcal{B} is a set containing a bid $b_i \in [0, v_{max}]$ for each auction $1 \leq i \leq M$ (the bids may be different for different auctions). For ease of exposition, we introduce the cumulative distribution function for the first-order statistics $G(b) = F(b)^N \in [0, 1]$, denoting the probability of winning a specific auction conditional on placing bid b in this auction, and its probability density $g(b) = dG(b)/db = NF(b)^{N-1}f(b)$. Now, the expected utility U for a global bidder with global bid \mathcal{B} and valuation v is given by:

$$U(\mathcal{B}, v) = v \left[1 - \prod_{b_i \in \mathcal{B}} (1 - G(b_i)) \right] - \sum_{b_i \in \mathcal{B}} \int_0^{b_i} yg(y)dy \quad (1)$$

Here, the left part of the equation is the valuation multiplied by the probability that the global bidder wins *at least* one of the M auctions and thus corresponds to the expected benefit. In more detail, note that $1 - G(b_i)$ is the probability of *not* winning auction i when bidding b_i , $\prod_{b_i \in \mathcal{B}} (1 - G(b_i))$ is the probability of not winning any auction, and thus $1 - \prod_{b_i \in \mathcal{B}} (1 - G(b_i))$ is the probability of winning at least one auction. The right part of equation 1 corresponds to the total expected costs or payments. To see the latter, note that the expected payment of a single second-price auction when bidding b equals $\int_0^b yg(y)dy$ (see [5]) and is independent of the expected payments for other auctions. Clearly, equation 1 applies to the model with *static* local bidders, i.e., where the number of bidders is known and equal for each auction (see Section 2.2). However, we can use the same equation to model *dynamic* local bidders in the following way:

Lemma 1: *By replacing the first-order statistic $G(y)$ with*

$$\hat{G}(y) = e^{N(F(y)-1)}, \quad (2)$$

and the corresponding density function $g(y)$ with $\hat{g}(y) = d\hat{G}(y)/dy = Nf(y)e^{N(F(y)-1)}$, equation 1 becomes the expected utility where the number of local bidders in each auction is described by a Poisson distribution with average N (i.e., where the probability that n local bidders participate is given by $P(n) = N^n e^{-N}/n!$).

PROOF To prove this, we first show that $G(\cdot)$ and $F(\cdot)$ can be modified such that the number of bidders per auction is given by a *binomial* distribution (where a bidder's decision to participate is given by a Bernoulli trial) as follows:

$$G'(y) = F'(y)^{\mathcal{N}} = (1 - p + pF(y))^{\mathcal{N}}, \quad (3)$$

where p is the probability that a bidder participates in the auction, and \mathcal{N} is the total number of bidders. To see this, note that not participating is equivalent to bidding zero. As a result,

$F'(0) = 1 - p$ since there is a $1 - p$ probability that a bidder bids zero at a specific auction, and $F'(y) = F'(0) + pF(y)$ since there is a probability p that a bidder bids according to the original distribution $F(y)$. Now, the average number of participating bidders is given by $N = p\mathcal{N}$. By replacing p with N/\mathcal{N} , equation 3 becomes $G'(y) = (1 - N/\mathcal{N} + (N/\mathcal{N})F(y))^{\mathcal{N}}$. Note that a Poisson distribution is given by the limit of a binomial distribution. By keeping N constant and taking the limit $\mathcal{N} \rightarrow \infty$, we then obtain $G'(y) = e^{N(F(y)-1)} = \hat{G}(y)$. This concludes our proof. \square

The results that follow apply to both the static and dynamic model unless stated otherwise.

3.2 Participation in Multiple Auctions

We now show that, for any valuation $0 < v < v_{max}$, a utility-maximising global bidder should always place non-zero bids in all available auctions. To prove this, we show that the expected utility increases when placing an arbitrarily small bid compared to not participating in an auction. More formally,

Theorem 1: *Consider a global bidder with valuation $0 < v < v_{max}$ and global bid \mathcal{B} , where $b_i \leq v$ for all $b_i \in \mathcal{B}$. Suppose $b_j \notin \mathcal{B}$ for $j \in \{1, 2, \dots, M\}$, then there exists a $b_j > 0$ such that $U(\mathcal{B} \cup \{b_j\}, v) > U(\mathcal{B}, v)$.*

PROOF Using equation 1, the marginal expected utility for participating in an additional auction can be written as:

$$U(\mathcal{B} \cup \{b_j\}, v) - U(\mathcal{B}, v) = vG(b_j) \prod_{b_i \in \mathcal{B}} (1 - G(b_i)) - \int_0^{b_j} yg(y)dy$$

Now, using integration by parts, we have $\int_0^{b_j} yg(y)dy = b_jG(b_j) - \int_0^{b_j} G(y)dy$ and the above equation can be rewritten as:

$$U(\mathcal{B} \cup \{b_j\}, v) - U(\mathcal{B}, v) = G(b_j) \left[v \prod_{b_i \in \mathcal{B}} (1 - G(b_i)) - b_j \right] + \int_0^{b_j} G(y)dy \quad (4)$$

Let $b_j = \epsilon$, where ϵ is an arbitrarily small strictly positive value. Clearly, $G(b_j)$ and $\int_0^{b_j} G(y)dy$ are then both strictly positive (since $f(y) > 0$). Moreover, given that $b_i \leq v < v_{max}$ for $b_i \in \mathcal{B}$ and that $v > 0$, it follows that $v \prod_{b_i \in \mathcal{B}} (1 - G(b_i)) > 0$. Now, suppose $b_j = \frac{1}{2}v \prod_{b_i \in \mathcal{B}} (1 - G(b_i))$, then $U(\mathcal{B} \cup \{b_j\}, v) - U(\mathcal{B}, v) = G(b_j) \left[\frac{1}{2}v \prod_{b_i \in \mathcal{B}} (1 - G(b_i)) \right] + \int_0^{b_j} G(y)dy > 0$ and thus $U(\mathcal{B} \cup \{b_j\}, v) > U(\mathcal{B}, v)$. This completes our proof. \square

3.3 Empirical Evaluation

In this section, we present results from an empirical study and characterise the optimal global bid for specific cases. Furthermore, we measure the actual utility improvement that can be obtained when using the global strategy. The results presented here are based on a uniform distribution of the valuations with $v_{max} = 1$, and the static local bidder model, but they generalise to the dynamic model and other distributions (these results consistently show the same trends, and are thus not included here to avoid repetitiveness). Figure 1 illustrates the optimal global bids and the corresponding expected utility for various M and $N = 5$, but again the bid curves for different values of M and N follow a very similar pattern. Here, the bid is normalised by the valuation v to give the bid fraction $x = b/v$. Note that, when $x = 1$, a bidder bids its true value.

As shown in Figure 1, for bidders with a relatively low valuation, the optimal strategy is to submit M equal bids at, or very close to, the true value. The optimal bid fraction then gradually decreases for higher valuations. Interestingly, in most cases, placing equal bids is no longer the optimal strategy after the valuation reaches a certain point. At this point, a so-called pitchfork bifurcation is observed and the optimal bids split into two values: a single high bid and $M - 1$ low ones. This transition is smooth for $M = 2$, but exhibits an abrupt jump for $M \geq 3$. In all experiments, however, we consistently observe that the optimal strategy is always to place a high bid in one auction, and an equal or lower bid in all others. In case of a bifurcation and when the

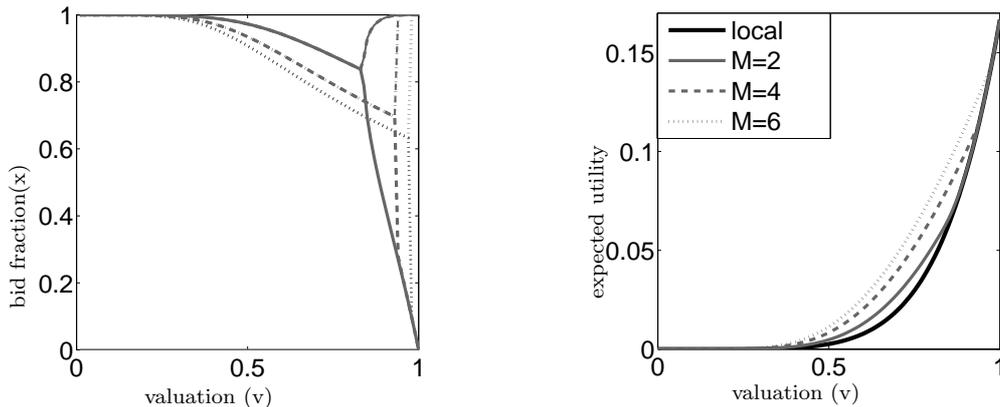


Figure 1: The optimal bid fractions $x = b/v$ and corresponding expected utility for a single global bidder with $N = 5$ static local bidders and varying number of auctions (M). In addition, for comparison, the solid line in the right figure depicts the expected utility when bidding locally in a randomly selected auction, given there are no global bidders (note that, in case of local bidders only, the expected utility is not affected by M).

valuation approaches v_{max} , the optimal high bid becomes very close to the true value and the low bids go to almost zero.

As illustrated in Figure 1, the utility of a global bidder becomes progressively higher with more auctions. In absolute terms, the improvement is especially high for bidders that have an above average valuation, but not too close to v_{max} . The bidders in this range thus benefit most from bidding globally. This is because bidders with very low valuations have a very small chance of winning any auction, whereas bidders with a very high valuation have a high probability of winning a single auction and benefit less from participating in more auctions. In contrast, if we consider the utility *relative* to bidding in a single auction, this is much higher for bidders with relatively low valuations (this effect cannot be seen clearly in Figure 1 due to the scale). In particular, we notice that a global bidder with a low valuation can improve its utility by up to M times the expected utility of bidding locally. Intuitively, this is because the chance of winning one of the auctions increases by up to a factor M , whereas the increase in the expected cost is negligible. For high valuation buyers, however, the benefit is not that obvious because the chances of winning are relatively high even in case of a single auction.

4 Multiple Global Bidders

As argued in section 2.2, we expect a real-world market to exhibit a mix of global and local bidders. Whereas so far we assumed a single global bidder, in this section we consider a setting where multiple global bidders interact with one another and with local bidders as well. The analysis of this problem is complex, however, as the optimal bidding strategy of a global bidder depends on the strategy of other global bidders. A typical analytical approach is to find the symmetric Nash equilibrium solution [8, 10], which occurs when all global bidders use the same strategy to produce their bids, and no (global) bidder has any incentive to unilaterally deviate from the chosen strategy. Due to the complexity of the problem, however, here we combine a computational simulation approach with analytical results. The simulation works by iteratively finding the best response to the optimal bidding strategies in the previous iteration. If this should result in a stable outcome (i.e., when the optimal bidding strategies remains unchanged for two subsequent iterations), the solution is by definition a (symmetric) Nash equilibrium.

4.1 The Global Bidder's Expected Utility

In order to find a global bidder's best response, we first need to calculate the expected utility given the global bid \mathcal{B} and the strategies of both the other global bidders as well as the local bidders. In the following, let N_g denote the number of *other* global bidders. Furthermore, let the strategies of

the other global bidders be represented by the set of functions $\beta_k(v), 1 \leq k \leq M$, producing a bid for each auction given a bidder's valuation v . Note that all other global bidders use the same set of functions since we consider symmetric equilibria. However, we assume that the assignment of functions to auctions by each global bidder occurs in a random fashion without replacement (i.e., each function is assigned exactly once by each global bidder). Let Ω denote the set of all possible assignments. Each such assignment $\omega \in \Omega$ is a (M, N_g) matrix, where each entry $\omega_{i,j}$ identifies the function used by global bidder j in auction i . Note that the cardinality of Ω , denoted by $|\Omega|$, is equal to $M!^{N_g}$. Now, the expected utility is the average expected utility over all possible assignments and is given by:

$$U(\mathcal{B}, v) = \frac{1}{|\Omega|} \sum_{\omega \in \Omega} v \left(1 - \prod_{b_i \in \mathcal{B}} (1 - \tilde{G}_{\omega_i}(b_i)) \right) - \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \sum_{b_i \in \mathcal{B}} \int_0^{b_i} y \tilde{g}_{\omega_i}(y) dy, \quad (5)$$

where $\tilde{G}_{\omega_i}(b) = G(b) \cdot \prod_{j=1}^{N_g} \int_0^b \beta_{\omega_{i,j}}(y) f(y) dy$ denotes the probability of winning auction i , given that each global bidder $1 \leq j \leq N_g$ bids according to the function $\beta_{\omega_{i,j}}$, and $\tilde{g}_{\omega_i}(y) = d\tilde{G}_{\omega_i}(y)/dy$. Here, $G(b)$ is the probability of winning an auction with only local bidders as described in Section 3.1, and $f(y)$ is the probability density of the bidder valuations as before.

4.2 The Simulation

The simulation works by discretising the space of possible valuations and bids and then finding a best response to an initial set of bidding functions. The best response is found by maximising equation 5 for each discrete valuation, which, in turn, results in a new set of bidding functions. These functions then affect the probabilities of winning in the next iteration for which the new best response strategy is calculated. This process is then repeated for a fixed number of iterations or until a stable solution has been found⁴.

Clearly, due to the large search space, finding the utility-maximising global bid quickly becomes infeasible as the number of auctions and global bidders increases. Therefore, we reduce the search space by limiting the global bid to two dimensions where a global bidder bids high in one of the auctions and low in all the others⁵. This simplification is justified by empirical results in Section 3.3 which show the optimal bid is either equal across auctions or the optimal global bid consist of two different values: a high bid in a single auction and a low bid in all other auctions.

The results reported here are based on the following settings.⁶ In order to emphasize that the valuations are discrete, we use integer values ranging from 1 to 1000. Each valuation occurs with equal probability, equivalent to a uniform valuation distribution in the continuous case. A bidder can select between 300 different equally-spaced bid levels. Thus, a bidder with valuation v can place bids $b \in \{0, v/300, 2v/300, \dots, v\}$. The local bidders are static and bid their valuation as before. The initial set of functions can play an important role in the experiments. Therefore, to ensure our results are robust, experiments are repeated with different random initial functions.

4.3 The Results

First, we describe the results with no local bidders. For this case, we find that the simulation does not converge to a stable state. That is, when there is at least one other global bidder, the best response strategy keeps fluctuating, irrespective of the number of iterations and of the initial state. The fluctuations, however, show a distinct pattern and alternate between two states. Figure 2 depicts these two states for $N_G = 10$ and $M = 5$. The two states vary most when there are at least as many auctions as there are global bidders. In that case, one of the best response states is to bid truthfully in one auction and zero in all others. The best response to that, however, is to bid an

⁴This approach is similar to an alternating-move best-response process with pure strategies [3], although here we consider symmetric strategies within a setting where an opponent's best response depends on its valuation.

⁵Note that the number of possible allocations still increases with the number of auctions and global bids. However, by merging all utility-equivalent permutations, we significantly increase computation speed, allowing experiments with relatively large numbers of auctions and bidders to be performed (e.g., a single iteration with 50 auctions and 10 global bidders takes roughly 30 seconds on a 3.00 Ghz PC).

⁶We also performed experiments with different precision, other valuation distributions, and dynamic local bidders. We find that the principal conclusions generalise to these different settings, and therefore we omit the results to avoid repetitiveness.

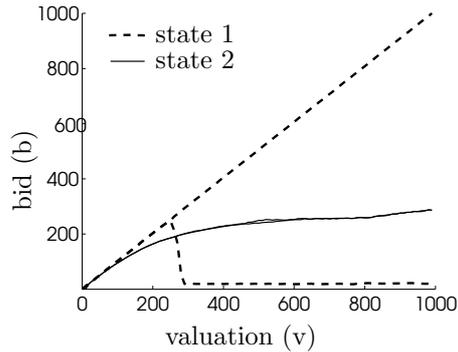


Figure 2: The two states of the best response strategy for $M = 5$ and $N_g = 10$ without local bidders.

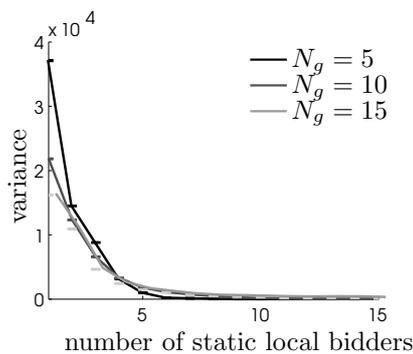


Figure 3: The variance of the best response strategy over 10 iterations and 10 experiments with different initial settings and $M = 5$. The errorbars show the (small) standard deviations.

equal positive amount close to zero in all auctions; this strategy guarantees at least one object at a very low payment. The best response is then again to bid truthfully in a single auction since this appropriates the object in that particular auction. As a result, there exists no stable solution.

As shown in Figure 2, a similar fluctuation is observed when the number of global bidders increases relative to the number of auctions. However, the bids in the equal-bid state (state 2 in Figure 2), as well as the low bids of the other state, increase. Moreover, if the number of global bidders is increased even further, a bifurcation occurs in the equal-bid state similar to the case without local bidders (see also Figure 1a).

We now consider the best response strategies when both local and global bidders participate. To this end, Figure 3 shows the average variance of the best response strategies. Here, the variance is a gauge for the amount of fluctuation and thus the instability of the strategy. As can be seen from this figure, local bidders have a large stabilising effect on the global bidder strategies. As a result, the best response strategy approximates a pure symmetric Nash equilibrium. We note that this result is obtained after only few iterations.

The results show that the principal conclusions in case of a single global bidder carry over to the case of multiple global bidders. That is, the optimal strategy is to bid positive in all auctions (as long as there are at least as many bidders as auctions). Furthermore, a similar bifurcation point is observed. These results are very robust to changes to the auction settings and the parameters of the simulation.

To conclude, even though a theoretical analysis proves difficult in case of several global bidders, we can approximate a (symmetric) Nash equilibrium for specific settings using a discrete simulation in case the system consists of both local and global bidders. Our experiments show that, even in the case of multiple global bidders, the best strategy is to bid in multiple auctions. Thus, our simulation can be used as a tool to predict the market equilibrium and to find the optimal bidding strategy for practical settings where we expect a combination of local and global bidders.

5 Related Work

Research in the area of simultaneous auctions can be segmented along two broad lines. On the one hand, there is the game-theoretic analysis of simultaneous auctions which concentrates on studying the equilibrium strategy of rational agents [2, 6, 7, 8, 10]. Such analyses are typically used when the auction format employed in the simultaneous auctions is the same (e.g. there are M Vickrey auctions or M first-price auctions). On the other hand, heuristic strategies have been developed for more complex settings when the sellers offer different types of auctions or the buyers need to buy bundles of goods over distributed auctions [1, 11, 4]. This paper adopts the former approach in studying a market of M simultaneous Vickrey auctions since this approach yields provably optimal bidding strategies.

In this case, the seminal paper by Engelbrecht-Wiggans and Weber provides one of the starting points for the game-theoretic analysis of distributed markets where buyers have substitutable goods. Their work analyses a market consisting of couples having equal valuations that want to bid for a dresser. Thus, the couple's bid space can at most contain two bids since the husband and wife can be at most at two geographically distributed auctions simultaneously. They derive a mixed strategy Nash equilibrium for the special case where the number of buyers is large. Our analysis differs from theirs in that we study simultaneous auctions in which bidders have different valuations and the global bidder can bid in all the auctions simultaneously (which is entirely possible for online auctions).

Following this, [6] then studied the case of simultaneous auctions with complementary goods. They analyse the case of both local and global bidders and characterise the bidding of the buyers and resultant market efficiency. The setting provided in [6] is further extended to the case of common values in [8]. However, neither of these works extend easily to the case of substitutable goods which we consider. This case is studied in [10], but the scenario considered is restricted to three sellers and two global bidders and with each bidder having the same value (and thereby knowing the value of other bidders). The space of symmetric mixed equilibrium strategies is derived for this special case, but again our result is more general.

6 Conclusions

In this paper, we derive utility-maximising strategies for bidding in multiple, simultaneous second-price auctions. We first analyse the case where a single global bidder bids in all auctions, whereas all other bidders are local and bid in a single auction. For this setting, we find the counter-intuitive result that it is optimal to place non-zero bids in all auctions that sell the desired item, even when a bidder only requires a single item and derives no additional benefit from having more. Thus, a potential buyer can achieve considerable benefit by participating in multiple auctions and employing an optimal bidding strategy.

Furthermore, we investigate a setting with multiple global bidders by combining analytical solutions with a simulation approach. We find that a global bidder's strategy does not stabilise when only global bidders are present in the market, but only converges when there are local bidders as well. We argue, however, that real-world markets are likely to contain both local and global bidders. The converged results are then very similar to the setting with a single global bidder, and we find that a bidder benefits by bidding optimally in multiple auctions. For the more complex setting with multiple global bidders, the simulation can thus be used to find these bids for specific cases.

In future work, we intend to investigate optimal strategies when a global bidder faces budget constraints and when the auctions are no longer symmetric. The latter arises, for example, when the number of (average) local bidders differs per auction or the auctions have different settings for parameters such as the reserve price.

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