Determination of the spatial position and orientation of the links of the robot anthropomorphic grip by the solution of the direct and inverse kinematics problem

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Abstract

A method of determining the spatial position and orientation of the finger of an anthropomorphic grip by a robot is presented in this paper with three degrees of mobility based on solving the direct and inverse kinematics problem. A feature of the gripping device is a common drive for the motion transfer to the executive group of links. The proposed method makes it possible to determine the spatial position of the gripping devise links which based on the direct and inverse kinematics problem. The method is based on the Denavit–Hartenberg representation; in this case, we can do the direct calculations of the nodes coordinates with minimal processing power. It is proposed to use a geometric method for determining the angles of the finger links orientation for the anthropomorphic gripping device of a robot.

1 Introduction

Nowadays, a development of technologies to increase the functioning efficiency of the anthropomorphic robotic systems is the one of a problem in the robotics field. The need for an increasing efficiency is due to the insufficient level of development the modern anthropomorphic robots to replace a person in complex jobs and situations involving a risk to life and occurring in conditions not suitable for life (space, rescue, aggressive zones). It is necessary to develop and improve the robot actuators – the anthropomorphic manipulators – to perform the targeted operations. An anthropomorphic manipulator consists of the nodes, connected by hinges, driven by electric motors, and an anthropomorphic grip.

The existing needs of the anthropomorphic manipulators application require the development of manipulator grips to the capabilities of the human hand, while the kinematic and force parameters of the grip must ensure work with objects and tools designed for human capabilities.
The creation of the anthropomorphic robots by space execution is an unconditional trend in scientific and practical aspects [Kut16]. In flight operations, the actions of astronauts outside of the seals are defined, including: taking the working tool out of the pile, preparing it for work, docking-undocking the cable connectors, working with the spanners, nippers, fastening and unfastening the carbines. The above operations are performed using fine motor skills. It is necessary to have not less than fifteen degrees of mobility to be able to perform similar actions anthropomorphic manipulator. Such operations are performed in conditions of limited hardware resources. Compliance with the requirements of such conditions is possible due to the use of new approaches to the construction and operation of anthropomorphic grip. One such approach is the use of a group drive.

The anthropomorphic grip contains five executive groups of links (EGL). Each EGL has three links. The group drive is used to ensure the movement of links around the parallel axes. The implementation of the cable transfer option provided the creation of grippers successfully used in the robots of the AR-600 (601), SAR-400 (401), FEDOR (Fig.1). The number of active drives is reduced to eight with a total number of degrees of mobility equal to seventeen in the developed and tested versions of the construction the transmission motion systems EGL using differential mechanisms. In that case, a sequence of motion of the output links is provided, sufficient to perform actions corresponding to fine motor behavior [Tay16].

![Figure 1: Basic type of a grip AR-600 with the executive group of links](image)

The purpose of the paper is a development the method of determining the spatial position and orientation of the finger of an anthropomorphic grip by robot based on solving the direct and inverse kinematics problem. It is necessary to complete the tasks to achieve the goal:

1. to carry out a critical analysis of the literature on the research topic
2. to develop a method for determining the spatial position of the finger links of the anthropomorphic grip by robot based on the direct kinematics problem
3. to develop a method for determining the orientation of the finger links of the anthropomorphic grip by robot based on the inverse kinematics problem

Let analyze the literature on the research topic to complete the first task. The variety of literature on the subject of the research is presented by numerous developments in this field and is due to the urgency of using these developments in the modern world. Problems arising in the development of solutions for determining the spatial position and orientation of the anthropomorphic grip of the robot on the basis of solving the direct and inverse kinematics problems are considered in the following papers.

Anthropomorphic grippers are used in most robot designs [Tay16, Wol08, Rot07, Wan16], as basic functional operations are performed by capturing objects of various shapes and sizes. The creation of anthropomorphic grips implies the maximum likeness of the kinematic diagram of the device with the human hand. However, the realization of all degrees of mobility using modern executive mechanisms is a complex and interesting task.

The researches aimed at developing new gripping devices are presented in papers [Sim16, Che13, Bor17]. In paper [Sim16], a non-anthropomorphic grip of a robot for performing tasks, like a human hand, was carried out. The gripping device is represented by a kinematic chain with a tree structure with five branches that have three joint joints. The authors formulated the equations of direct kinematics of relative displacements for each
The object to be captured is recognized using the SpatialVision application based on image analysis, and then the 3D based on the Motion Leap device and the software module: HandCommander, HandProcessor and HandSIM. The solution is designed to assemble industrial robot equipment. The solution is presented in papers [Mol17, Hsu17, Che16, Wan15]. In paper [Mol17], the authors presented a new solution for the management of five-finger anthropomorphic grip without the use of high-precision contact sensors.

In paper [Bir09], the authors consider the problem of reorienting the position repeatedly based on the solution of the direct and reverse kinematics problems in the mechanisms. They introduced a simple grip that can reorient the spatial position of the links of the gripping device with the grip object. A simple grip is presented, which can reorient the position repeatedly based on the solution of the direct and reverse kinematics problems. The presented solution is used to reorient the position of the links of the gripping device with the grip object. A simple grip is presented, which can reorient the position repeatedly based on the solution of the direct and reverse kinematics problems.

In paper [Sar17], the authors considered the applications of a group gripping device. They analyzed the objects of various shapes and sizes for grip and manipulation based on a kinematic model. In paper [Eht17], the authors presented a new grip device with the possibility of reconfiguration. The device has two degrees of freedom on each finger and can support a sufficient payload for production operations. At the same time, a simple kinematics makes it possible to quickly determine the spatial position of the grip links. The authors detail all the principles and concepts used to design this grip. The physical models are given as the result of the project. In paper [Bor17], the authors describe the construction of a gripping device for processing heavy steel pipes with varying physical properties, such as a diameter, a mass and a length. This exciting device is an alternative solution for expensive and complex anthropomorphic seizures. The research focuses on the development of hardware and conceptual design issues of the device.

The researches aimed at the kinematic and dynamic analysis of the gripping device are given in papers [11-17]. In paper [Ram15], the authors presented a hybrid kinematic model of a hand prosthesis that takes into account the different positions of the hand in accordance with the conditions of interaction with the environment. The presented model uses the positions of the phalanges of the finger, calculated using the Denavit-Hartenberg method, mixed with the representation of quaternions. The authors pursued the goal of conducting a detailed grip study to provide an in-depth step-by-step demonstration of the design process and to illustrate the interactions between its stages. Firstly, a geometric model is established that allows one to determine the spatial position of the final effector in the apparatus of dual quaternions. The path of the hand is planned using a hybrid global and generalized coordinates. The Jacobi matrix is determined on the basis of this for calculating the parameters of the device kinematic model. The dynamic model is determined using the Lagrange equations.

In paper [Kre15], the authors gave the results of analysis and research of the kinematic model of anthropomorphic grip with 22 degrees of freedom. They described the process of computer modeling and experimental research of the effectiveness of the grip various objects. The presented analysis technique makes it possible to compare different variants of kinematic grip schemes and to determine the adequacy of the choice of a specific kinematic scheme for optimal grip of a given set of objects. This approach is important in the development of manipulator grip, especially when there are restrictions on the number of controlled degrees of freedom. For example, the task of reducing weight and, accordingly, the number of degrees of freedom is relevant in the development of bionic prostheses. In paper, the authors gave a comparison of two kinematic diagrams for capturing a set of geometric primitives: the human diagram - the thumb is opposite to the little finger and the monkey’s hand diagram - the thumb is opposed to the middle finger. In paper [Has16], the authors carried out a research of the process of modeling robots and optimizing their structure. This process is illustrated by the example of studying the robot grip mechanism, which has a structure with a closed loop and a single degree of freedom (DOF). The authors pursued the goal of conducting a detailed grip study to provide an in-depth step-by-step demonstration of the design process and to illustrate the interactions between its stages. Firstly, a geometric model is established that allows one to determine the spatial position of the final effector and generalized coordinates. The Jacobi matrix is determined on the basis of this for calculating the parameters of the device kinematic model. The dynamic model is determined using the Lagrange equations.

In paper [Hei17], the authors presented a method for describing the kinematics of robot grip for work in indeterminate environments. The goal of the authors is to improve the kinematic scheme of the gripping device in order to enable the grip of large objects with the minimum necessary effort for working in space. In paper [Elt17], the authors presented the research of the kinematic and dynamic properties of the previously developed gripping device. They analyzed the objects of various shapes and sizes for grip and manipulation based on a modified version of the Grubler formula. In paper [Sar17], the authors considered the applications of a group drive that implements the movement of elements in kinematic pairs with parallel axes of rotation. The authors raised an analytical research of the mechanism of group drive, the expression of geometric relationships in vector form was compiled for kinematic analysis, and then a system of scalar equations was obtained. As a result, the angular graphs change from the stroke of the slider, the position plans and the trajectories of the node points of the mechanisms are created and their angular velocities are determined. These speed plans allow you to get the permissible load on the working group of the elements. In paper [Bir09], the authors consider the problem of reorienting the spatial position of the links of the gripping device with the grip object. A simple grip is presented, which can reorient the position repeatedly based on the solution of the direct and reverse kinematics problems without the use of high-precision contact sensors.

The researches aimed at studying the reliability of the griped object and manipulating it are presented in papers [Mol17, Hsu17, Che16, Wan15]. In paper [Mol17], the authors presented a new solution for the management and control of five-finger anthropomorphic grip designed to assemble industrial robot equipment. The solution is based on the Motion Leap device and the software module: HandCommander, HandProcessor and HandSIM. The object to be captured is recognized using the SpatialVision application based on image analysis, and then the 3D
model is loaded into the GraspIT application. The user’s gesture is recognized and sent to the grip test module and the RoboHand component to grip the preconfigured objects. The object is gripped in a physical environment by the RoboHand component, an anthropomorphic grip with five fingers. In paper [Hsu17], the authors do research of the reliability the grip of the object. The solution is proposed by developing an intelligent self-locking mechanism installed parallel to the drive that starts automatically when the object is grip. This design uses an adjustable power distribution between the grip and the brake via a differential gear. The advantages of adaptive and strong coupling and energy-saving capabilities of the proposed model are demonstrated experimentally with the help of a prototype gripper. In paper [Che16], the authors consider the task of adaptability of the gripping device for capturing objects of various shapes. The solution is based on the decomposition of the problem into four stages: an identification of the size and a shape of the object, a determination of the initial spatial position of the grip, a calculation of the trajectory of the motion of the grip links, a calculation of the speed of the links for capturing the object. In paper [Wan15], the authors proposed a method of manipulating a gripped object by planning a trajectory of motion based on graph theory. The emphasis is on the operation of capturing small objects, corresponding to the fine motor skills of the human hand.

Analysis of existing solutions reflects the individuality of the use of the developed methods for a specific situation and device. Thus, the determination of the spatial position and orientation of the anthropomorphic grip links by the robot on the basis direct and inverse kinematics problems is an actual problem.

2 Methods

2.1 Problem Statement

There is a kinematics diagram of the gripping device finger of the anthropomorphic robot on Fig. 2. The diagram is kinematically similar to the human finger for the simplicity of interpreting the angles of rotation of rotational pairs.

![Diagram of the ratio of the rotational pairs of the anthropomorphic grip finger](image)

The points A, B, C are the joints of the proximal, middle and distal phalanges of the finger. The point D is a finite point of the finger of a brush in accordance with which you can put a finger pad of a person’s finger. The proximal, middle and distal phalanges are designated for the finger of the gripping device as links AB, BC, CD, respectively.

The mathematical formalization of the problem for determining the spatial position based on the solution of the direct kinematics problem has the following form: it is necessary to find the coordinates of the end point of the robot’s finger $D(x_D; y_D; z_D)$ at the known orientation angles $\alpha_A, \alpha_B, \alpha_C$ and the links length AB, BC, CD.

The problem for determining the orientation of the links of an anthropomorphic robot’s finger on the basis of solving the inverse kinematics problem has the following form: it is necessary to find the orientation angles $\alpha_A, \alpha_B, \alpha_C$ at the known node’s points coordinates of the robot finger $A(x_A; y_A; z_A)$, $B(x_B; y_B; z_B)$, $C(x_C; y_C; z_C)$, $D(x_D; y_D; z_D)$ and the links length AB, BC, CD.

2.2 Solving the direct kinematics problem

The initial parameters for solving the direct kinematics problem are the Euler angles given in the joints of the finger ($\alpha_A$ is an angle of proximal phalanges, $\alpha_B$ is an angle of middle phalanx, $\alpha_C$ is an angle of distal phalanx) and the links length (AB, BC, CD). The output parameters are the coordinates of the nodal points of the anthropomorphic grip finger: $A(x_A; y_A; z_A)$ is a node of proximal phalanx, $B(x_B; y_B; z_B)$ is a node of middle phalanx, $C(x_C; y_C; z_C)$ is a node of distal phalanx, $D(x_D; y_D; z_D)$ is a finite node.

Due to the fact that all degrees of mobility are exclusively rotational, the description of the kinematic diagram reduces to specifying the connected angles, as well as linear and angular displacements. In this case, let use the matrix Denavit-Hartenberg representation. Assume that the references point of the system of absolute
coordinates coincides with the joint A. In this case, the coordinate systems associated with each of the links, equally oriented in pairs with each other and with the global coordinate system (Fig. 3).

![Diagram of the ratio of the rotational pairs of the anthropomorphic grip finger]

In this case, the description of this kinematic diagram for the Denavit-Hartenberg representation is shown in the table 1.

<table>
<thead>
<tr>
<th>Joint</th>
<th>Linear displacement</th>
<th>Angular misalignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>(l_{AB})</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>(l_{BC})</td>
<td>0</td>
</tr>
</tbody>
</table>

In this table \(l_{AB}\) and \(l_{BC}\) are the links length AB and BC. Let compose the transformation matrix from the coordinate system the previous link to the next link's coordinate system, as a product of elementary transformations of shear and rotation.

\[
R_x(\alpha) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\alpha) & -\sin(\alpha) & 0 \\
0 & \sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[(1)\]

\[
R_y(\alpha) = \begin{bmatrix}
\cos(\alpha) & 0 & \sin(\alpha) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(\alpha) & 0 & \cos(\alpha) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[(2)\]

\[
R_z(\alpha) = \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[(3)\]

\[
T(\vec{d}) = \begin{bmatrix}
1 & 0 & 0 & a_x \\
0 & 1 & 0 & a_y \\
0 & 0 & 1 & a_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[(4)\]

\(R_x(\alpha)\) is a transformation of an elementary rotation on the axis Ox to the angle \(\alpha\);

\(R_y(\alpha)\) is a transformation of an elementary rotation on the axis Oy to the angle \(\alpha\);

\(R_z(\alpha)\) is a transformation of an elementary rotation on the axis Oz to the angle \(\alpha\);

\(T(\vec{d})\) is a transformation of an elementary shift to \(\vec{d}\) vector.

In order to obtain the transformation matrix \(A^0_A\) from the coordinate system of the joint A to the global coordinate system, it is necessary to perform elementary rotations according to the sequence x - y' - z" of the Euler angles:
\[ A_A^0 = R_x(\alpha_A) * R_y(0) * R_z(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_A) & -\sin(\alpha_A) & 0 \\ 0 & \sin(\alpha_A) & \cos(\alpha_A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (5)

Let perform an elementary shift along the \( O_z \) axis by a length \( l_{AB} \) to convert the coordinate system of the joint B to the coordinate system of the joint A and then rotate according to the sequence of x-y'-z" Euler angles:

\[ A_B^A = T(l_{AB}, 0, 0) * R_x(\alpha_B) * R_y(0) * R_z(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_{AB} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_A) & -\sin(\alpha_A) & 0 \\ 0 & \sin(\alpha_A) & \cos(\alpha_A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_c) & -\sin(\alpha_c) & 0 \\ 0 & \sin(\alpha_c) & \cos(\alpha_c) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_A) & -\sin(\alpha_A) & 0 \\ 0 & \sin(\alpha_A) & \cos(\alpha_A) & l_{AB} \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (6)

Let perform an elementary shift along the \( O_x \) axis by a length \( l_{BC} \) to convert the coordinate system of the joint C to the coordinate system of the joint B and then rotate according to the sequence of x-y'-z" Euler angles:

\[ A_C^B = T(l_{BC}, 0, 0) * R_x(\alpha_C) * R_y(0) * R_z(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_{BC} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_A) & -\sin(\alpha_A) & 0 \\ 0 & \sin(\alpha_A) & \cos(\alpha_A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_c) & -\sin(\alpha_c) & 0 \\ 0 & \sin(\alpha_c) & \cos(\alpha_c) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_A) & -\sin(\alpha_A) & 0 \\ 0 & \sin(\alpha_A) & \cos(\alpha_A) & l_{BC} \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (7)

It is necessary to apply consistently the previously derived transformations To obtain the transformation matrix from the coordinate system of the joint C to the global coordinate system

\[ A_C^0 = A_A^0 A_B^A A_C^B \] (8)

The coordinates of nodal points can be obtained by the following formulas:

\[ T_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \] (9)

\[ T_{CD} = \begin{bmatrix} 0 \\ 0 \\ l_{CD} \\ 1 \end{bmatrix} \] (10)

\[ A = A_A^0 * T_0, \] (11)

\[ B = A_B^A A_B^A * T_0, \] (12)

\[ C = A_A^0 A_B^A A_C^B * T_0, \] (13)

\[ D = A_A^0 A_B^A A_C^B * T_{CD}. \] (14)

Thus, the determination of the spatial position of the links of the anthropomorphic grip by the robot on the basis of the solution of the direct kinematics problem is carried out by the formulas (5-14), the result is the calculation of the coordinates of the nodal points of the finger of the gripper \( A(x_A, y_A, z_A) \) is a node of proximal phalanx, \( B(x_B, y_B, z_B) \) is a node of middle phalanx, \( C(x_C, y_C, z_C) \) is a node of distal phalanx, \( D(x_D, y_D, z_D) \) is a finite node.
2.3 Solving the inverse kinematics problem

The second kinematics problem has two conditions: the finger kinematic diagram of the gripping device is given, the position and orientation in the coordinate system associated with the gripper base are known. It is required to determine the angles of orientation of the finger links of the anthropomorphic grip of the robot.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>α_A</td>
<td>0° - 75°</td>
</tr>
<tr>
<td>α_B</td>
<td>0° - 90°</td>
</tr>
<tr>
<td>α_C</td>
<td>0° - 90°</td>
</tr>
</tbody>
</table>

Table 2: Limits of finger rotation angles

Due to the cable transfer of the movement to the links of the finger, the movement can be regarded as uniform. In this case, according to the table's 2 parameters, it is possible to determine the relationship between the angles of rotation of the finger links of the anthropomorphic gripping device

\[
\begin{align*}
\alpha_B, & 0 \leq \alpha_B \leq 90 \\
\alpha_C & = k_{\alpha C} \alpha_B, k_{\alpha C} = 1 \\
\alpha_A & = k_{\alpha A} \alpha_B, k_{\alpha A} = 0.83 \\
\end{align*}
\]

(15)

\(k_j\) is a coefficient of rotation limitation of the phalanx finger. The angle \(\alpha_B\) is taken as the reference one due to the fact that due to the special design of the gripper device, the finger contains only one encoder installed in the node B.

Calculate the angle \(\alpha\) by the length of vectors \(\overrightarrow{AB}\) and \(\overrightarrow{BC}\). The Fig 4. shows how is this angle looks like. Consider a triangle ABC.

![Figure 4: Position of the \(\alpha_B\) angle](image)

The lengths of vectors \(\overrightarrow{AB}\) and \(\overrightarrow{BC}\) are known, and if the points coordinates \(A(x_A, y_A, z_A)\) and \(C(x_C, y_C, z_C)\) are known as well, then calculate the vector's length \(\overrightarrow{AC}\) by formula

\[
l_{AC} = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2 + (z_C - z_A)^2}.
\]

(16)

Calculate the angle \(\alpha_B\) by the law of cosines:

\[
\alpha_B = \arccos\left(\frac{l_{AB}^2 + l_{BC}^2 - l_{AC}^2}{2l_{AB}l_{BC}}\right) \tag{17}
\]

Thus, the angles \(\alpha_A\) and \(\alpha_B\) are founded by formula (15).

So, if the coordinates of the finger points of the robot’s finger are known and the link lengths are known as well then the determination of the angles of orientation of the finger links of an anthropomorphic robot gripping device is possible by the formulas (15-17).

3 Results

3.1 Results of solving the direct kinematics problem

Calculate the coordinates of the spatial position of the gripper fingers \(A(x_A, y_A, z_A), B(x_B, y_B, z_B), C(x_C, y_C, z_C), D(x_D, y_D, z_D)\) by the parameters in the tables 3.
Table 3: Input parameters for solving the direct kinematics problem

<table>
<thead>
<tr>
<th>No</th>
<th>Description</th>
<th>Initial parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The lengths of the gripper fingers (mm)</td>
<td>$l_{AB} = 5.8; l_{BC} = 4.2; l_{CD} = 3.3;</td>
</tr>
<tr>
<td>2</td>
<td>Angles of orientation of the links of the gripping device</td>
<td>$\alpha_A = 49^\circ; \alpha_B = 60^\circ; \alpha_C = 60^\circ$</td>
</tr>
</tbody>
</table>

Calculate the matrices $A_n$ and $T_{CD}$ by table 3 parameters.

$$A_A^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_B^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5.8 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_C^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{CD} = \begin{bmatrix} 0 \\ 0 \\ 3.3 \\ 1 \end{bmatrix},$$

In this case, the coordinates of the finger nodes are the following

$A(x_A; y_A; z_A) = (0; 0; 0);$

$B(x_B; y_B; z_B) = (0; -4.011; 4.189);$

$C(x_C; y_C; z_C) = (0; -8.208; 4.033);$

$D(x_D; y_D; z_D) = (0; -10.307; 1.486);$

There are results of solving the direct kinematics problem on the Fig 5.

Thus, it is possible to define coordinates of a spatial position of nodal points of a finger anthropomorphic grip by kinematic diagram and the given orientation of its links. In this case, the matrix methods are used to avoid cumbersome expressions.

3.2 Results of solving the inverse kinematics problem

Calculate the orientation of the finger links of the grip device $\alpha_A, \alpha_B, \alpha_C$ by the initial parameters in the table 4.

Calculate the angle $\alpha_B$ by the vectors length $\overrightarrow{AB}$ and $\overrightarrow{BC}$, consider the triangle ABC.
Table 4: Initial parameters for solving the inverse kinematics problem

<table>
<thead>
<tr>
<th>No</th>
<th>Description</th>
<th>Initial parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The lengths of the gripper fingers (mm)</td>
<td>( l_{AB} = 5.8; l_{BC} = 4.2; l_{CD} = 3.3; )</td>
</tr>
<tr>
<td>2</td>
<td>Coordinates of the gripper finger points</td>
<td>( A(x_A; y_A; z_A) = (0; 0; 0), )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( B(x_B; y_B; z_B) = (0; -5.624; 1.418), )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C(x_C; y_C; z_C) = (0; -7.043; 5.371), )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D(x_D; y_D; z_D) = (0; -7.043; 5.371) )</td>
</tr>
</tbody>
</table>

The vectors length are known, let calculate the length of the vector \( \overrightarrow{AC} \): \[
l_{AC} = \sqrt{(0 - 0)^2 + (7.043 - 0)^2 + (5.371 - 0)^2} = \sqrt{78.4515} = 8.857.
\]

Let calculate the angle \( \alpha_B \) by the law of cosines \[
\alpha_B = \arccos\left(\frac{l_{AB}^2 + l_{BC}^2 - l_{AC}^2}{2l_{AB}l_{BC}}\right) = \arccos\left(\frac{5.8^2 + 4.2^2 - 8.857^2}{2 \times 5.8 \times 4.2}\right) = 77.
\]

According to the dependence defined by formula (15), calculate the remaining angles of orientation of the links \( \alpha_A \) and \( \alpha_A \)

\[
\alpha_A = k\alpha_A \alpha_B = 0.83 \times 77 = 63.91; \alpha_C = \alpha_B = 77.
\]

There are results of solving the inverse kinematics problem on the Fig.6.

Figure 6: Results of solving the inverse kinematics problem

Thus, the position and orientation of the links of the anthropomorphic grip finger were found by the kinematic diagram and coordinates of the nodal points.

4 Discussion

The originality of the work lies in the fact that a method developed specifically for anthropomorphic capture with an executive group of links allows us to determine the spatial position of the links of the gripping device on the basis of solving the direct kinematics problem by calculating the Cartesian coordinates of the nodal points of the gripping device. The method is based on the use of the Denavite-Hartenberg representation, which allows direct calculations of the coordinates of nodes with minimum computing power. At known coordinates of nodal points and lengths of links for definition of angles of orientation of links of a finger of an anthropomorphic capture device of the robot it is offered to use a geometrical method. The developed method can be adapted and applied to gripping devices characterized by the number of fingers, links and degrees of mobility, which indicates the flexibility of the method.
5 Conclusion

Based on the results of solving the problems posed in the paper, we can formulate the following conclusion. During the analysis of the literature on the research topic, it was established that existing methods and algorithms are developed and specialized exclusively for certain devices. Therefore, they are not applicable to the target device considered in this article. Thus, the method for determining the spatial position and orientation of the links of anthropomorphic grip by the robot on the basis of solving the direct and inverse kinematics problem is described in this paper. This method allows to determine the spatial position of the links AB, BC, CD of the gripping device on the basis of the solution of the direct kinematics problem, by calculating the Cartesian coordinates of the anchor points of the gripping points: \( A(x_A,y_A,z_a) \) is the joint of the proximal phalanx, \( B(x_B,y_B,z_B) \) is the joint of the middle phalanx, \( C(x_C,y_C,z_C) \) is the joint of the distal phalanx; \( D(x_D,y_D,z_D) \) is the end of the finger. The proposed method is based on the calculation of transformation matrices according to the Denavit-Hartenberg representation. The formulas (5-14) were found analytically to solve the problem of calculating the Cartesian coordinates of nodal points of the grip device. It is proposed to use a geometric method for determining the angles of orientation of the finger links of the anthropomorphic gripping device by the robot by coordinates of node points and link lengths based on which the formulas (15-17) were obtained.

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