The adaptive control system of quadrocopter motion

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Abstract

In this paper we present a system for automatic control of a quadrocopter based on the adaptive control system. The task is to ensure the motion of the quadrocopter along the given route and to control the stabilization of the quadrocopter in the air in a horizontal or in a given angular position by sending control signals to the engines. The nonlinear model of a quadrocopter is expressed in the form of a linear non-stationary system.

1 Introduction

The UAV (Unmanned aerial vehicle) can be controlled by an occasional command or continuously, in the latter case the UAV is called a remotely piloted aircraft (RPA). The main advantage of the UAV / RPA is the significantly lower cost of its production and control (under the condition of equal efficiency in the performance of the tasks).

Nowadays unmanned aerial vehicles (UAVs), which are mainly flying robots, constitute an important part of scientific research in military, civil and space fields. Replacing manned vehicles, UAVs have an advantage in complex and dangerous environments. Their reliability in severe conditions for humans is much higher.

In the last decade, studies on various types of unmanned multi-rotors have received much attention in the field of automatic control. Quadrotor is the most popular type of multi-rotor UAV due to its simple mechanics and high maneuverability such as the fast vertical take-off and landing. In addition, the implementation of stable stationary flight is a valued opportunity for a quadrotor. However, controlling the motion of a quadrotor is a difficult and challenging task because of its nonlinearity under controlled dynamics. Many methods for controlling quadrotors were proposed [Alt03], and some strategies were developed for solving the issues which follow the problems of this system type. Early control strategies are traditional PID control, feedback linearization and LQR methods. Recently, nonlinear controllers such as TSMC [Bes07], backstepping control [MBe06] and model predictive control (MPC) [Ma16] have been used to improve tracking accuracy and resistance to model

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uncertainty and external disturbances. In addition, adaptive controllers are designed for a low-power non-linear quadrotor system to eliminate the tracking error, despite disturbances.

In spite of the fact that the mentioned controllers have the reliable tracking the trajectories, they are just suitable for implementation on unmounted stationary the control units. On the other hand, they need fast and heavy computing devices because of their complex and time-consuming control laws. In addition, they are limited by long-range applications, when the delay of communication with the stationary control unit disrupts the real-time operation of the quadrotor. The conventional proportional integral-derivative (PID controller) is the most preferred control method for on-board implementation of the control unit in autonomous systems. However, it is very difficult to adjust the PID gain to a number of operating points or for different continuous trajectories. In addition, PID regulators are not very successful for controlling non-linear systems with reduced voltage. Therefore, several methods are used to develop a PID controller with improved tracking characteristics, which may be suitable for on-board implementation. Fuzzy logic is a powerful tool that is often used to obtain excellent results compared to a conventional PID controller.

Consequently, there is a need for a mathematical model which could describe the control of a quadcopter. The difficulty is that a quadrocopter has 6 degrees of freedom, while we can control only 4 parameters: the angular velocities of the engines.

The next important task is to build a stabilizing algorithm. Controlled by four spaced apart engines, a quadrocopter is an unstable dynamic system which, due to the nonlinearity of the mathematical model, must be stabilized by the complex control algorithms.

2 Related work

Many modeling approaches have been presented [Erg07],[Bou04] and various control methods have been proposed [Alt02], [Alt03]. First of all, several backstepping controllers have been developed. Madani studied a full-state backstepping technique based on Lyapunov stability [Mad06], [MBe06]. E. Altug presented backstepping control using single and double cameras as visual feedback [Alt02], [Alt03]. Other backstepping control methods were used by Castillo. He used this controller with a saturation function and it performed well under perturbation [Cas06]. Also, Metni used backstepping technique in order to obtain adaptive nonlinear tracking law for quadrotors system [Met07].

Feedback linearization controller was implemented by Altug [Alt02]. A PD controller was designed to control y and and feedback linearization controller was implemented to control x and z. A. Benalleuge presented feedback linearization high-order sliding mode observer for a quadrotor. The algorithm had shown robustness for wind disturbances and noise [Ben06].

The method of quaternions for position stabilization was presented in [Tay06]. With compensation of the Coriolis and gyroscopic torques, the controller guaranteed exponential stability while a classical PD controller without compensation of the Coriolis and gyroscopic torques could guarantee only asymptotic stability. A sliding mode disturbance observer was shown [Bes07] and designed as the robust controller for quadrocopters. This controller showed the robustness for external disturbances, model uncertainties, and engine's errors. The robust adaptive fuzzy control was applied in [Coz06]. This controller showed a good performance against sinusoidal wind disturbances. Mokhtari presented in [Mok04] robust dynamical feedback controller of Euler angles which used the estimation wind parameters.

3 Proposed method

3.1 The matrix of rotation

The rotation around axis OX is described by the matrix:

$$\mathrm{R_x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix} \tag{1}$$

Around axis OY

$$\mathrm{R_y} = \begin{pmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{pmatrix} \tag{2}$$

And around axis OZ

$$R_z = \begin{pmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The matrix $D = R_x * R_y * R_z$ describes the transition from Earth-fixed frame to the Body-fixed frame. This matrix is presented as:

$$D = \begin{pmatrix} \cos\Theta\cos\Psi & \cos\Theta\sin\Psi & -\sin\Theta \\ \sin\vartheta\sin\Theta\cos\Psi - \cos\varphi\sin\Psi & \sin\vartheta\sin\Theta\cos\Psi + \cos\vartheta\cos\Psi & \sin\vartheta\cos\Theta \\ \cos\vartheta\sin\Theta\cos\Psi + \sin\vartheta\sin\vartheta & \cos\varphi\sin\Theta\cos\Psi + \sin\vartheta\cos\vartheta & \cos\vartheta\cos\Theta \end{pmatrix} \tag{4}$$

The transition from in Earth-fixed body to the velocities in Body fixed system is defined by equations:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = D^{-1} \begin{pmatrix} u \\ v \\ \omega \end{pmatrix} \tag{5}$$

Transformation angles velocities for the transition from one frame system to an-other one will be defined in the following way:

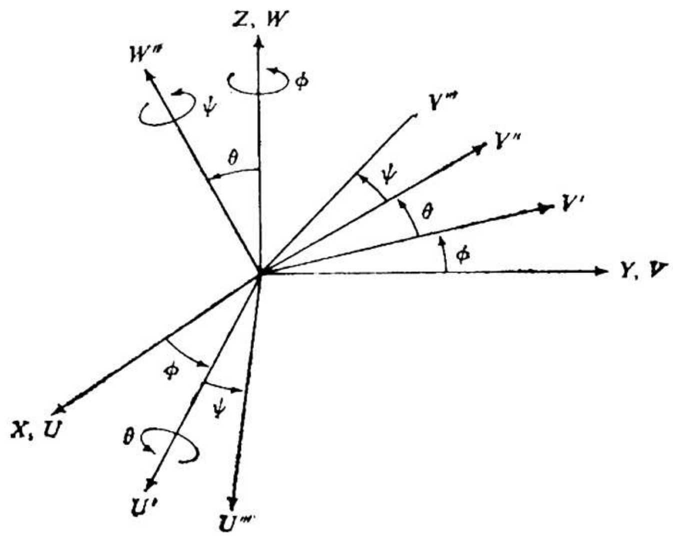

Figure 1: Euler's angles

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = E \begin{pmatrix} \dot{\varphi} \\ \dot{\Theta} \\ \dot{\Psi} \end{pmatrix} \tag{6}$$

E matrix is represented

$$E = \begin{pmatrix} 1 & 0 & \sin\Theta \\ 0 & \cos\vartheta & \sin\vartheta\cos\Theta \\ 0 & -\sin\vartheta & \cos\vartheta\cos\Theta \end{pmatrix} \tag{7}$$

3.2 The equation of moving

The written second law of Newton in Earth-fixed frame:

$$\mathrm{F} = \mathrm{m}\dot{\mathrm{V}} \tag{8}$$

Here m – a mass of a vehicle, which is constant. V – velocity of UAV in the Earth-fixed frame. Writing the total derivative [Cas05] of velocity for time, we will rewrite the law:

$$\mathrm{F} = \mathrm{m}\dot{\mathrm{V}} + \mathrm{m}(\overrightarrow{\omega} * \overrightarrow{V}) \tag{9}$$

or

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = m \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{\omega} \end{pmatrix} + m \begin{pmatrix} p \\ q \\ r \end{pmatrix} * \begin{pmatrix} u \\ v \\ \omega \end{pmatrix} \tag{10}$$

After a performance of cross product [Che03], the notation of Newton's second law will be obtained:

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = m \begin{pmatrix} \dot{u} + q\omega - rv \\ \dot{v} + ru - p\omega \\ \omega + p\dot{v} - qu \end{pmatrix} \tag{11}$$

Neglecting all forces beside thrust force of propeller [Coo97] T and force of gravity [Lel11], the equation (11) is writing as:

$$\begin{pmatrix} 0 \\ 0 \\ mg - T \end{pmatrix} = m \begin{pmatrix} \dot{u} + q\omega - rv \\ \dot{v} + ru - p\omega \\ \omega + p\dot{v} - qu \end{pmatrix} \tag{12}$$

Transition to Body fixed frame with the help of transfer matrix D, we obtain equation:

$$\begin{gathered} \dot{u} = rv - q\omega - g\sin\Theta \\ v = p\omega - ru + g\sin\Theta\sin\varphi \\ \dot{\omega} = qu - pv + g\cos\varphi\cos\Theta - \tfrac{1}{m}. \end{gathered} \tag{13}$$

The thrust force of all engines can be shown in the following way [Bou04]:

$$T = b(\Omega_1^2\Omega_2^2\Omega_3^2\Omega_4^2) \tag{14}$$

Where b – the trust coefficient [Dor07] and Ω_i - the velocity of each engine ($i = 1, 2, 3, 4$). In this case, the equation (13) is rewritten as:

$$\begin{gathered} \dot{u} = rv - q\omega - g\sin\Theta \\ v = p\omega - ru + g\sin\Theta\sin\varphi \\ \dot{\omega} = qr - pv + g\cos\varphi\cos\Theta - \frac{b}{m}(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \end{gathered} \tag{15}$$

3.3 Rotating motion

The torques of quadrotor's rotation represented as:

$$M = \dot{H} \tag{16}$$

Total derivative of vector H is written as:

$$M = \dot{H} + \omega * H \tag{17}$$

Let

$$H = I * \omega \tag{18}$$

Here I – torque of quadrocopter's inertia $\omega = (p, q, r)$ - vector Tensor of torque inertia is written as:

$$I = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_y \end{pmatrix} \tag{19}$$

Then the equation (17) is rewritten:

$$M = I\dot{\omega} + \omega * I\omega$$

After substitution:

$$\begin{gathered} M_x = \dot{p}I_x + qr(I_zI_y) \\ M_x = \dot{q}I_y + pr(I_xI_z) \\ M_x = \dot{z}I_z + pq(I_yI_x) \end{gathered} \tag{20}$$

By symmetry relative planes XZ and YZ

$$I_x = I_y$$

And equation (20) becomes simpler

$$\begin{gathered} M_x = \dot{p}I_x + qr(I_z - I_y) \\ M_x = \dot{q}I_y + pr(I_x - I_z) \\ M_x = \dot{r}I_z \end{gathered} \tag{21}$$

Considering the thrust force and drag force the torques will be rewritten as:

$$\begin{gathered} M_x = lb(\Omega_2^2\Omega_4^2) \\ M_y = lb(\Omega_1^2\Omega_3^2) \\ M_z = d(\Omega_2^2\Omega_4^2\Omega_1^2\Omega_3^2) \end{gathered} \tag{22}$$

d - drag coefficient [Fra05], l – the length of the propeller Transforming the system (22) the last equations of moving are presented in the following way:

$$\begin{gathered} \dot{p} = \frac{lb}{I_x}(\Omega_2^2\Omega_4^2) - qr\frac{I_z - I_x}{I_x} \\ \dot{q} = \frac{lb}{I_y}(\Omega_1^2\Omega_3^2) - pr\frac{I_x - I_z}{I_y} \\ \dot{r} = \frac{d}{I_z}(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{gathered} \tag{23}$$

3.4 The dynamic of the engine [Hof07]:

Kirchhoff's equation and the second law of Newton represent the equations of quadrocopter's engines.

$$\begin{cases} J_r\ddot{\omega}_m + b\dot{\omega}_m = K_ti, \\ L\frac{d_i}{d_t} + R_i = U - K_e.\dot{\omega_m} \end{cases} \tag{24}$$

Here
J_r – torque of inertia shaft,
b – coefficient of viscous friction,
K_e – coefficient EMF,
K_t – torque of engine rotation,
R – electric resistance,
L – inductivity.

4 Adaptive control

Adaptive control is a method used for automatic control of moving in the real time. It uses the online estimating of external parameters and automatic control. The system of equations describes the kind of control presented:

$$\dot{x}(t) = A_m x(t) + b(u(t) + K_x^T x(t)), \quad x(0) = x_0 \tag{25}$$

$$y(t) = C^T x(t) \tag{26}$$

Here Am known matrix, $x(t)$ condition vector of system, b and c known constants, K_x – vector of unknowns. Let

$$U_{norm}(t) = -k_x^T x(t) + k_g r(t) \tag{27}$$

$$k_g = \frac{1}{c^T A_m^1 b} \tag{28}$$

Due to simplification the following equations for ideal system are obtained:

$$\begin{gathered} \dot{x_t}(t) = A_m x(t) + bk_g r(t), \quad x_m(0) = x_0 \\ y_m(t) = C^T x(t) \\ e(t) = x_m(t) - x \end{gathered} \tag{29}$$

The law of adaptive control

$$\dot{K_x} = -\Gamma x(t) \tilde{e}^T(t) pb, \quad K_x(0) = K_{x0} \tag{30}$$

Matrix P is defined from equation of Lyapunov

$$A_m^T P + P A_m = -Q \tag{31}$$

The diagram of such control type has been represented:

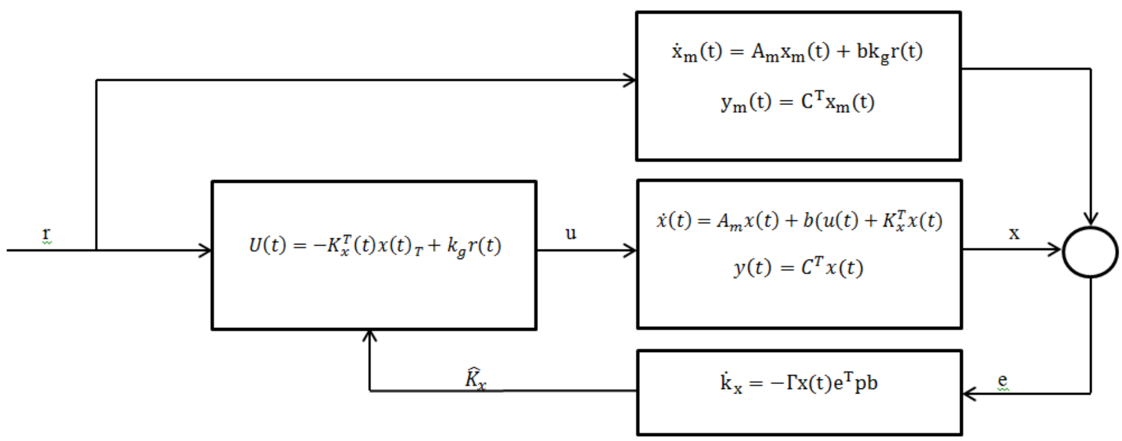

Figure 2: The diagram of adaptive control

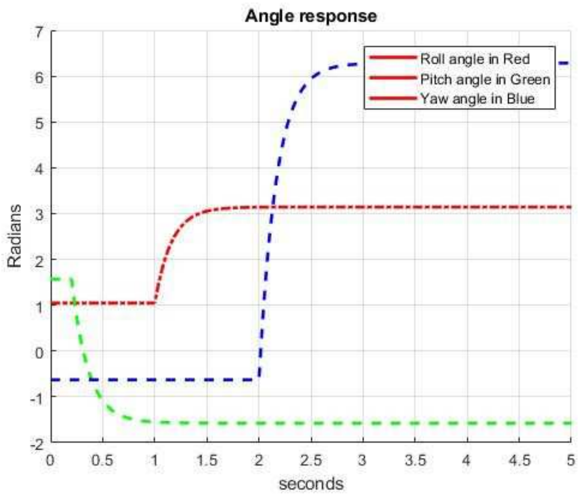

Figure 3: Angle response

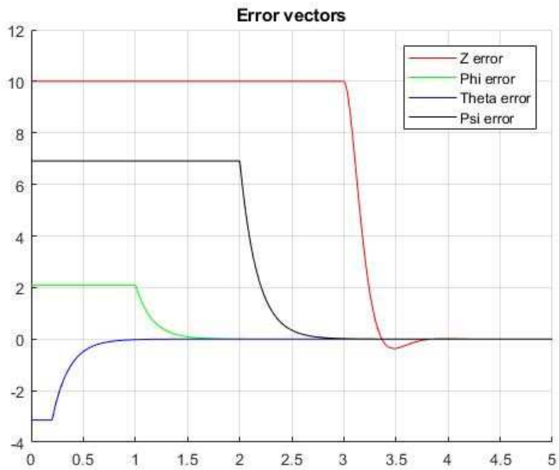

Figure 4: Error response

4.1 Transfer function

The system of equations (24) describes the work of engines. In order to obtain the transfer-function [Kiv06], the transformation of Laplace was implemented in the system (24):

$$\begin{cases} S(J_s S + b)C(s) = K_t I(s) \\ (L_s + R)I(s) = u(s) - K_e S C(s) \end{cases} \tag{32}$$

$$W(s) = \frac{\dot{C}(s)}{u(s)} = \frac{k_t}{(J_s S + b)(L_s + R) + k_e k_t} \tag{33}$$

Here $\dot{C}(s)$-the velocity of the motor shaft; $u(s)$ - voltage input. The result of modeling is represented on the graphics: 3, 4, 5.

Conclusion

The adaptive control as a control system has been considered. The evaluation has been used in the design of the controller. Comparative results of simulation tests have been carried out in MatLab / Simulink. The

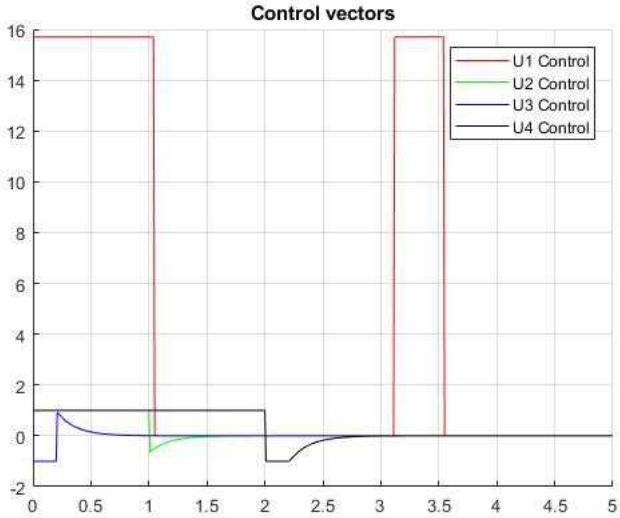

Figure 5: Control vectors

results of simulations demonstrate that the quadrocopter is successfully stabilized, keeping the desired position and altitude using the proposed controller, thereby showing the effectiveness of the proposed system under conditions of uncertainty.

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