Financial sustainability evaluation of higher education institutions using “compatible” cognitive maps

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Abstract. The paper considers the features of monitoring the effectiveness of financial and economic activities of higher education institutions. Integral (group) indicators are identified and the main approaches to obtaining evaluations for them are considered. The distinctive features of the financial sustainability group indicator are considered. The evaluation method of the financial sustainability indicator for higher education institutions is presented on the basis of fuzzy cognitive modeling taking into account the different compatibility levels between the concepts and different weights of mutual influence. A set of convolution operations corresponding to various degrees of compatibility between concepts has been selected and justified. Based on the results of the simulation, it is possible to determine: the facts of exceeding by a certain moment the concepts criterial values; predictive values and trends in the values of different concepts; effects evaluation of direct and mediated external influences on concepts.

1. Introduction
The effectiveness of financial and economic activities of higher education institutions is determined largely by the quality of the planning organization and budgeting processes in the institution. In order to control the effectiveness of spending budget funds, the Ministry of Education monitors a number of integrated (group) indicators of financial and economic activity, including:

- planning quality indicator;
- financial sustainability indicator;
- strategic planning indicator;
- distribution of the organization's funds;
- accrued salary fund;
- scientific research costs.

As a methodological support for obtaining evaluations for the selected integrated indicators, it is proposed to use two main approaches.

- Application of deterministic algorithms that provide an exact solution. These algorithms allow to obtain values of integral quality indicators based on averaging of weighted partial evaluations (average values of monitoring indicators or average value of the final quality indicator depending on the task setting). This approach to obtaining estimates of the financial and economic activities effectiveness is used for most group indicators.

- Application of fuzzy cognitive modeling algorithms to obtain values. The fuzziness can be caused by the absence in some cases of input primary data exact values (monitoring indicators), for example, due to the lack of indicator values for the estimated period (quarter,
month) in the information array. The predicted values are characterized by uncertainty, probability; subjective assessments of experts can be applied. Described types of uncertainty can be found in the financial sustainability integral factor, which is one of the main monitoring indicators of the higher education institutions financial activities effectiveness.

Actively developed methods of cognitive analysis [1-3] and modeling provide advanced opportunities for qualitative and quantitative analysis and modeling under conditions of stochastic and non-stochastic uncertainty, allowing to solve a wide range of both analytical problems [4] (stability analysis, identification of undesirable cycles, evaluation of system characteristics, analysis of direct, aggregated and mediated influence of system factors on each other, scenario analysis under various impacts, and the availability of target situations, the forecast of the development of the state of systemic factors), and modeling tasks [7,8] (modeling of system dynamics without and in the presence of external influences, modeling states in conditions of limited resources).

At the same time, in the case of the financial sustainability indicator, it is also necessary to take into account the compatibility of system factors - concepts - both in determining the direct influence between the concepts, and in accumulating the influence of several concept-sources on the concept-receiver. But, simultaneously, the accounting mechanism of the systemic factors compatibility, realized in generalized production fuzzy cognitive maps: first, it is oriented only to the representation of the concepts meanings and relations between them in the form of linguistic terms; secondly, it is quite complicated for formalization and expert interpretation; thirdly, it requires rather complicated procedures for constructing production rules for modelling system dynamics.

The paper considers the method that allows to generalize the solution of the problem of the system factors compatibility accounting with different levels of influence between them on the example of the higher educational institutions financial sustainability indicator.

2. Fuzzy cognitive model for financial sustainability integral indicator

A fuzzy cognitive map, taking into account the compatibility of its concepts, represented as follows:

$$ FCM = \{K, W, C\}, $$

where $K = \{K_1, ..., K_N\} \rightarrow$ set of concepts; $W = \{w_{ij}\} \rightarrow$ set of concepts influence weights on each other; $C = \{c_{ij}\} \rightarrow$ set of compatibility levels of concepts pairs, $i, j = 1, ..., N [9]$. 

Depending on the nature of problem being solved, [5] compatibility of concepts can be interpreted as correlation, mutual influence, simultaneous achievement of their criterial values, etc.

Hereinafter, the compatibility levels of map concepts are taken into account in the choice of operations both for determining the direct spread of influence between concepts, and for accumulating the influence of several concept-sources on the concept-receiver.

As an example of using the proposed method of fuzzy cognitive modeling to obtain evaluations, the construction of a possible compatible cognitive model for calculating the integral financial sustainability indicator based on the accumulation of the influence of private concepts is considered.

Based on the normative documents regulating private indicators of monitoring the educational institutions financial activities, in constructing a fuzzy cognitive model, a structure of concepts was formed, consisting of financial sustainability integral indicator of educational institution $K_6$ (target concept); and five private concepts of a compatible cognitive map that influence one another and directly on the target concept:

- $K_1$ – the share of income from income-generating activities in the total income from income-generating activities and subsidies for financial support for the fulfillment of the state task (autonomy indicator);
- $K_2$ – the increase in income from income-generating activities in relation to the previous year;
- $K_3$ – institution dependence on borrowed financing sources (debt ratio);
- $K_4$ – the presence of the overdue accounts payable;
• $K_5$ – the presence of accounts receivable to collection.

The structure of the obtained fuzzy cognitive model of the financial sustainability integral indicator is presented on figure 1.

**Figure 1.** Fuzzy cognitive model of the financial sustainability integral indicator.

### 3. Construction of the mutual influence weights table

Concepts meanings and their mutual influences representation in the form of fuzzy numbers, sets, and relations makes it possible to use for their evaluation methods of fuzzy arithmetic, fuzzy sets and relations, fuzzy logic for their evaluation.

Consider, for example, fuzzy cognitive maps, characterized by the following features [5]. Concepts $K_i$ take values from the range $[0, 1]$. The levels of influence between each concept pair $K_i$ and $K_j$ from $K = \{K_1, ..., K_\nu\}$ are represented in the form of directed arcs with weights $w_{ij} \in [-1, 1]$.

Value $w_{ij} = -1$ characterizes the most negative, and $w_{ij} = 1$ – maximally positive impact of the concept $K_i$ on concept $K_j$, and value $w_{ij} = 0$ means the absence of direct influence of the concept $K_i$ on concept $K_j$. With the positive influence of the concept $K_i$ on the concept $K_j$ ($w_{ij} > 0$) and increasing $K_i$ means that the value $K_j$ grows, and decreasing – diminish. When $w_{ij} < 0$ value increasing of $K_i$ will reduce the value $K_j$ and, vice versa.

It is also possible to simultaneously influence the pair of concepts against each other with different values $w_{ij}$ and $w_{ji}$.

According to the expert survey results, the mutual influence weights of the considered concepts are represented on table 1.
### Table 1. Mutual influence weights of private indicators.

<table>
<thead>
<tr>
<th>№</th>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K_1$</td>
<td>0</td>
<td>0.85</td>
<td>0.72</td>
<td>-0.22</td>
<td>-0.34</td>
<td>0.87</td>
</tr>
<tr>
<td>2</td>
<td>$K_2$</td>
<td>0.63</td>
<td>0</td>
<td>0.44</td>
<td>-0.31</td>
<td>0</td>
<td>0.69</td>
</tr>
<tr>
<td>3</td>
<td>$K_3$</td>
<td>-0.19</td>
<td>0.40</td>
<td>0</td>
<td>-0.12</td>
<td>-0.21</td>
<td>-0.35</td>
</tr>
<tr>
<td>4</td>
<td>$K_4$</td>
<td>0</td>
<td>-0.43</td>
<td>-0.36</td>
<td>0</td>
<td>-0.36</td>
<td>0.56</td>
</tr>
<tr>
<td>5</td>
<td>$K_5$</td>
<td>0.45</td>
<td>0.44</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
</tr>
<tr>
<td>6</td>
<td>$K_6$</td>
<td>0.91</td>
<td>0.79</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4. Construction of the mutual influence weights table

To determine the compatibility levels of the fuzzy cognitive map concepts, both direct and inverse methods can be used. When using direct methods, these values are either set by experts or obtained as a result of experiments.

Inverse methods are used in the case of the direct evaluation complexity of the concepts compatibility levels. For example, a method based on a pairwise comparison of directly interacting concepts and filling the so-called compatibility tables between the linguistic values of these indicators can be applied [10].

For fuzzy cognitive maps considered in [2], when assessing the concepts compatibility levels, one can also use the results of the relationships analysis of their mutual influence on the basis of indicator of consonance (coherence) of the concepts influence on each other.

Based on the compatibility tables analysis for all pairs of private concepts, six criteria compatibility levels are defined: {1 – “No compatibility”, 2 – “Low level”, 3 – “Below average level”, 4 – “Above average level”, 5 – “High level” 6 – “Full compatibility”}.

Table 2 presents a fuzzy relation of the concepts compatibility of the cognitive model in question as a compatibility matrix.

### Table 2. Fuzzy compatibility matrix.

<table>
<thead>
<tr>
<th>№</th>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K_1$</td>
<td>–</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>$K_2$</td>
<td>5</td>
<td>–</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>$K_3$</td>
<td>2</td>
<td>3</td>
<td>–</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>$K_4$</td>
<td>–</td>
<td>3</td>
<td>3</td>
<td>–</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>$K_5$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>–</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>$K_6$</td>
<td>6</td>
<td>5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

### 5. The fuzzy convolution operations basis

To directly spread influence from the concept $K_i$ to the concept $K_j$ as operations of fuzzy influences “weighing” (considering the compatibility of these concepts) it is advisable to use the family of parametrized operations $med$, which satisfy the axioms of normalization, nondecreasing, continuity, bisymmetry [6]. Depending on the problem situations nature, the following expressions can be used:

$$K_j = med(K_i; w_{ij}; c_{ij}); \quad K_j = K_j \pm med(\Delta K_i, w_{ij}; c_{ij}); \quad K_j = K_j \pm med(K_j; w_{ij}; c_{ij}),$$

where $K_j$ – state (value) of the j-th concept-receiver; $K_i$, $\Delta K_i$ – state (value) and increment of the i-th concept-receiver; $i, j \in [1, ..., N], c_{ij} \in [0, 1]$.

To compare the compatibility levels of aggregated concepts with the operations of convolution, as a rule, direct methods are used, in which the expert establish such conformity.
When evaluating and selecting alternatives, different strategies can be used, the extreme variants of which are: achieving the lowest value from all indicators or reaching a maximum value for at least one of the indicators.

For the two-place case, the following convolution operations correspond to these extremal strategies: min\(k^*, k^*\) = med\(k^*, k^*; 0\) and max\(k^*, k^*\) = med\(k^*, k^*; 1.0\).

Thus, it can be concluded that the whole set of compromise strategies provides a parametrized family of convolution operations of the type: med\(k^*, k^*; \alpha\), \(\alpha \in [0, 1]\).

To determine the degree of compromise of two-fold convolution operations, it is proposed to use the parameter \(\theta \in [0, 1]\), in this case, the smaller the value of the parameter, the less the degree of compromise of indicators.

The values of the parameter \(\theta\) are calculated in accordance with the expression:

\[
\theta = \frac{V - V_{\min}}{V_{\max} - V_{\min}},
\]

where \(V\) – the “volume” value under the surface of the function formed as a result of applying the corresponding convolution operation; and \(V_{\min}, V_{\max}\) – as a result of operations min\(k^*, k^*\) = med\(k^*, k^*; 0\) and max\(k^*, k^*\) = med\(k^*, k^*; 1.0\), respectively.

Parameter \(\theta\) values can be used to determine parameter \(\alpha\) of this convolution and characterize the compatibility levels of the aggregated concepts. Figure 2 shows the dependence \(\theta\) on the \(\alpha\).

![Figure 2](image)

**Figure 2.** The dependence of \(\theta\) on the \(\alpha\) for the med\(k^*, k^*; \alpha\) convolution operation.

Using the proposed approach, the selected compatibility levels of indicators were compared to the following convolution operations:

\[
\begin{align*}
h_1(k', k^*) &= \text{med}(k', k^*; 0) & h_2(k', k^*) &= \text{med}(k', k^*; 0.43) & h_3(k', k^*) &= \text{med}(k', k^*; 0.71) \\
h_4(k', k^*) &= \text{med}(k', k^*; 0.29) & h_5(k', k^*) &= \text{med}(k', k^*; 0.56) & h_6(k', k^*) &= \text{med}(k', k^*; 1.0)
\end{align*}
\]

The set of operations directly influencing the concept \(K_i\) to the concept \(K_j\) considering the compatibility levels of these concepts is represented on the table 3.
6. The strategies for the influences accumulation

If the number of private concept-sources is more than 2, the result may depend on the order of accumulation of their influences.

The following strategies for accumulating the effects of several concept-sources on one concept-receiver are applicable: either from the least to the greatest compatibility level of concepts, or from the greatest to the least compatibility level of concepts.

Thus, for the example presented in Figure 1, the expression for accumulating the concepts influence on the target concept of the financial sustainability integral indicator (at the accumulation order from the least to the greatest concepts compatibility level \( (((K_1 \leftrightarrow K_4) \leftrightarrow K_3) \leftrightarrow K_4) \leftrightarrow K_2 \)) takes the form:

\[
K_6 = K_2 + \text{med}(K_4 - \text{med}(K_4 - \text{med}(K_4, w_{13}; 0.29), w_{13}; 0.43)), w_{25}; 0.56 \).
\]

The concepts compatibility here is interpreted as the simultaneous attainability of the criterial concepts values.

Modelling the influence propagation on the map can be performed under conditions of self-situation, or with external force moments at various values of concepts, which all reduces to the following sequence:

- setting the initial concepts values fuzzy cognitive map;
- the simulation start in accordance with the selected expression to change the concepts values;
- setting the influences on the map concepts at certain moments (changing their values);
- completion of the simulation when the selected criterion is fulfilled.

Based on the results of the simulation, it is possible to determine: the facts of exceeding by a certain moment the concepts criterial values; predictive values and trends in the values of different concepts; effects evaluation of direct and mediated external influences on concepts.

7. Conclusion

The article considers the features of monitoring the effectiveness of financial and economic activities of higher education institutions. Integral (group) indicators are identified and the main approaches to obtaining evaluations for them are considered. The distinctive features of the financial sustainability group indicator are considered.

An approach allowing to generalize the problem solution of the system factors compatibility accounting in the transfer of influence between them for different fuzzy cognitive maps is considered.
on the example of the financial sustainability integral factor. A set of convolution operations corresponding to various degrees of compatibility between concepts has been selected and justified.

8. References


Acknowledgments

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