Simulation of Thermal Processes in Permafrost: Parallel Implementation on Multicore CPU

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Abstract. The processes of seasonal changes in upper layers of permafrost soil are described. A thermal conductivity equation is considered and algorithm is presented for solving it. An approach of computational optimization is suggested. The proposed parallel algorithm is implemented for multicore processor using OpenMP technology.

Keywords: thermal conductivity, numerical simulation, permafrost, parallel computing, MPI, OpenMP

1 Introduction

Permafrost occupies 35 million km^2 (about 25% of the total land area of the world [1, 2]. In Russia, more than 60% of its territory is occupied by permafrost, and 93% natural gas and 75% oil are produced in these regions. Human activity, production and transportation of oil and gas have a significant effect on permafrost. The warm oil heated the pipes in wells and pipelines and other processes may lead to permafrost degradation. Therefore, the problem of reducing the intensity of thermal interactions in the "heat source — permafrost" zones has a particular importance for solving problems of energy saving, environmental protection, safety, cost savings, and enhance operational reliability of various engineering structures [3, 5, 4, 6].

We consider new three-dimensional model which allows one to describe the heat distribution in upper layers in permafrost soils with taking into consideration not only most significant climatic factors (seasonal changes in temperature and intensity of solar radiation due to geographic location of the field) and physical factors (different thermal characteristics of non-uniform ground that change over the time), but also engineering construction features of the production wells, and other types of technical systems such as ripraps, tanks, pipelines, flare systems, and others [7–10]. Considered numerical algorithms are justified and approved on the base of data for 12 Russian oil and gas fields. When the source of heat in the frozen ground was a well, a comparison was made between numerical data on the distribution of the boundary of the melting of frozen ground (zero isotherm) and experimental data. The detailed calculations of the long-term prediction of the permafrost boundary changes demand considerable computing power, the parallel approach to solving such problems is also required [11]. The complete simulation of all technical systems located in a well pad makes it necessary to solve such problems in a significally larger area with a three-dimensional computational grid. This leads to an essential encreasing time of computations (up to 100 hours).

In numerical simulations with using finite-difference methods, an original problem is often reduced to numerical solution of systems of linear algebraic equations (SLAE). In papers [12–15], for solving SLAE with the block-three- and block-fivediagonal matrices, direct and iterative parallel algorithms are designed. Those are: the conjugate gradient method with preconditioner, the conjugate gradient method with regularization, the square root method, and the parallel matrix sweep algorithm. In papers [16–18], the iterative gradient methods were used for solving SLAE with dense matrices arising in inverse gravity problems. These algorithms are implemented on multi-core processor with good efficiency.

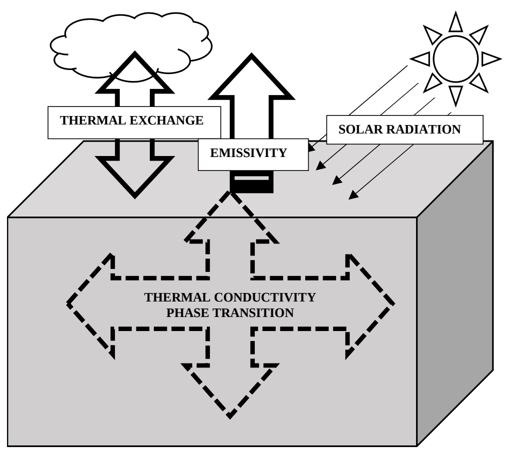


Fig. 1. A basic scheme of simulated area and heat fluxes

However, the considered model of heat distribution is assumed a nonlinear boundary condition of heat fluxes balance at the soil–air boundary that increases the computational comlexity and does not allow the direct application of the parallel methods of SLAE solving. In this paper, an appoach of effective code implementation of a splitting method by spatial variables is presented.

2 Statement of problem

Simulation of processes of heat distribution in the permafrost soil is reduced to solution of a three-dimensional diffusivity equation with non-uniform coefficients.

The equation includes the localized heat of phase transition; *i.e.*, an approach to solve the problem of the Stefan type without the explicit separation of the phase transition in 3D area (Fig. 1) [3]. The equation for the temperature $T(t, x, y, z)$ at the instant t at the point (x, y, z) has the form

$$\rho\big(c_\nu(T) + k\delta(T - T^*)\big)\frac{\partial T}{\partial t} = \nabla\left(\lambda(T)\varDelta T\right), \tag{1}$$

with initial condition

$$T(0, x, y, z) = T_0(x, y, z). \tag{2}$$

Here, ρ is the density [kg/m^3], T^* is the temperature of the phase transition [K],

$c_\nu(T) = \begin{cases} c_1(x, y, z),\, T < T^*, \\ c_2(x, y, z),\, T > T^* \end{cases}$ is the specific heat [J/(kg · K)],

$\lambda(T) = \begin{cases} \lambda_1(x, y, z),\, T < T^*, \\ \lambda_2(x, y, z),\, T > T^* \end{cases}$ is the thermal conductivity coefficient [W/(m · K)],

$k = k(x, y, z)$ is the specific heat of phase transition, δ is the Dirac delta function.

Balance of the heat fluxes at the surface $z = 0$ defines the corresponding nonlinear boundary conditions

$$\gamma q + b(T_{air} - T(x, y, 0, t)) = \varepsilon\sigma(T^4(x, y, 0, t) - T^4_{air}) + \lambda\frac{\partial T(x, y, 0, t)}{\partial z}. \tag{3}$$

To determine the parameters in the boundary condition (3), an iterative algorithm is developed that takes into account the geographic coordinates of the area, lithology of soil, and other features of the considered location [10].

3 Method for solving the problem

An effective scheme of through computations with smoothing the discontinuous coefficients was developed for the equation of thermal conductivity by temperature in the neighborhood of the phase transformation. This scheme is applied to numerical simulations. On the basis of ideas in [3], a finite difference method is used to solve the problem with splitting by the spatial variables in three-dimensional domain. We construct an orthogonal grid, uniform, or condensing near the ground surface or to the surfaces of internal boundaries.

The original equation for each spatial direction is approximated by a locally additive implicit pattern, and to solve SLAE, a combination of the sweep and Newton method is used. At the upper boundary $z = 0$, there is an algebraic equation of the fourth degree, which is solved by the Newton method. Solvability of the same difference problems approximating (1)–(3) is proved in [5].

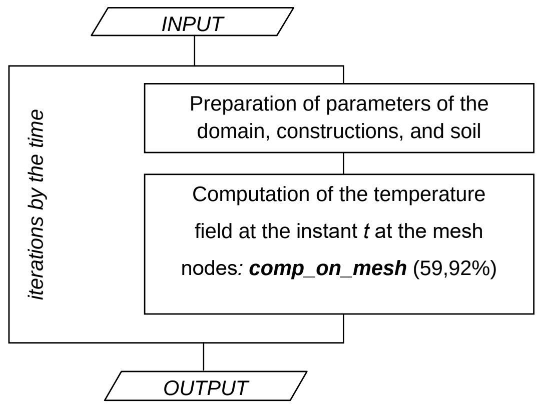


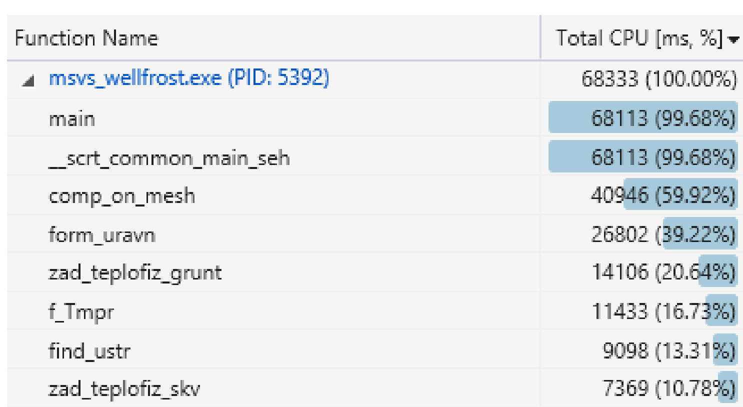
Fig. 2. Principal computations algorithm

4 Parallel algorithm

A basic scheme of the computational algorithm is presented in Fig. 2

Let us consider an approach to construction of the parallel algorithm for multicore CPU using OpenMP technology.

To analyse the serial program, the Visual Studio 2017 Performance Profiler tool was used. The results presented in Fig. 3 show that the most time-costly procedures are computation of the temperature field 'comp_on_mesh()' and preparation of the domain parameters ' zad_teplofiz_grunt ()' and ' zad_teplofiz_skv ()'.



Function Name	Total CPU [ms, %]
msvs_wellfrost.exe (PID: 5392)	68333 (100.00%)
main	68113 (99.68%)
__scrt_common_main_seh	68113 (99.68%)
comp_on_mesh	40946 (59.92%)
form_uravn	26802 (39.22%)
zad_teplofiz_grunt	14106 (20.64%)
f_Tmpr	11433 (16.73%)
find_ustr	9098 (13.31%)
zad_teplofiz_skv	7369 (10.78%)

Fig. 3. Profiler results

The temperature computation procedure consists of three steps of forming and solving SLAEs for each spatial direction. We can solve each SLAE independently within one direction. For example, when forming and solving SLAEs along the X axis, we can split the YZ domain into a number of horizontal bars and distribute them between a number of threads. We do this by using '**#pragma** omp parallel **for**' for outer of two nested loops. After all SLAEs are solved for one direction, we should synchronize threads by '**#pragma** omp barrier' to avoid update conflicts.

Figs. 4 and 5 show the serial algorithm and the parallel variant, respectively.

The procedures for domain parameters consist of three nested loops; so, we can just use ' parallel **for**' for the outer one.

Profiling of parallel program shows that the proportion of the parallel code is 75%.

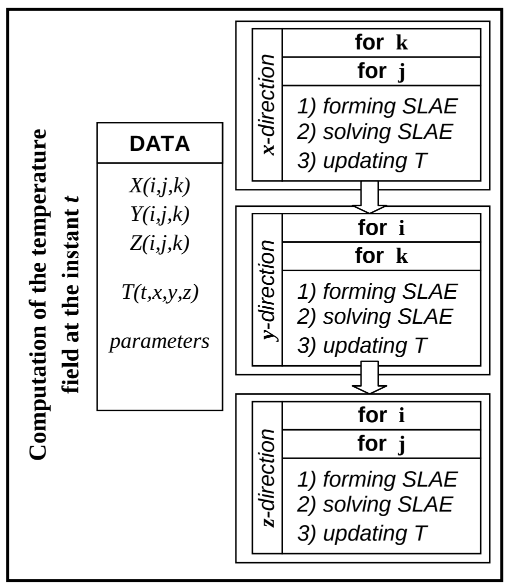


Fig. 4. Serial algorithm

5 Numerical experiments

As a model problem, let consider a seasonal freezing–thawing of the upper layers of a permafrost soil. The permafrost temperature lower than the area of influence of the seasonal changes (lower than 10 meters) is -0.7°C. The basic

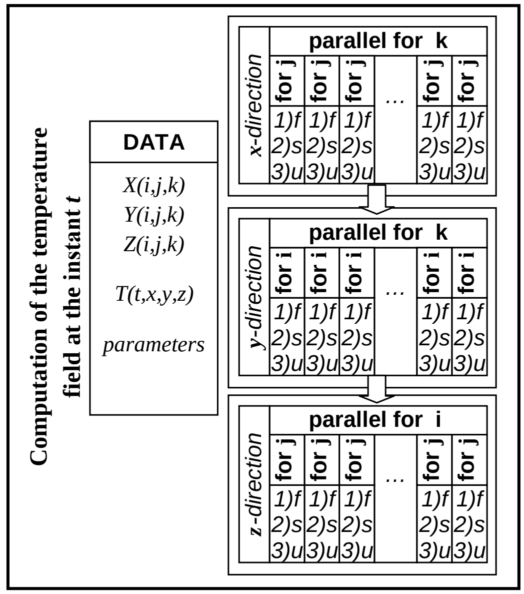


Fig. 5. Parallel algorithm

thermal parameters of the soil are in the following: the thermal conductivity is 1.82 and 1.58 W/(m·K), the volumetric heat is 2130 and 3140 kJ/(m^3·K) for frozen and melted soil, respectively, the volumetric heat of phase transition is 1.384· 10^5 kJ/(m^3·K).

Table 1 shows the data for the considered area. The other parameters in equation (1) and conditions (2) and (3) are determined as a result of the geophysical research.

Table 1. Basic annual climatic parameters

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Aver. temperature, °C	-28.1	-26.3	-20.0	-10.8	-2.0	8.5	15.1	11.5	5.1	-6.5	-20.8	-26.2
Solar radiation, W/m^2	0	112	282	567	809	865	889	639	355	122	34	0

The experiments are carried out using the six-cores AMD Ryzen 5 1600X CPU. The grid size for problem is $91 \times 91 \times 51$. The time interval was 10 days with the step of 1 day.

Table 2 shows the computing times T_p, speedup $S_p = T_p/T_1$, and efficiency $E_p = S_p/p$ for solving the fixed problem size by various numbers p of OpenMP threads (strong scaling). It also contains the theoretical speedup calculated using the Amdahl law [19]:

$$\overline{S_p} = \frac{1}{\alpha + \frac{1-\alpha}{p}}, \tag{4}$$

where α is proportion of the serial code (25% for our program).

Table 2. Results of strong scaling experiments

Number of threads p	Calculation time T_p, sec	Theoretical speedup $\overline{S_p}$	Speedup S_p	Efficiency E_p
1	620	-	-	-
2	390	1.6	1.59	0.79
4	310	2.3	2	0.5
6	250	2.7	2.5	0.41

Table 3 shows computing times and efficiency $W_p = T_1/T_p$ for fixed problem size *per thread* (weak scaling). The grid size is adjusted automatically, so the dimensions are not exactly scaled.

Table 3. Results of weak scaling experiments

Number of threads p	Grid size	Calculation time T_p, sec	Efficiency W_p
1	$91 \times 91 \times 51$	620	-
2	$181 \times 91 \times 51$	820	0.76
4	$379 \times 85 \times 51$	1210	0.51
6	$585 \times 85 \times 51$	1500	0.41

By extrapolating these calculation times, we can say that the serial program would take 6.3 hours to solve the problem for 1 year interval and 188 hours (8 days) for 30 years interval. Using the parallel computing, we can cut this time to approximately 2 days.

6 Conclusion

Thus, the developed mathematical model and software allow one to carry out detailed numerical calculations on long-term forecasting the temperature field changes from different technical systems in the near-surface layer of soil in the permafrost zone. Effective and quite simple OpenMP approach for a multicore system allows one to reach approximately 90% of the theoretical speedup. The results of numerical experiments show that by using the six-core processor, the computation time is reduced up to 2.5 times.

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