Scalable Facet Model and Forest Terrain Radar Image Processing in Range-Doppler Coordinates

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Abstract. This paper describes the radar image processing model for modern synthetic aperture radars and other airborne radar systems. The main attention in the paper is paid to elaboration of the method of combination of various well-known and enhanced mathematical models of forest surface types, radar signals, radar scene and scenario, etc. The method is applied to the effective radar image calculation in the range-Doppler coordinates. Also, benefits of the radar echoes computation algorithm of the radar scene are discussed. In conclusion, an overview of the model features and some future investigations are presented.

Keywords: radar image, forest surface, reflected signal, airborne radar system, doppler frequency, mathematical model, digital signal processing

1 Introduction

Nowadays computer modeling technologies and digital signal processing are ubiquitous and extremely useful in many various radar applications [1], [2], [3]. They are usually used together, and, under circumstances of rapidly increasing requirements to different onboard radar systems, it can be a step change to create precision measurements with high informativity about illuminated surfaces for safety flying. Reflected (backscattered) radar signals can give accurate navigation coordinates of all reflective objects such as trees, buildings, cars, fences, pipe- and electric nets, etc.

Solution of the autonomous navigation problem for different types of aircrafts is very important. Especially it is so for aircraft that fly at extremely low altitudes above typical Earth surfaces, that include forest terrain. There are many known works on the study of signals reflected from various natural surfaces [3], [4], the creation of forest models, and similar layered structure models [4], [5], [6]. These works allow us to build a uniform universal model that can be adapted to mathematical simulation of signal formation and processing the different radar systems.

So, a mathematical model of the reflected signal of a pulsed radio altimeter that uses a rectangular waveform is considered in the paper on the example of operating under various forest surfaces in the centimeter range of radio waves for a flight altitude of about 100 meters and below. The first feature of the model is the attempt to solve the problem of image synthesizing for the vertical illumination of a surface. The second feature is the calculation of Earth radar images of forest vegetation in the range-Doppler coordinates (in the range-Doppler domain).
2 Choice and justification of mathematical models

To create a mathematical model of the Earth surface radar response for the low altitude radar system with an active monostatic radiolocation method and pulsed signals, it is necessary to successively solve the following problems.

1. *Construction of a mathematical surface model.* For the mathematical description of the surface with forest vegetation, the facet model is chosen. It is the most universal for various types of concentrated, surface and volume-distributed targets. It is based on a geometric model of propagation and reflection of the radio waves (which here are similar to the light beams) from the whole set of illuminated reflectors, which are presented in the form of conditional facets, i.e., simplified flat areas with dimensions less than the radar resolution. So, this provides the adequate and experimentally confirmed results under circumstances of correct definition of the modeled facet number and reflection parameters [5], [6].

2. *Development of a model for the reflected signal formation.* The modeling method used in the work assumes the joint application of a phenomenological approach and sets of empirical data [4]. Within the framework of this approach, the facets of the illuminated surface are considered statistically independent with the random reflectance coefficients, the mean values of which depend on the type of the reflector material (substance). This is provided by using the empirical data [4], [8] about surface type and its backscattering characteristics. Particularly, we note that small nonuniformity and roughnesses of a real surface are excluded from the geometric model of a surface, because it is taken into account furtherly by using non-specular reflection, i.e., the diffuse backscattering. The reflected signal power is calculated as the total power of the reflections from each of these elementary reflectors. Of course, here, we also must take into account the receiving and transmitting antenna patterns and the relative facet angles that are the local incidence/observation angles of the electromagnetic waves.

3. *Calculation of the radar image of the previously simulated surface in the range-Doppler coordinates.* Here, our solution of this problem is based on an algorithm similar to the one described in [9]. The reflected signal received on the previous observation stage is described by three arrays containing information about the slant range, Doppler frequency shift, and power for each elementary reflector. Each element of the image is the radar image pixel that contains the value that is proportional to the "radio brightness". So, the total radar cross section (RCS) of all facets referred (close in range and Doppler frequency shift) to the corresponding element of the radar image.

4. *Calculation of the received radar signal.* To calculate the signal that is the radar response for each emitted pulse, we find the sum of low frequency replicas of the emitted pulse, then attenuate and shift them by the delay and frequency due to the corresponding radar image pixel value and their position inside the radar image. So, as it was mentioned above, the amplitude of each copy corresponds to the radio brightness of the pixel of the previously obtained radar image. The number of components is equal to the number of non-zero or the most "bright" elements of the radar image corresponding to the given radar antenna direction and pattern.
The above sequence of operations makes it possible to significantly reduce the amount of computation, especially, for the complex signals with the intra-pulse modulation. Methods of the digital signal generation in the model are effectively implemented by matrix calculations in the used MATLAB system. Let us now consider the solution of these problems using the example of forest terrain modeling.

3 Stages of modeling

3.1 Forest surface modeling

Our choice of the forest terrain for research is based on a classification by the type of prevailing vegetation. It is so due to the presence of appropriate tree models from the standard set of primitives presented in the Autodesk 3ds Max. Moreover, a sufficient amount of empirical data is taken on a corresponding vegetated ground echoes [4], [8]. Based on these criteria, two types of the forest terrain were chosen: deciduous (leafy) woods based on the elm tree model, and coniferous (pine) woods based on the spruce (fir) tree model, respectively.

The block diagram of constructing the mathematical model of the forest terrain and the results of modeling are shown in Figs. 1 and 2, respectively [7].

![Flowchart for constructing the model of the forest terrain](image1)

**Fig. 1.** Flowchart for constructing the model of the forest terrain

![Examples of test models of the forest surfaces](image2)

**Fig. 2.** Examples of test models of the forest surfaces: deciduous elm woods (on the left) and coniferous spruce woods (on the right)
Simulation of the reflected signals for the canopy, branches, and trunks of the tree triangular facets is possible with the reduction (by random selection) of the number of elementary reflectors. It is made in order to leave about 100 facets per element of the modeled radar image (the resolution will be given below). Therefore, here, in order to save computational resources, a simplified discarding of a part of the facets can be additionally performed. For the remaining facets, a proportional scale conversion is performed to increase their relative power.

3.2 Calculation of a set of facets parameters

Simulation of the reflected signal begins with a sequential calculation of the slant range, Doppler frequency shift, and power for each elementary reflector the facet. The power of a reflected signal from a facet is found from the radar equation and according to the expression

\[ P = \frac{P_0 \lambda^2 K_{ap}^2 K_{ref} K_{bp}}{(4\pi)^3 \frac{R^4}{\Delta S \eta}}, \]

where \( P_0 \) is the transmitter power; \( \lambda \) is the wavelength of the emitted signal; \( K_{ap} \) is the coefficient of antenna pattern; \( K_{ref} \) is the coefficient of reflection to account the specific surface RCS; \( K_{bp} \) is the coefficient of accounting for the backscattering pattern; \( \Delta S \) is the facet area; \( R \) is the slant range, that are demonstrated for the sample \( i \)-th facet in Fig. 3; \( \eta \) is the propagation loss factor (including some tuneable shading because of propagation through the vegetated layers).

Values of \( K_{ref} \) and \( K_{bp} \) take into account specific types of surface material of the facets. In the general case, they are differently set to represent the canopy leaves (needles), branches, tree trunks, and, also, the terrain ground layer model. They are commonly used by selecting corresponding features for desired carrier frequency in order to achieve the expected results that will correspond to the experimental studies. More accurate signal shadowing by the canopy can
be resolved by implementing the backward raytracing method. But this is too
difficult, since there are so many facets in our tree models.

Slant range $R$ is the physical absolute distance from the onboard radar to the
facet. It is obviously determined for each facet from the geometry of the model
by the differences in the relative coordinates (dimensions)

$$R = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2},$$

(2)

where $\Delta x$, $\Delta y$, and $\Delta z$ are the relative coordinates.

The calculation of the Doppler frequency shift of the facet reflected signal is
carried out according to the expression

$$\Delta f = \frac{2V_r \lambda}{\cos(\alpha) \cos(\beta)},$$

(3)

where $V_r$ is the radial velocity of the radar platform in the direction of the facet;
$\alpha$ and $\beta$ are the angles to account the individual direction of the facet (from the
radar antenna relatively to the velocity vector $V$, which are also demonstrated
for the sample facet in Fig. 3) in the horizontal and vertical planes.

### 3.3 Construction radar images of forest surfaces in the
range-Doppler coordinates

For radars such as the synthetic aperture radar (SAR), the mean value of the
resolution in the along-track dimension is determined by the synthesizing time
according to the expression [1–3, etc.]

$$\Delta \theta = \frac{\lambda}{2VT_s \sin(\theta_{\text{view}})},$$

(4)

where $\Delta \theta$ is the azimuth resolution; $V$ is the aircraft speed; $T_s$ is the time of
synthesizing (accumulation of reflected pulses); $\theta_{\text{view}}$ is the mean viewing angle.

Under the vertical illumination (because of the small value of the denomina-
tor in (4)), the resolution is relatively low for the qualitative (or the best) radar
application. Therefore, the vertical illumination mode is not traditionally used
for ground-mapping radars. However, take into account the fact that the width
of the antenna beam of a typical radio altimeter at half power is about 40-60 de-
grees. So, the reflected signal from each of the 4 quadrants of the illuminated zone
in spatial coordinates can be considered as a reflected signal in the front/rear-
side-looking observation (the squint surveillance). Therefore, it is possible to use
a typical radio altimeter for synthesizing the aperture, to extract and analyze
additional radar information [9], although, with a relatively low resolution.

The radar images in the range-Doppler coordinates are useful from the point
of view of the correspondence/equality of the discrete step of the radar image pix-
els to the radar resolution. So it becomes suitable for detecting the radio-contrast
objects with dimensions near to the actual resolution of the radar system.

For constructing the radar images in the range-Doppler coordinates (dimen-
sions), the underlying surface is divided into separate areas called the resolution
elements or cells. These calls are bounded by the iso-range lines (lines of the same range) and isodopes (lines of the same Doppler frequency) as it is simplified shown in Fig. 4 for the horizontal motion of the radar platform under a plane surface.

![Fig. 4. Division of the surface by frequency and range into area cells](image)

Each area cell (resolved by an imaging radar) has its own power proportional to the area RCS. In such a grid, the pixel step of the radar corresponds to the radar resolution in the frequency and range. Notice for further that when using such a radar system, it is necessary to take into account the variable dimensions of the cells, the nonlinearity of distances, and the mutual overlapping of the facets located at the same range to the left and right of the flight axis (for example, points \(A\) and \(A^*\) in Fig. 4). Practically the field of view with high resolution is formed by the radar motion from one synthesis interval to another with the single beam formation at each synthesis interval [1], [2], [9]. Here, to simplify our model, the square area under the aircraft is initially taken.

Algorithm for the formation of the radar image of the Earth surface in the range-Doppler coordinates consists of the following steps.

1) Determination of surface resolution elements in the range-Doppler coordinates. The resolution of the frequency and range is determined by the parameters of the emitted signal and the principles of the signal processing. For radars with the synthesis of the antenna aperture and the compression of the received chirp pulses, the following expressions can be used:

\[
\Delta f_{\text{dop}} = \frac{1}{T_s}, \quad \Delta r = \frac{c}{2W},
\]

where \(\Delta f_{\text{dop}}\) is the frequency resolution; \(\Delta r\) is the resolution in slant range; \(c\) is the speed of light; \(W\) is the bandwidth of the linear-FM pulse carrier frequency.

2) Distribution-grouping the facets according to the cell midpoints that now correspond to the radar image pixels.

3) Calculation of the total power of each resolution element, i.e., the image pixel.

4) Threshold digital image processing for the selection of some tall trees and analysis their properties.
In Fig. 5 an example of constructing the radar image for a test surface with a single tree is shown. The Doppler frequency is plotted along the abscissa axis and the slant range along the ordinate. The flight altitude is 50 m, the speed of the aircraft is 100 m/s, and the carrier frequency is 10 GHz. Here and further to increase the contrast, the zero values of the image pixels are replaced by a white (transparent) color.

![Fig. 5. Example of construction the radar image; the top view of the test terrain (on the left) and the radar image in the range-Doppler coordinates (on the right)](image)

The simulation results for models of the forest terrain depicted earlier in Fig. 2 are presented in Fig. 6. The simulation parameters are similar to the previous test.

![Fig. 6. Example of the radar images of the forest terrain: deciduous elm woods (on the left), coniferous spruce woods (on the right)](image)

Based on the results of modeling, it is possible to distinguish the following features of the radar images of the forest surfaces:

- The prominent trace of the soil layer on the radar image is characteristic only of the deciduous forest model. In the coniferous forest, the attenuation created by the canopy (treetops) significantly weakens the relative level of the soil signal.
- In the absence of the large-scale relief, the soil layer signal forms a figure bounded by two hyperbolas. The bottom hyperbola is the nadir track.
- There is ambiguity in the azimuth that is resulted in the fact that the trees located symmetrically relatively to the velocity vector of the aircraft on the radar image are overlayed.
There is a relationship between the clarity, the scale of the image, and the viewing angle. This is resulted, for example, in the fact that the side-located trees (located at the angles about 90 degrees relatively to the velocity vector) have clear outlines of the treetops and trunks in contrast to trees located along the flight direction.

We note that the process of obtaining such radar image is, in fact, close to finding an impulse response of a surface. Then, it allows us to calculate the response of the surface by using one of the convolution methods with the emitted pulse. In our approach, all information about delays, amplitudes, and Doppler frequency shifts for all reflectors on the surface remains in the radar image. Therefore, using on this information, we can then calculate the reflected signal for the radar.

### 3.4 Modeling the reflected signal

The process of reflected signal modeling assumes a consecutive calculation of the reflected signal for each emitted pulse. To do this, for each emitted pulse, the low frequency replicas of the emitted pulse (or, possibly, only its envelope) are summed by the delayed and frequency-shifted values at the corresponding position of the radar image pixel. The amplitude corresponds to the radio brightness (equal to the square root of the total facet backscattering) of the pixel as the radar image element. So, a complex form of the signal with some additive noise for practical using is calculated as

\[
S(t) = \sum_{i, \text{where } U_i > U_{\text{min}}} U_i A \left( t - \frac{2R_i}{c} \right) e^{i2\pi \Delta f_i t} + N(t),
\]

where \( U_i \) is the amplitude corresponding to the pixel, which must be greater than attainable level \( U_{\text{min}} \); \( A(t) \) is the emitted waveform, which also should be defined in a complex form; \( 2R_i/c = \tau_i \) is the delay for corresponding slant range \( R_i \) for the \( i \)-th pixel; \( \Delta f_i \) is the Doppler frequency shift for the \( i \)-th pixel; \( N(t) \) is the white/band noise with the desired signal-to-noise ratio corresponding to the radar receiver.

The number of emitted pulse replicas \( A(t) \) is equal to the number of non-zero radar image elements. Otherwise, we can use only the “bright” \( (U_i > U_{\text{min}}) \) elements in the radar geometric region of our interest and assuming that when the signal is processed by the radar receiver, a part of the partial signals can be effectively filtered, suppressed, or gated.

Also note here that one can see the saving of computing resources when calculating the reflected signal, since in (6) the signals are summed not by the number of facets in the surface model, but by the number of resolution elements in the radar image.

Further, for the radar functioning, a typical or special signal processing is possible. For example, an altitude measurement of an aircraft over the forest terrain is measured by catching the relative position of the rising slope of the first echo impulse [3, 7]. In Fig. 7, the modeled envelopes of the reflected pulse with the altitude equal to 50 m are shown.
The dense deciduous forest model is characterized by the separation of the reflected signal into two commensurable layer responses: the forest canopy, and the soil layer. This fact allows one to measure the flight altitude of aircraft not only over the treetops, but, also, above the soil layer. Here, with the flight altitude of 50 m and a tree height of 28 m, the distance to the tops of the trees is 22.5 m, and the height above the soil layer is 50 m.

For the model of dense coniferous forest, there is a clear predominance of reflection from the canopy, which is explained by the high closeness of the trees’ canopy and the high level of the signal attenuation in it.

When constructing the radar image with one emitted pulse, information about the volume distribution of reflectors is lost. However, based on the received reflection signal sequence for the radar, this information can be recovered with some precision. Also, note here that the range migration information is contained in the sequence of the reflected pulses collected through the image synthesis interval. But this is a separate and complex goal, which concerns the signal processing by the SAR techniques.

4 Conclusions

As a result, the mathematical model of the layered surface was constructed, the algorithm for calculating the radar image and the reflected signal of a pulsed radio altimeter is built. The model allows taking into account the difficult structure of natural surfaces, where forests are both typical and complex Earth cover (other terrain models are under considering). Also, we can vary the parameters of the aircraft and emitted signal in order to perform the analysis of the radar operation and to improve the signal processing parameters including the synthesis of the antenna aperture. The radar image in the range-Doppler coordinates is visual and informative mean for detecting radio-contrast objects. Moreover, this method is useful and practical from the point of view of the correspondence of the discrete pixel size and the radar resolution.

One of our main goals is to save computing resources. When calculating the response signal from a surface, we use properties of the partial grouped signals. So, the total complex signal reflected from all the scene facets is calculated economically, but in accordance with the desired radar resolution. Additional
computational resources can be saved by reducing the number of considered model facets when calculating each pixel of a radar image. Such scalable methods of the surface representation are also promotional for implementation in the HIL-simulators [10]. Note that there the real-time computing mode needs to produce the radar echoes, which is used to simulate operation of the radar system in different flight circumstances.

The general appearance and characteristics of the modeled images in the X-band, for example, the presence of graininess in the image, correspond to the expectations for the selected simulation conditions. This fact confirms the correctness of the proposed method and the implementation of the modeling algorithm.

The results can be used to investigate the possibilities of applying the radio altimeter to the reflected signal processing and radar image constructing, particularly, in the range-Doppler coordinates. In the future, in order to increase the compliance of the modeled radar images with the experimental ones, it is proposed to calculate the amplitude and phase of the total signal for each element of the radar image and, also, to elaborate the algorithm for accounting the shading depending on the modeled thickness and moisture content of the forest canopy. This will allow better simulating the presence of the speckle noise on the image. It will be useful in calculating the radar ground-returns with intrinsic amplitude fluctuations, and fading effects of the backscattered signal.

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References