An Entropy Model of the Aircraft Gas Turbine Engine Blades Restoration Method Choice

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Abstract: **The paper goal is to investigate the influence of a few specified methods of the gas-turbine engine blades repair upon a choice of a preferable blades geometry restoration technology. There is a scientific proof for the good blades shape restoration method choice that fits the customer needs, being taking into account the subjective preferences of the available technologies and extremizing the preferences uncertainty.**

Keywords: **mathematical model, alternative, economic activity, subjective preference, subjective entropy.**

I. Introduction

For an active system economic activity management it is used the theory of subjective analysis [1]. A system governed by a decision making person (or a group of such persons in a certain hierarchical structure) is considered as active system since the system is under the managerial influence of the individuals (they are deemed to be reckoned with as the active elements of their own active system). The foundation stone of the subjective analysis theory (the theory of individual subjective preferences optimal distribution upon a certain, taken by an individual into her/his consideration, set of some attainable alternatives) is the entropy paradigm, the so-called Subjective Entropy Maximum Principle (SEMP), which is going to be used in the presented paper as a research tool, although with the application into an objectively existing optimum sphere rather than just to the subjectively preferred matters and only. This creates a background for the information and analytical support of the economic activity.

In aviation industry, in particular, in aeronautical engineering design and its further operation modes, the individuals' subjective preferences play a crucial role. The same is to the aircraft as a whole, as well as to its power plant specifically. Definitely, those individuals' subjective preferences are intruding the fields of both aircraft and their powerplants maintenance and repair as that follows the references [2, 3].

The other area of subjective preferences application here is alternatives in technologies. Concerning an aviation gas-turbine engine (AGTE) repair it deals with the techniques proposed in multiple and developed, described, and discussed in publications of both directions of practitioners/engineers and academicians [4–6].

The objectives of the presented paper are to demonstrate the multi-optional optimality doctrine newest developments applicability initiated in works [7–11] to the problems of aeroengines technical operation.

The developed herein concept seems promising to the variety of adjacent scientific areas applications, for instance, like for those ones considered and discussed in publications [12–19].

II. Mathematical Modeling and Developed Methods

A gas-turbine engine (GTE) blades apparatus is designed on one hand to ensure the required compression of the air supplied with the necessary parameters to the combustion chamber by the compressor part of the engine and on the other hand the blades apparatus of the turbine part allows getting enough work and gas parameters for the compressor driving and aircraft propulsion.

The important thermodynamic parameter here is the polytropic process index n which magnitude must lie within the very narrow designed value diapason. In operation the blades wear out, their geometrical shape distorts, as a result of this the engine cannot perform the designed work.

The restoration of the blades geometry form has to be made.

A. Theoretical Problem Setting

Polytropic process index n approximate mean value, in real engines thermodynamic processes (such as, for instance, expansion of combustion gasses in the cylinder of an internal combustion engine, or in our case, the other examples are compression and expansion of gasses in an AGTE) calculations, can be found from polytropic process equation:

$$pV^n = \mathrm{Const}, \tag{1}$$

where p is pressure; V is volume; provided the values of the pressure p_1, p_2 and volume V_1, V_2 are known at some points 1 and 2 of the process.

Indeed

$$p_1 V_1^n = p_2 V_2^n. \tag{2}$$

$$\ln p_1 + n \ln V_1 = \ln p_2 + n \ln V_2,$$

$$n \ln V_1 - n \ln V_2 = \ln p_2 - \ln p_1,$$

$$n = \frac{\ln p_2 - \ln p_1}{\ln V_1 - \ln V_2}. \tag{3}$$

This simplest method of (1) – (3) for polytropic process index n determination can be found from practically any reference, guidance or study book on either theoretical or engineering thermodynamics either heat engines.

B. Optional Functions Entropy Problem Setting

The other concept also proposed in references [7–11] and hereinafter is based upon an optimization principle close to SEMP [1] application in the context close to the described in papers [9, 10].

Let us consider the thermodynamic states 1 and 2 of a gas in polytropic process as some optional states in a certain respect. Thus we come to a multi-optional problem.

Now, the other sub-problem of the polytropic process given description is to discover the options' objective effectiveness functions related to those two optional states. Let us presuppose the objective effectiveness functions for the considered two-optional problem of the polytropic process considered description are $\ln V_1$ and $\ln V_2$. This might be reasonably natural with regards to apparent perception of the obvious quantitative characteristic of the existing reality.

With the use of the supposed multi-optional optimality, likewise in subjective analysis [1, 10] conditional optimality of the individual's subjective preferences distribution, with extremizing subjective entropy, that is applying the doctrine analogous to SEMP concept, we have the right to write down the postulated functional in the view of:

$$\Phi_h = -\sum_{i=1}^{2} h_i(V_i)\ln h_i(V_i) + n\sum_{i=1}^{2} h_i(V_i)\ln V_i + \gamma\left[\sum_{i=1}^{2} h_i(V_i) - 1\right], \tag{4}$$

where $h_i(V_i)$ are specific hybrid-optional effectiveness functions, similar to the preferences functions of [1, 10], however in this problem setting the assumed specific hybrid-optional effectiveness functions $h_i(V_i)$ are not relating with anybody's preferences or choice; γ is normalizing coefficient (function).

The first member of expression (4) is the hybrid-optional effectiveness functions entropy (like subjective entropy of the preferences).

The necessary conditions for the functional (4) extremum existence:

$$\frac{\partial \Phi_h}{\partial h_i(V_i)} = 0\,, \tag{5}$$

yield

$$\frac{\partial \Phi_h}{\partial h_i(V_i)} = -\ln h_i(V_i) - 1 + n\ln V_i + \gamma = 0\,, \quad \forall i = \overline{1,2} \tag{6}$$

This inevitably means in turn

$$\ln h_1(V_1) - n\ln V_1 = \gamma - 1 = \ln h_2(V_2) - n\ln V_2\,. \tag{7}$$

From where

$$\ln h_1(V_1) - \ln h_2(V_2) = n(\ln V_1 - \ln V_2)\,. \tag{8}$$

And

$$n = \frac{\ln h_1(V_1) - \ln h_2(V_2)}{\ln V_1 - \ln V_2}\,. \tag{9}$$

III. Analysis

A. Polytropic Equation Setting

Thus, we have got a parallel result to the law of subjective conservatism if the values of parameters n, V_1, and V_2 are given.

In case as in work [9]:

$$h_1(V_1) = xp_2\,, \qquad h_2(V_2) = xp_1\,, \tag{10}$$

where x is unknown, uncertain multiplier in type of the Lagrange one, we obtain with the help of the procedure considered through (4) – (10) the needed polytropic process index (3).

Indeed, substituting equations (10) for their values into expression (9) it yields

$$n = \frac{\ln \frac{xp_2}{xp_1}}{\ln V_1 - \ln V_2}\,, \tag{11}$$

finally, formula (3).

The sense of the uncertain multiplier x becomes obvious with the use of the normalizing condition of the initial functional (4). That is

$$xp_1 + xp_2 = 1\,. \tag{12}$$

Hence,

$$x = \frac{1}{p_1 + p_2}\,. \tag{13}$$

Remarkable here is that the multi-optional hybrid function has the view of

$$n\frac{\sum_{i=1}^{2} p_i \ln V_{\bar{i}}}{\sum_{i=1}^{2} p_i} \quad \text{or} \quad n\frac{\sum_{i=1}^{2} p_{\bar{i}} \ln V_i}{\sum_{i=1}^{2} p_i}\,, \tag{14}$$

where subscript $\bar{i}$ means: "*pertaining not to the i^{th} but to the other option of the two-optional situation*".

Moreover, in case expressed with (4) – (14) the sought polytropic process index n has been got for the given values of hybrid-optional effectiveness functions $h_i(V_i)$ and at this the found value of n can make the hybrid-optional effectiveness functions of $h_i(V_i)$ also be optimal for the objective functional (4).

B. Aeroengine blades alternative restoration technique problem solution on the subjective entropy paradigm basis

Now, in order to retain polytropic process index within the required designed value interval, so that to ensure the desired engine performance, periodical restoration of the specified GTE blades apparatus geometry has to be executed.

Supposedly, there are competing methods. These are, for example, plasma (Pl), laser (La), and electro-arc (El). Each of which are the alternatives for a GTE repair plant to implement. Here we have a three-alternative problem. Then, there are corresponding subjective effectiveness functions: Pl(·), La(·), and El(·) that have relations to each of the being considered alternatives.

Let us apply although simplified, rough, however possible model for the objective effectiveness functions Pl(·), La(·), and El(·).

The results of the numerical modeling (in conditional units) are shown in Fig. 1–3.

In the considered example the objective effectiveness functions Pl(·), La(·), and El(·) have five independent variables. But in actual result presented in Fig. 1 only one: # 4 (productivity of the alternative technology: *P*) is being variated. The rest of the parameters are the corresponding constant values accepted for the: 1) thickness of the metal layer welded onto the blades prepared surface; 2) thickness of the metal layer removed out from the blades bodies down to the nominal size; 3) number of blades undergoing the treatment; and 5) is the number of the laborers supposed to be involved into the alternative technological process.

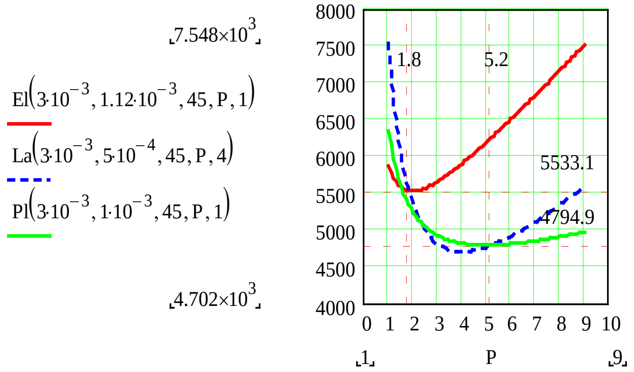

Fig.1. Objective effectiveness functions related to corresponding alternatives

C. *Results of the Numerical Experiment*

For the available three alternatives: plasma, laser, and electro-arc, all the described approach variables may also be functions of their arguments. Even with the simplified set of the independent variables the situation depending upon the arguments combination remains uncertain. The inter-influence of the parameters can lead to variants when at some circumstances it is hard to give the preference to a specific alternative.

The preferences obtained by the objective functional treatment like (4) – (6) are visible in Fig. 2.

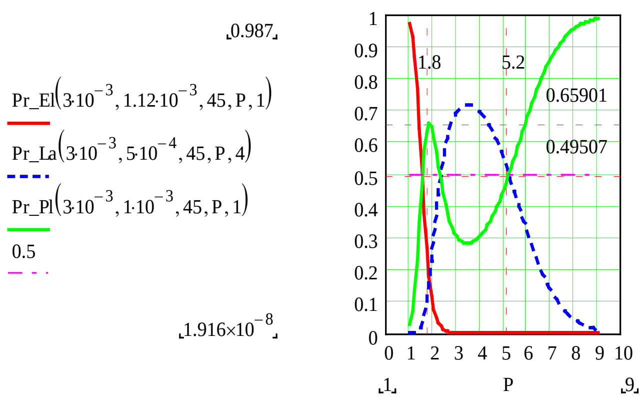

Fig.2. Subjective preferences of corresponding alternatives

In Fig. 2 it is depicted, with the Pr characters, the subjective preferences functions distributed optimally, accordingly with the procedures of (4) – (6) [1], upon the set of the alternative AGTE blades restoration technologies: Pl(·), La(·), and El(·), correspondingly.

For the preferences, their four arguments out of the five introduced are fixed exactly as for the corresponding effectiveness functions (see designations in Fig.1 and 2).

Subjective entropy of preferences is presented in Fig. 3.

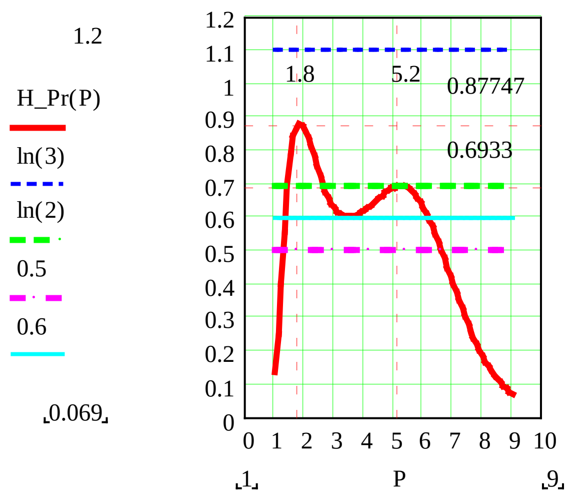

Fig.3. Subjective entropy of corresponding alternatives preferences

From Fig. 3 it is visible that there are areas with respect to the productivity *P* of the blades restoration methods from approximately 1.5 up to 6.5 kg/h where it is almost impossible to make a decision concerning which technology is better to apply (see and compare multi-alternativeness knots of the individual subjective preferences, noticeable in Fig. 2 at $P = 1.8$ and 5.2 kg/h, with the subjective entropy climaxes appeared in Fig. 3). It might be suspected because of the effectiveness functions positioning (also see Fig. 1 at those values).

The additional information is required to decrease the subjective entropy. In the framework of the presented model and developed doctrine the mathematical expressions constructed and the major parameters being considered will undoubtedly lead to factual realizations. This process of the modifications creation is a challenging task; and it is absolutely clear that the necessary additional information needed to decrease the uncertainty of the alternative AGTE blades restoration technologies (subjective entropy of the available preferences) lies in the sphere of the technological processes intrinsic values selected each time by the researcher to compare the alternatives. Therefore, the essential parameters of the processes models, as well as the models' own plausible mathematical constructions, for every stated problem setting, embody the significant information that finally decreases the entropy value.

Thus, if we suppose existence of the different thresholds of the entropy: 0.5; 0.6; 0.7 (see Fig. 3) for the corresponding choice, then the subjective entropy as the measure of uncertainty will be higher or lower than mentioned thresholds. More to the point, there are two extremums at

$P = 1.8$ and 5.2 kg/h; the latter says of practically two-alternative situation since the uncertainty almost coincides with the two-alternative situation uncertainty maximal value ln2 (see Fig. 3).

IV. Conclusion

In view of the stated problem, the developed doctrine allows formulating the following new scientific results.

The proposed approach gives a possibility of the alternative aviation gas-turbine engine blades restoration technologies comparison based upon the application of the entropy extremization principle. The doctrine implementation reflects the results of the goal achieving for both objectively existing and subjectively preferable optimums, as well as for their combinations.

Thus, for the absolutely objectively existing polytropic process, the pressure in polytropic process is the optimal hybrid-optional function, measured with certain units, of the "*logarithmic measureless volume*". Furthermore the polytropic process index is obtained on the basis of the multi-optional entropy conditional optimality doctrine rather than on the absolutely thermodynamic derivations.

For the subjective component the preferences functions allow the alternatives assessing with the uncertainty measure.

In further research it should be considered some other effectiveness functions and their variables, as well as found more theoretical results and applicable areas of the hybrid-optional optimality doctrine.

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