Roles Contradictions Play in Logical Models of Metaphors and Presuppositions

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Abstract. The article considers new approaches to logical analysis of metaphors and presuppositions. Arguments are given in favor of the necessity for a contradiction to be present in metaphors. It is established that logical model of presuppositions also contains a contradiction, but, unlike in metaphors, here it can be eliminated without distortion the meaning of the phrase.

Keywords: metaphor, presupposition, logical model, contradiction.

1 Introduction

In natural languages, including scientific texts and journalism, and especially in subsets of natural languages used in computers, any contradiction is considered to be an undesirable component, which should be avoided as much as possible. At the same time, contradictions are inevitable and, as a rule, they stimulate criticism and development of our knowledge. However, situations are widely known where contradictions are considered not disadvantages, but advantages of a language. These include metaphors, which will be shown to not exist without a hidden or obvious contradiction. In addition, we will put forward arguments in favor of the necessity for a contradiction to be contained in the logical model of presupposition. Conversely to the metaphor, in this case it can be eliminated without distorting the meaning of the text.

Let us clarify what is meant here by a contradiction. In formal logic, a contradiction is defined as an identically false logical formula in which any substitutions for any interpretation are false. For example, formulas $A \wedge \neg A$ and $(A \supset B) \wedge (A \supset \neg B) \wedge A$ contain contradictions. In natural reasoning, the concept of contradiction is broader: a situation is considered a contradictory one when an object is supposed to exist within the situation and this object has incompatible properties. Consider an example when the following premises are given:

1) All my friends are braggarts.
2) All my friends are not rowdies.
3) All braggarts are rowdies.

To analyze this reasoning, we can use the means of propositional calculus [1] or partially ordered sets [2], but we will use a relatively simple system of logical inference described in [3] and based on *QC*-structures.

A ***QC*-structure** (abbreviated from quasi-complement) is a partially ordered set (poset) that has the *smallest* (**0**) and the *largest* (**1**) elements and a quasi-complement operation with the following properties:

(i) for any element A of a poset, there exists or can be computed a single element $\overline{A}$ called the ***quasi-complement*** of A;

(ii) for any element A, the equality $\overline{\overline{A}} = A$ is satisfied;

(iii) for any two elements A and B, if $A \leq B$, the contraposition $\overline{B} \leq \overline{A}$ is correct.

This mathematical system completely describes properties of all types of partially ordered sets, multisets, and fuzzy sets. This system is proved to not comply with the law of the excluded middle, which is typical, in particular, for fuzzy sets and multisets. But if we extend the axioms of quasi-complement with the property:

(iv) for any element A, the relation $A \leq \overline{A}$ is admissible only for the case when A = **0** and $\overline{A}$ = **1**,

we obtain a poset that has all properties of the inclusion relation in algebra of sets. Such kind of *QC*-structures is called ***Euler's logical structure*** (the name is due to the fact that these structures correspond to the properties of Euler's circles (or diagrams)) abbreviated as ***E*-structure**. In *E*-structures, the order relation is usually denoted by the symbol "$\subseteq$".

Universal affirmative propositions of the type "All A are B" or "The property B is inherent to the object A" can be represented as set inclusions: $A \subseteq B$. Universal negative propositions of the type "All A are not B" we model as $A \subseteq \overline{B}$. Unlike Aristotelian syllogistics, the reasoning system [3], in accordance with a more accurate modern conception of logical deduction, admits arranging premises in an arbitrary order (in syllogistics, conclusions change in some cases, when the order of premises changes), and the first premise of a statement may be negative, which is not recommended in syllogistics.

If there are no particular judgments in a system of premises [3], it is sufficient to use only two laws of algebra of sets as rules of inference, namely:

1) *contraposition* ($A \subseteq B$ is equipotent to $\overline{B} \subseteq \overline{A}$) and

2) *transitivity* ($A \subseteq B$ and $B \subseteq C$ infer $A \subseteq C$).

To reduce complexity of calculations when working with a large number of initial premises of the type $X \subseteq Y$, it is recommended to apply the contraposition law for all premises of reasoning first, after which all other consequences can be obtained by using the law of transitivity.

A contradiction in the broad sense (a paradox collision [3]) is stated here in the case when an inference results in at least one premise of the type $X \subseteq \overline{X}$.

Denote F for my friends, B as braggarts, R as rowdies. Then the premises can be written as logical formulas: 1) $F \subseteq B$; 2) $F \subseteq \overline{R}$; 3) $B \subseteq R$. Consider the corollaries. By the contraposition rule, $F \subseteq \overline{R}$ derives $R \subseteq \overline{F}$, and $B \subseteq R$ infers $\overline{R} \subseteq \overline{B}$. According to the transitivity rule, $F \subseteq B$ and $B \subseteq R$ yield $F \subseteq R$, $F \subseteq \overline{R}$ and $\overline{R} \subseteq \overline{B}$ infer $F \subseteq \overline{B}$, while $F \subseteq R$ and $R \subseteq \overline{F}$ deduce $F \subseteq \overline{F}$.

If we translate the resulting consequences into a natural language, we will see that "my friends" have opposite properties: they are "braggarts" and "not braggarts", "rowdies" and "not rowdies", and eventually it turns out that "my friends" are "not my friends".

If we use the language of propositional calculus for this reasoning system, the inclusion $X \subseteq Y$ shall be replaced with the implication $X \supset Y$, complement of the set $\overline{X}$ is equal to the negation $\neg X$, and the totality of premises shall be united by using the symbol of conjunction ($\wedge$).

Then the enumerated premises can be expressed in the form of the logical formula: $(F \supset B) \wedge (F \supset \neg R) \wedge (B \supset R)$. Analysis that is not included here shows that this formula is not identically false. Hence, there is no formal contradiction, but the object "my friends" does not exist here. In natural reasoning, such a broad-sense contradiction denotes non-existence of a presumably existing object. In [3], it is called a paradox collision. It would be a good idea to use this term in order not to confuse the broad-sense contradiction with a formal one, but we propose to keep the more common name in this article. We define this case as a paradox since the object F is assumed to be true (that is, I have friends), while the premises yield that this object is false. This situation can be reduced to a formal contradiction, if we add what is meant (in this example, the formula F), to the initial premises. Then, it is easy to prove that the formula

$$F \wedge (F \supset B) \wedge (F \supset \neg R) \wedge (B \supset R),$$

which includes the implicitly expressed premise, is formally contradictory. Indeed, as proved above, the sub-formula $(F \supset B) \wedge (F \supset \neg R) \wedge (B \supset R)$ infers $\neg F$, and the formula $F \wedge \neg F$ is contradictory.

The simplest logical model of a contradiction of the given type is defined by the formula $A \wedge (A \supset B) \wedge (A \supset \neg B)$, which can be expressed by the following phrase: "The object A is true; at the same time it has the property B and does not have the property B". This formula is easy to prove to be equivalent to the formula $\neg A$, but this does not mean that the given formula is identically false, since it contains a dummy variable B, whose value can be either true or false. In general case, we will consider a reasoning ***contradictory*** (in the broad sense), if its formalization and logical analysis reveal a variable X that denotes a presumably existing object and assumes the false value only.

2 Logical Model of Metaphors

A metaphor is a word (in general, an expression) that is intentionally used in the text instead of another (replaced) word (expression) based on some incomplete coincidence of meanings of these words (expressions). Such incomplete coincidence of meanings in the definition of a metaphor is essential; otherwise it is difficult to distinguish a metaphor from a synonym. Sometimes a metaphor is defined as an action. For example, "Metaphor is the transfer of a name from one subject or phenomenon to another based on their similarity in some respect." As O. N. Laguta [4] noticed, definitions of this kind use a metaphor (the word "transfer" is a metaphor). Note that usage of metaphors in any science is more likely a rule than an exception (for instance, locutions like "the effect of gossip" in chemical reactions, "black hole", "solar corona", "vertebral column", "computer virus", "lattices" in mathematics, etc.).

The concept of metaphor was known even in ancient Greece. Here is the Aristotle's definition: "Metaphor is a transfer of a word with an altered meaning from a genus to a species, from a species to a genus, or from a species to another species, or in the form of proportion." Metaphor is considered in numerous scientific works throughout the course of human history, beginning with antiquity. At present, the growth of research interest to the metaphor is associated with formation of cognitive science [4 – 7].

Interest to the metaphor becomes more intense and rapidly widens, capturing different fields of knowledge, namely philosophy, logic, psychology, psychoanalysis, hermeneutics, literary criticism, philology, theory of fine arts, semiotics, rhetoric, linguistic philosophy, and various schools of linguistics [5]. Due to this increased interest, a new science has emerged, whose name is "metaphorology" [4].

Let us consider some logical models of the metaphor. In [4], a logical model based on the "deviatological approach" is considered, where the metaphor manifests itself as some logical anomaly. Within this approach, a deviation from logical norms is detected when a metaphor is a convoluted deduction (enthymeme), i.e., an inference with a missed premise. As an example, in [4] the metaphor "Admiralty Needle" (in the quoted text, it is "the Needle of the Admiralty") from Pushkin's poem "The Bronze Horseman" is used. Obviously, the "needle" replaces the word "spire" in this case. The following inference is proposed.

The minor premise:

This spire (architectural element) (S) is very long in relation to its own diameter, straight, with a point (M).

The major premise:

[Everyone knows that] some (tools) are long in relation to their own diameter, straight, with a point (M); they are needles (P).

Conclusion: The spire (S) is a needle (P).

Note that in this example, "deviation" is not only a "convolution" of the inference. According to the rules of syllogistics and formal logic, the conclusion "A spire is a needle" cannot be deduced from the initial premises, even if you do not take into account the bracketed differences ("architectural element" and "tool"). The reason is that the major premise is formulated as a private assertion (it contains the word "some"), so the syllogism turns out to be wrong, and the given conclusion is nondeductive.

In order to correctly formulate a logical model of the metaphor, it is necessary to recognize that such a model necessarily contains a contradiction. Traditionally, it is considered that only some properties of an object or a phenomenon, denoted by a metaphor, coincide with properties of an object denoted by a replaced word. At the same time, in the numerous definitions of the metaphor, the differences in meanings of the word-metaphor and the replaced word are not sufficiently emphasized; just this feature of the metaphor determines many of its remarkable attributes and, moreover, distinguishes it from another linguistic phenomenon – synonymy.

Consider the same example. We designate S as a spire, M as a very long object with respect to its own diameter, straight, with a point, P denotes a needle, A means an architectural element, and T is a tool.

Let us formulate the correct premises taking into account the above distinction. Then we obtain:

$$S \supset M;\ P \supset M;\ S \supset A;\ P \supset T;\ A \supset \neg T.$$

The last premise affirms that properties "architectural element" and "tool" are incompatible. To analyze these premises, we use the inference system [3] again. Analysis shows that there are no contradictions in the totality of the above premises.

Let us construct some corollaries. By the contraposition rule, the judgment $P \supset T$ yields $\neg T \supset \neg P$, and the statements $S \supset A$, $A \supset \neg T$ and $\neg T \supset \neg P$ infer $S \supset \neg P$ by the transitivity rule (i.e., a spire is not a needle). Hence, it is clearly impossible to deduce the proposition "a spire is a needle" as a consequence of these premises. And if we add the statement $S \supset P$ (that is, the proposition that defines the metaphor) to the system of premises, we will obtain the expression $\neg S$ as one of the consequences (i.e., the logical variable "spire" takes the value "false"). To obtain a formal contradiction, it suffices to add the formula S to the premises, which means that the replacing word "spire" is true.

If we use the means of mathematical logic, then this and any other metaphor is restored and reproduced by the logical formula

$$S \wedge (S \supset M) \wedge (P \supset M) \wedge (S \supset A) \wedge (P \supset T) \wedge (A \supset \neg T) \wedge (S \supset P), \qquad (1)$$

where S is a metaphor, P is a replaceable word, M stands for the matching properties of objects denoted by S and P, A is a property of the object S, T is a property of the object P.

It is not difficult to prove that the latter formula is formally contradictory.

The question is as follows: what role does the contradiction play in the metaphor? Part of the answer to this question can be found in the work of P. Ricoeur [6]. He believes that the contradiction in metaphors creates tension between terms, which is the essence of metaphorical meaning.

Hence, in order to increase this "tension" and, accordingly, the aesthetic attractiveness of a metaphor, the difference in values between the metaphor word and the replaceable word must be as large as possible; not just different gradations of the values for one property (for example, like "architectural element" and "tool " in the above metaphor), but the values should be close to the level of antonyms (a small needle and a huge spire). This is one of the main features of metaphors, in which a "strong" contradiction is a necessary component.

From the point of view of logical analysis, this situation occurs not only in metaphors. For example, in reasoning by analogy, properties of one object are matched with properties of another object based on coincidence of certain properties. Similar identification occurs in some models of case-based reasoning [8]. With this in mind, it makes sense to generalize the paradox arising in metaphors to the numerous cases of matching for various objects by means of replacing their names. To do this, let us consider the paradox of identification. Assume there exist an initial object O and its analogue A, and these objects have a common set of properties PC. The object O is also known to have properties PO, and object A has some incompatible properties PA, which can be expressed by the formula $PA \supset \neg PO$. Then the logical model of identification can be expressed using the formula

$$A \wedge (A \supset PC) \wedge (O \supset PC) \wedge (A \supset PA) \wedge (O \supset PO) \wedge (PA \supset \neg PO) \wedge (A \supset O),$$

in which the subformula $A \supset O$ denotes the procedure for replacing the original object with an analogue, and the subformula A at the beginning is an assertion of the trueness of the analogue. It is easy to verify that this formula, as well as its similar formula (1), is contradictory.

The paradox of identification does not refute frequently encountered and very useful reasoning by analogy or case-based reasoning. This paradox is valid only in cases where the original object and its analogue are identified and investigation reveals incompatibility of some their properties.

3 Presupposition

It is easy to find the term "presupposition" (the term "assumption" is preferably used in papers in English) in publications on logic and philosophy [9 – 15], linguistics [16], cognitive science [17], neuro-linguistic programming (NLP) [18, 19], etc. This concept has several differing definitions. We will hold this one: Presupposition is an assertion stipulated (or considered to be true) when analyzing a major assertion or question, while negation or falsity of the major statement does not influence trueness (or falsity) of the presupposition.

For example, the proposition "John has returned back in his family" presupposes that he had gone from the family one day. Evidently, the phrase inverse to the major statement, i.e., "John has not returned to his family") does not change the trueness of this presupposition. Conversely, the sentence "Peter had enough money to buy a smartphone" cannot be correctly used as a presupposition for the proposition "Peter bought a smartphone in a store" since the negation of the major statement, namely "Peter did not buy a smartphone", can be caused in particular by the reason that Peter did not have enough money to pay at that time. Presuppositions are often included in the major statement explicitly. For instance, the phrase "Richard did not know that wolves were found in this forest" clearly presupposes that "There are wolves in this forest."

Hidden presuppositions often cause subconscious perception of some assertions. Sometimes, this is used to manipulate attitudes of people, i.e., to manipulate consciousness [18]. The same can be applied for advertising and in disputes for asking "tricky" questions implicitly presupposing a misdeed of the adversary. Such questions can look like "Do you continue to beat your father?" or "Are you going to return the stolen goods?".

The concept of presupposition was also studied in detail by E. V. Popov [20] during his research in artificial intelligence (AI). His study of communication with a computer in natural languages displayed that omitting presuppositions during automatic translation can lead to distortions in meaning of texts. In [21], D. A. Pospelov showed how important is to consider presuppositions in models of inference. Modern publications on AI rarely deal with presuppositions. For instance, this concept is absent in fundamental AI monographs like [22, 23]. In accordance with [11], we will further name a major statement as assertion.

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4 Logical Analysis of Presuppositions

Connection problems between explicit and implicit information stimulated logicians in the Middle Ages already [21]. Conversely, linguists consider G. Frege one of the first researchers who drew scientific attention to hidden statements in logical analysis. Particularly, he analyzed distinctions between the assertion in a statement and presupposition(s) for this assertion [24]. He understood presuppositions fairly simply yet, namely only as statements about existence of a referenced entity. For instance, he regarded existence of a person Mozart as an only presupposition for the phrase "Mozart died in poverty".

P. Strawson [9] and B. van Fraassen [10] logically analyzed presuppositions in detail. Strawson proposed the following definition for presuppositions: a sentence P is a presupposition of S, if trueness of P is a necessary condition for S to be true (i.e., S can be either true or false). If P is false, S has no value.

Van Fraassen studied relationships between presupposition and implication. One his proposed definitions stated as follows:

P is a presupposition of S, if and only if:

(a) if S is true, then P is true,

(b) if (non-S) is true, then P is true.

In propositional calculus, this definition corresponds to the formula:

$$\text{P is a presupposition of S if } (S \supset P) \text{ and } (\neg S \supset P).$$

Some studies define presupposition by using the notion of "corollary" rather than implication: "A statement P is a presupposition of S, if it is a corollary from both S and from the negation of S." However, such definitions can cause problems. For instance in [11], the author noted that the formula $(S \supset P) \wedge (\neg S \supset P)$ is equipotent to P, that is such interpretation yields fictitiousness of S. Moreover, many examples of presuppositions show that it usually is a precondition for an assertion, and the opposite interpretation is wrong. Events used in an assertion are mostly a prolongation of the events comprising a presupposition, so the former events may not be considered as preconditions/antecedents.

Sometimes, logical analysis for obtaining a presupposition P for a given statement S reminds derivation a corollary. For example, the reasoning: "The fact John used to beat his father (P) can be derived from the fact that John continues to beat his father (S)" looks likely. However, the more thorough analysis shows that here we have restoring of a former event rather than deducing P from S.

The above-stated leads us to logically define the presupposition unlike the mentioned authors do. Suppose we have the assertion (S) "Anthony was late for school." Evidently, the statement (P) "Anthony was going to school" is a presupposition of S. The latter sentence is also true if Anthony was not late. If we suppose P to be false, S has no sense at all. We cannot consider it false since its negation ("Anthony was not late for school") is actually false too.

The formal approach results in a paradox when presupposition is treated as a precondition. As the formula $(P \supset S) \wedge (P \supset \neg S)$ is equivalent to $\neg P$, the identical falsity of the presupposition can be stated, although it is assumed to be true according to the meaning of the statements. At the same time, no paradox results from informal analysis of all examples of presuppositions. To study this controversy, we need to closer investigate examples of presuppositions. For the assertion "Anthony was late for school", the fact "Anthony was going to school" is obviously a preceding event for Anthony's being late (or not late).

Within classical logic, we have to add a new factor into reasoning in order to explain presupposition as a precondition. Evidently when Anthony went to school, he could have different reasons to be late (oversleeping, meeting with friends and talking with them, helping an old woman in crossing a road, and so on). Conversely, if no such interfering factors occur, Anthony would not be late. So a presupposition can serve a correct precondition of a major assertion, if we introduce one or more new factors (attributes, variables) into reasoning. In our case, this can be a logical variable R clarifying existence or not existence of reasons for Anthony to be late for school. Obviously, such a factor is necessary to substantiate some strange features" of presupposition.

To get rid of the paradox, we formulate the following hypothesis. Let an assertion S and its presupposition P be given, and we add a new variable R called the relay of an assertion. Within propositional calculus, we obtain:

Hypothesis. If P is a presupposition of a sentence S, then there exists and can be found a logical variable R such that the expression $P \wedge R$ is a prerequisite of the sentence S, and the expression $P \wedge \neg R$ is a prerequisite of the sentence $\neg S$.

Then the argument containing the presupposition and the relay of the assertion can be written by the formula $((P \wedge R) \supset S) \wedge ((P \wedge \neg R) \supset \neg S)$.

We can find that this formula contains no dummy variables, hence, all variables (namely, the assertion S, the presupposition P and the relay R) are not fictitious. Moreover, P can become true or false in this formula, so there is no paradox in such reasoning.

This hypothesis can be favored by the fact that the trueness of the presupposition is preserved for any value of the trueness of the assertion. Hence, it seems quite possible to surmise that the assertion changes its values of trueness depending on some other factor(s). In addition, analysis of numerous examples of presuppositions shows that such a factor (i.e., the relay of an assertion) can always be found.

For instance, the sentence "Alex did not pass the contest to an institute" can have a presupposition "Alex tried to enter the Institute." The negation of the original sentence does not influence the trueness of this presupposition. To substantiate reasons why Alex passed (or did not pass) this contest, we will need at least one more attribute (for instance, the level of Alex's capabilities, term of his practicing, etc.).

On the other hand, this technique does not explain another "peculiarity" of presupposition. Namely, if we false or deny a presupposition, the sense of both assertion and its negation is lost. For example, the sentence "Jones has hitherto been sick" can serve a presupposition for the assertion "Jones has recovered." If we deny the presupposition, neither assertion nor its negation makes sense since Jones did not recover. Other examples of presupposition display the same feature.

Explanation of this phenomenon is assumed to be beyond the binary logic, i.e., a non-classical logic should be used as an analysis tool [11, 15]. However, it is possible to solve this problem within the framework of classical logic by using a rule-based knowledge representation system.

Consider what happens when denying a presupposition. The presupposition is implicitly present in the values of both assertion and its negation (both "somebody was late to school" and "somebody was not late to school" imply that "someone was going to school"). Thus we can explain why negating a presupposition leads to inconsistency of both the assertion and its negation to the meaning of the modified presupposition: the meaning of the presupposition has changed, and the meanings of the assertion and its negation still imply the former meaning of the presupposition. At the same time, we can assume that when changing a presupposition, the assertion shall also be changed in order to either imply the modified presupposition or to skip a premise associated with the new assertion and the initial presupposition. Hence, the new assertion has to admit a value that differs from the values of the initial assertion and its negation. Then the previous values of the assertion are not the negations of each other, but simply incompatible situations (in our example, the value "did not come" becomes possible in addition to the values "was late" and "arrived on time").

If an assertion is represented as a logical variable, many researchers consider non-classical logic a necessary tool for modeling presuppositions. In this model, assertion can take three values rather than two. To remain in the classical framework, it is sufficient to assume that the variables P, R, and S, which respectively stand for presupposition, relay of assertion and assertion itself, are not logical variables, but unary predicates that have 2, 2, and 3 possible values, correspondingly. To model presupposition within this model, we can use a rule-based knowledge base that can be represented by means of predicate calculus.

Let ranges of predicates values be given as: $P = \{0, 1\}$, $R = \{0, 1\}$, $S = \{p, q, r\}$. Then the logical model of presupposition will look like this:

If P is a presupposition of S, the following rules are taking place.

(1) if P(1) and R(1), then S(a);

(2) if P(1) and R(0), then S(b);

(3) if P(0), then S(c).

Rule (3) models the "inanity" of the assertion in case its presupposition is false. Hence, if Jones was not sick that corresponds to P(0), he is neither recovered ($\neg$S(a)), nor continues to be sick ($\neg$S(b)) now. For P(0) situations are possible in which Jones continues to be healthy (S(c)) or he fell ill all of a sudden. We can describe the latter situation, if we first add another possible value c1 to the values of the predicate S. Sure, we do not wish Jones this option.

The above-proposed technique allows analyzing most examples of presuppositions.

5 Relation of Presupposition to Contradiction Anomalies in KBs

Many papers ([25 – 27, etc.]) deal with anomalies in KBs, which formalize some rules of reasoning. Research on KBs' verification [28] yielded some first methods to recognize and eliminate KBs' anomalies, which can relate to integrity violations (wrong definitions of types and values of attributes) or to consistency violations (mistaken rules themselves). Consider one anomaly of the second category, namely the contradiction anomaly.

Usually, a rule rp looks like $B1 \wedge B2 \wedge \ldots \wedge Bn \rightarrow A$. The part to the left of the arrow is called the antecedent of the rule, and the right part is the consequent of this rule.

Contradiction anomaly. Let the following two rules be given:

r1: $B1 \wedge B2 \wedge \ldots \wedge Bn \rightarrow D$;

r2: $C1 \wedge C2 \wedge \ldots \wedge Cn \rightarrow F$.

Besides, $Ci \subseteq Bi$ for each i ($i = 1, 2, \ldots, n$) and $D \cap F = \varnothing$. In such a case, the listed rules contain an anomaly of contradiction.

Rules with contradicting consequents and coinciding antecedents describe a particular example of contradictory rules. For instance, the KB of a robot can include following two rules:

rp: $B1 \wedge B2 \rightarrow D$;

rq: $B1 \wedge B2 \rightarrow F$.

Here B1 corresponds to the statement "there is an obstacle ahead", B2 states that "the target is behind the obstacle", D advises to bypass the obstacle on the right, and F wants the robot to pass the obstacle on the left.

Both rules are executable and contradict each other nevertheless. If we consider them contradictory ones, some revising of the KB becomes necessary, in particular, deletion of one of the rules. Conversely, such a removal can result in poor consequences in some cases, when this anomaly does not manifest an error in a KB. Some rules that look conflicting in the course of reasoning can become absolutely correct after considering some data, which had been possibly missed. In this respect, the contradiction anomaly corresponds to the presupposition.

Consider the above-introduced example of two contradicting rules:

rp: if an obstacle is ahead, and the goal is behind the obstacle, go around the obstacle on the right;

rq: if an obstacle is ahead, and the goal is behind the obstacle, go around the obstacle on the left.

As we said this case looks more like a lacking presupposition than an anomaly. So, no change of the KB is needed, rather, we have to define or find an assertion relay (see Definitions 1 and 2), i.e., an additional attribute that describes different obstacles located on the right and left of the main obstacle, namely, their list and locations.

As we saw, contradictory rules and presuppositions have almost the same definitions. However, the precondition of a presupposition includes one variable, and matching (or nested) preconditions of contradicting rules can comprise several attributes. Sometimes, this allows us to consider contradiction anomalies as a sign for searching an assertion relay rather than declaring a paradox. Then it is necessary to finding some additional variables that solve the paradox.

Now, we propose some techniques to search for a relay of an assertion. Let conflicting rules in a KB be given:

rD: B1 $\wedge$ B2 $\wedge \ldots \wedge$ Bn $\rightarrow$ D;

rF: B1 $\wedge$ B2 $\wedge \ldots \wedge$ Bn $\rightarrow$ F,

and D $\wedge$ F = False.

Here each disjunct Bi is a couple "attribute – its value(s)". For instance, the condition "If Xi = a or Xi = b, then ..." means that Bi includes the set of values {a, b}. The atoms D and F belong to the same attribute, and the equality D $\wedge$ F = False means that the sets of their values do not overlap.

When describing a system in terms of "attribute – its value(s)", every rule is defined within a certain relation diagram, which is determined by a set of attributes. The relation diagram of the antecedent of a rule rm is named Ant(rm), Cons(rm) will mean the relation diagram of the consequent for this rule, and Val(Xi, rm) is the value of the attribute Xi in this rule. Suppose we have two contradicting rules rD and rF, and XContr is the set of attributes in their antecedents, so Ant(rD) = Ant(rF) = XContr.

To find an assertion relay for the above rules, we propose the following algorithm.

1) Determine the subset S of the rules rm present in a KB for which Cons(rm) = {Y};

2) Within the set S, determine subsets of rules SD $\subseteq$ S and SF $\subseteq$ S with values of the attribute Y equal to Val(Y, rD) and Val(Y, rF) respectively;

3) For the above-obtained subsets SD and SF, remove rules for which the correlations XContr $\subset$ Ant(rD) and XContr $\subset$ Ant(rF) are not satisfied (strict inclusion means that antecedents of the selected rules contain other attributes besides XContr);

4) From the obtained sets SD and SF, exclude the rules in which the attributes values differ from the corresponding values in the conflicting rules rD and rF;

5) From the sets SD and SF, form the set P of rules pairs (rm, rn) such that rm $\in$ SD, rn $\in$ SF, and Ant(rm) $\cap$ Ant(rn)) \ XContr $\neq \varnothing$ (i.e., their antecedents have other common attributes apart from XContr);

6) For every pair (rm, rn) of P and every attribute Xi of the set (Ant(rm) $\cap$ Ant(rn)) \ XContr, check the correlation Val(Xi, rm) $\cap$ Val(Xi, rn) = $\varnothing$;

7) If the intersection in step 6 is empty for an attribute Xi, this attribute is an assertion relay for conflicting rules rD and rF.

If the introduced algorithm gives no positive result, a similar search can be applied for pairs, triples, etc. of attributes to check whether they can serve assertion relays rather than single attributes.

There is a simple example of seemingly contradictory rules in a KB:

(i) if a bull goes towards a man, and the man shows the bull a red rag, then there is a big risk of causing significant damage to the health of the man;

(ii) if a bull goes towards a man, and the man shows the bull a red rag, then there is no risk of causing significant damage to the health of the man.

Consider how this algorithm works. Let X denote the attribute "direction of the bull" with values "to the subject", "from the subject"; Y is the attribute "red rag in the hands of the subject" with values "true" and "false"; and Z means the risk of causing significant damage to the health of the subject with values "big" and "small".

In accordance with the algorithm, we form sets of rules SD and SF, which have Z as the consequent, while its value is high risk in SD and small risk I SF (Steps 1 and 2).

In the sets SD and SF, we retain only rules whose antecedents contain other attributes in addition to X and Y (Step 3).

In the selected rules, we retain only those for which the value of the attribute X is "to the subject", and the value of the attribute Y is "true" (Step 4).

From the sets of rules SD and SF, we form a set P of pairs (rm, rn) of rules with different values of the consequent (attribute Z); these rules can contain the same attributes Wj in their antecedents and those attributes differ from X and Y (among such attributes, an assertion relay may be found) (Step 5).

In each pair of the set P, we compare the values of the attributes from the set Wj; if for an attribute its values in different rules are incompatible, this attribute is an assertion relay (Steps 6 and 7).

For example, such an attribute may be the location of the subject (the subject, in particular, may be standing in a clear field, or be in the cab of an armored personnel carrier). Another possible option is the attribute "subject's profession"; if he is a bullfighter, the value of the attribute Z is "small", otherwise it is "big".

As we can see from the description of the algorithm, its computational complexity is evidently polynomial (not higher than the second degree of the total number of rules in the knowledge base). It becomes more complex, if the solution of the problem requires for the search for intermediate conclusions (for example, "if the bull goes towards the subject and the subject shows the bull a red rag, then the bull is expected to attack the subject"). This algorithm is under development.

6 Conclusion

For metaphors, we developed a model applicable in propositional calculus and beyond it. This model allows keeping the difference in values between the metaphor word and the replaceable word as large as possible up to the level of antonyms, which is good to increase the aesthetic attractiveness of a metaphor.

For presuppositions, we analyze their connection with contradiction anomalies in knowledge bases. To explain presupposition as a precondition within the framework of classical logic, it is suggested to supply reasoning with an assertion relay that is new factor(s) formalizing presence or absence of reasons for trueness or falsity of an assertion. To simulate the "inanity" of an assertion when its presupposition is denied, it is proposed to use the model of predicate calculus instead of propositional calculus in order to define the assertion as a logical variable with more than two values. For contradicting rules, we introduced an algorithm to determine possible assertion relays.

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