



# Research directions for the rHEALPix Discrete Global Grid System

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## ABSTRACT

Discrete Global Grid Systems (DGGs) are important to Digital Earth, the Open Geospatial Consortium (OGC), and big data research. A promising OGC conformant quadrilateral-based approach is the rHEALPix DGGs. Despite its advantages over hexagonal- or triangular-based DGGs, little research is being done to explore or advance our understanding of it. In this paper, we briefly review existing work and then present several important directions for future research related to harmonic analysis, discrete line generation, pure- and mixed-aperture DGGs, and DGGs-based distance/direction metrics.

## 1. Introduction

A Discrete Global Grid System (DGGs) consists of a hierarchy of discrete global grids at multiple resolutions. DGGs represent a class of spatial data structures that directly address the earth's surface via a topologically equivalent approximation such as the sphere or ellipsoid (Sahr and White 1998). Since their introduction, DGGs have slowly gained prominence in the geospatial community and in 2015, they were identified as the foundation of modern Digital Earth frameworks (Mahdavi-Amiri et al. 2015). In 2017, they were adopted by the Open Geospatial Consortium (OGC) with the aim of standardizing the DGGs model, increasing awareness, and increasing interoperability between DGGs (OGC 2017a). More recently, DGGs were suggested as a solution to big spatial vector

data management (Yao and Lin 2018) and as the de facto global reference system for geospatial big data (OGC 2017b).

Over the years, several different DGGs have been created each with advantages and disadvantages. However, only a subset are deemed appropriate under the OGC DGGs Abstract Specification (OGC 2017a). Importantly, an OGC conformant DGGs must utilize a method that partitions the earth's surface into a uniform grid of equal area cells. Currently, the most popular method is the Icosahedral Snyder Equal Area (ISEA) projection and a considerable amount of research has focused on hexagonal- or triangular-based DGGs that adopt this approach.

That being said, quadrilateral-based DGGs have several advantages over hexagonal-or triangular DGGs, such as compatibility with existing data structures, hardware, display devices, and coordinate systems (Sahr et al. 2003; Amiri et al. 2013). Moreover, recent work has demonstrated benefits of using a quadrilateral approach in various domains, such as data transmission and rendering (Sherlock 2017) and point cloud handling (Sirdeshmukh 2018).

The rHEALPix DGGs (Gibb et al. 2016) is a promising OGC conformant quadrilateral-based approach with many interesting properties. Despite this, DGGs research remains focused on hexagonal- or triangular-based approaches and little work is being done to explore or advance the rHEALPix DGGs. In this paper we review existing work and highlight several important directions for future research. We hope this work will promote the benefits of

the rHEALPix DGGS and stimulate more researchers to explore it.

## 2. Review of existing work

In short, the rHEALPix DGGS is constructed by projecting an ellipsoid (e.g. WGS84) onto the faces of a cube, partitioning each face into square grids, and then inversely projecting the result back to the ellipsoid (Figure 1.0). Besides the theoretical definition presented in Gibb et al. (2016), there are only three works directly related to the rHEALPix DGGS which we now briefly review.

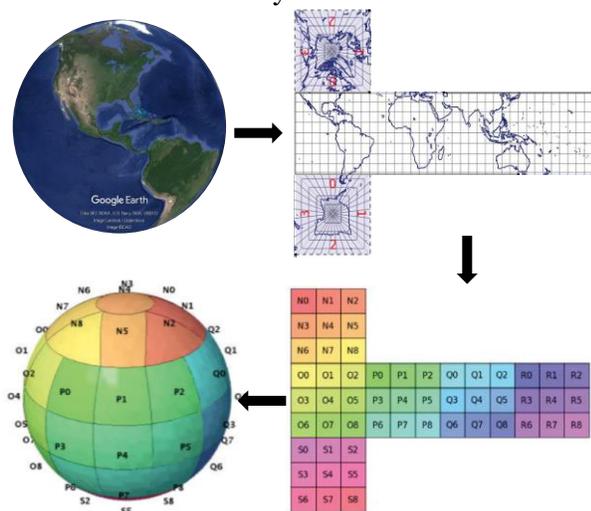


Figure 1.0. Constructing the rHEALPix DGGS (grid resolution 1 is shown) (derived from Gibb 2016).

Gibb (2016) advances our understanding in three ways. Firstly, he describes how cell identifiers uniquely address cells across all resolutions of the rHEALPix DGGS using a cell addressing scheme that has both space filling and hierarchical properties. Secondly, he explores cell adjacency and describes a method to determine cell ID's of adjacent cells at any resolution using a combination of base 3 and base 4 math. Lastly, he presents methods to determine DE-9IM topological relationships (e.g., within, contains, and touches) by manipulating cell ID's. Importantly, this work shows how cell adjacency and topological relationships can be efficiently determined using cell ID's directly rather than geodetic coordinates.

Bowater and Stefanakis (2018a) consider the rHEALPix DGGS from a Canadian perspective and discuss how varying cell shape and cell orientation are key considerations in the polar region (i.e.,  $|\varphi| > 41.9^\circ$ , where  $\varphi$  is geodetic latitude). In addition, they describe how these variations can be avoided or exploited for small regions of interest (e.g., provinces) by rotating the grid cells. In particular, they show how triangular dart cells can be avoided for regions with longitudinal extent less than approximately  $90^\circ$ , and how north-south aligned quadrilateral grids can be created in the polar region. This is important because north-south aligned quadrilateral grids are familiar to users and link to similar grids used in remote sensing and environmental modelling (Gibb 2016).

Bowater and Stefanakis (2018b) present an open-source web service that enables users to create grids based on the rHEALPix DGGS. In their paper, the authors describe the implementation, including issues and limitations, and demonstrate how both discrete global grids and regional grids can be created. This is important work because it provides an easily accessible tool for experimenting with grids based on the rHEALPix DGGS, thereby promoting its use in future research. In addition, it supports interoperability studies with other DGGSs which is an important aim of the OGC DGGS Abstract Specification.

## 3. Directions for future work

Evidently, the body of research related to the rHEALPix DGGS is small. In comparison to hexagonal- and triangular-based DGGSs, the rHEALPix DGGS is largely unexplored which means several directions exist for future work. In this section, a number of them are presented that we feel deserve attention.

Arguably, the most unique property of the rHEALPix DGGS is the distribution of cell nuclei along rings of constant latitude

(often referred to as isolatitude distribution). Gorski et al. (2005) states that this property is essential for computational speed in operations involving spherical harmonics, which means the rHEALPix DGGS is a good choice for applications (e.g., gravitational field modelling) that involve harmonic analysis (Gibb et al. 2016). To our knowledge, no other OGC conformant DGGSs have this property. However, it has not yet been fully explored. Therefore, future work should explore this property further (e.g., is it possible to determine isolatitude ring directly from cell ID to facilitate harmonic computations), and real-world applications involving harmonic analysis should be investigated.

In a DGGS, vector data (i.e., points, lines, and polygons) are rasterized into cells. Therefore, if we consider linear features, such as roads or rivers, the generation of the discrete line is an important problem (Du et al. 20108). Although solutions for hexagonal (Tong et al. 2013) and triangular (Du et al. 2018) DGGSs have been presented, discrete line generation on the rHEALPix DGGS has not been studied. Furthermore, the typical approach involves solving the problem in the plane and then projecting the result to the ellipsoid. But this causes issues in the plane when the linear feature intersects more than one base cell. Moreover, regarding the rHEALPix DGGS, square cells in the plane project to different cell shapes on the ellipsoid. Consequently, the ideal discrete line in the plane may not be so on the ellipsoid. Therefore, an interesting direction for future work is to consider discrete line generation in the plane but also directly on the ellipsoid.

It may be generally unknown that the definition of the rHEALPix DGGS presented in Gibb et al. (2016) describes a general class of DGGSs rather than a single, unique approach. Specifically, the definition holds for any integer  $N_{side} \geq 2$ , where each planar square is divided into  $N_{side} \times N_{side}$  sub-squares at successive resolutions. In work thus far,  $N_{side} = 3$  has been chosen because it is the smallest

integer that produces aligned hierarchies. However, if we want to maximize the number of resolutions under a fixed number of cells (i.e., to provide a smooth transition), then  $N_{side} = 2$  is a better approach. In addition, one-to-four refinement is exactly encoded using 2 bits, as opposed to one-to-nine refinement which requires 4 bits (although only 9 of the 16 possible values are actually needed). Note that refinement, sometimes called aperture, simply refers to the process of subdividing a cell into smaller cells. Therefore, we see two interesting directions for future work: (i) investigate the  $N_{side} = 2$  rHEALPix DGGS and make comparisons with the  $N_{side} = 3$  approach, and (ii) explore the possibility of a mixed-aperture rHEALPix DGGS. Unlike pure-aperture DGGSs, mixed-aperture DGGSs need not have the same aperture across all resolutions. Therefore, they provide greater control over cell area at each resolution and have been successfully implemented for hexagonal DGGSs (Sahr 2013).

Our last direction for future research is not specific to the rHEALPix DGGS - it is relevant to all DGGSs. Currently, there is no way to determine the distance (or direction) between two cell IDs without recourse to geodetic coordinates (Sirdeshmukh 2018). Just as a DGGS simplifies integration of heterogeneous data sets on a global scale, a DGGS-based distance (or direction) metric would simplify vector data analysis by removing the dependence on geodetic coordinates and complex ellipsoidal formulae. In this way, DGGSs would become a more complete solution to all our geospatial needs. While this may not be possible, future work should attempt to find out.

## 4. Conclusion

The rHEALPix DGGS is a promising quadrilateral-based approach that has several advantages over hexagonal- or triangular-based DGGSs. However, the body of research that explores the rHEALPix DGGS is small. In this paper, we briefly

reviewed existing work, and then proceeded to highlight several directions for future work related to harmonic analysis, discrete line generation, pure- and mixed-aperture DGGs, and DGGs-based distance/direction metrics. We believe these areas are interesting, challenging, and necessary to advance our understanding of the rHEALPix DGGs.

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