# Generalized Model of Functioning of Generator Equipment of Synchronization Network with the Limited Reliability of its Elements

Andrew K. Kanaev Department of Electrical Communication, Emperor Alexander I St. Petersburg State Transport University Saint Petersburg, Russia kanaevak@mail.ru Andrew A. Privalov Department of Electrical Communication, Emperor Alexander I St. Petersburg State Transport University Saint Petersburg, Russia aprivalov@inbox.ru

Valery V. Sapozhnikov Department of Autom. Remote Control, Emperor Alexander I St. Petersburg State Transport University Saint Petersburg, Russia TrES-4b@yandex.ru

### Abstract

In the given article the estimation of stationary characteristics of reliability of the generator equipment of telecommunication system (TCS) with use semi-Markov models is resulted. A generalized semi-Markov model of the process of functioning of generator equipment is proposed, with the separation of states reflecting the main operating modes of the TCS generator equipment. Based on the developed semi-Markov model, the stationary reliability characteristics of the generator equipment are calculated, which reflect its integral state.

## **1** Introduction

Modern information systems taking into account the applied information technologies in their work are based on telecommunication systems (TCS). One of the key TCS subsystems that support its operation is the synchronization network.

Copyright © by the papers' authors. Copying permitted for private and academic purposes.

Eugene V. Oparin Communication department, "Giprotranssignalsvyaz" - branch of JSC "Roszheldorproekt" Saint Petersburg, Russia onapuh@mail.ru

The main purpose of the synchronization network is to maintain the coordinated interaction of the digital communications equipment of the telecommunications system. This coordinated interaction is ensured by the generation, transmission and delivery of the synchronization signals necessary to provide and maintain synchronous operation of the TCS equipment [Dav04].

The occurrence of failures in the synchronization network and the deviation of the quantitative and qualitative characteristics of the clock signals can cause a significant deterioration in the quality of the transmitted data, up to the total refusal to provide telecommunications services. Given this feature of the synchronization network, an important role is played by the process of ensuring a given reliability of its elements, as well as solving problems of evaluation and prediction of reliability. An important role is played by the process of managing the synchronization network as an integral part of the TCS, ensuring the required performance of its operation. Particularly relevant are the issues of managing the synchronization network during periods of failure, including as a result of external influences. In these states, it is necessary to restore the functioning of the synchronization network within a given time to ensure the stability of the entire TCS. In the process of managing the synchronization network, an important task is not only to evaluate the current generator equipment and

In: B. V. Sokolov, A. D. Khomonenko, A. A. Bliudov (eds.): Selected Papers of the Workshop Computer Science and Engineering in the framework of the 5 th International Scientific-Methodical Conference "Problems of Mathematical

and Natural-Scientific Training in Engineering Education", St.-Petersburg, Russia, 8–9 November, 2018, published at http://ceur-ws.org

other elements of the synchronization network, but also to forecast the states and reliability indicators in order to prevent failures and emergencies.

There is a complex of methods for assessing the reliability of the elements of the TCS and the process of functioning of the TCS, among which a special place is occupied by semi-Markov methods for assessing reliability. The advantage of using semi-Markov methods of reliability evaluation is the ability to produce simulations in cases where the law of distribution of the occurrence of failures differs from the exponential distribution law. Semi-Markov models are also used in cases where there is an independence of the probabilities of a transition from one state of the process of functioning to another from previous transitions between states, and also when the law of distribution of the residence time in states does not depend on previous stages of functioning [Shu12, Ush85]. In view of the foregoing, in most cases, the processes of the functioning of the subsystems of the TCS can be considered as semi-Markovian, for which it is possible to estimate and predict the reliability indicators.

# 2 Types Of Generating Equipment Used In Telecommunication System Synchronization Networks

As sources of synchronization signals on digital communication networks, primary reference sources (PRS) are used, the characteristics of which are governed by international Recommendations [ITU98/1, ETS98/1]. The performance requirements determine the permissible long-term deviations of the PRS frequency from their nominal value, the limiting values of the wanderings of the output signals, expressed in terms of the maximum time interval error (MTIE), deviation of the time interval (DTI), and permissible changes in the duration of the clock interval with all possible internal switching. In accordance with the Recommendations for PRS output signals, "the maximum permissible frequency deviation from the nominal value should not exceed for all practicable environmental conditions and observation times of not less than one week". On the synchronization networks. various types of PRS can be used, both forming the sync signals independently, and using reference clock signals transmitted by other systems not directly connected to communication systems. As PRS, creating a sync signal for communication systems, can serve as cesium or hydrogen generators. Reference synchronization signals can also be obtained by signal receivers transmitting earth stations operating on long waves or signals from satellite navigation systems. To synchronize communication networks, not individual PEIs are used, but a complex consisting of several PEIs, called the primary reference generator (PRG). In small-scale networks, it is sometimes permissible to use separate PRSs in conjunction with a secondary master oscillator (SMO).

The primary reference generator is a complex of equipment containing three PEI and SMO. As a rule, PRG is the main source of reference clock signals on the synchronization network. The equipment that is part of the PRG can be completely autonomous and use only cesium or hydrogen PRS. The PRG output signals should not change their duration for any switching in the equipment by more than 1/8 of the clock interval.

The output signals of PRG equipment are usually the 2.048 MHz and 2.048 Mbit/s clock signals, as well as other reference signals, the need for which may appear when the digital network is synchronized (64, 100 kHz, 1, 5 and 10 MHz).

The main characteristics of the PRS are regulated and outlined in ITU-T Rec. G.811 [ITU98/1].

To restore and maintain the necessary quality of sync signals on the synchronization network, there are the SMO and the local master oscillator (LMO). Due to the fact that the LMO have slightly worse characteristics than the SMO, they are applied only on the sections of the synchronization network, from which the synchronization signals do not flow to other LMO or SMO. To ensure reliable synchronization, the synchronization network of almost any telecoms operator should be based on own equipment of the SMO or LMO.

In many cases, the SMO is additionally connected with the PRS in order to more reliably reserve the reference clock signals. Thus, the SMO and the LMO are widely used on the digital network and are the main element of the synchronization network. Availability on the network of equipment of the SMO and LMO allows to organize the system of control and monitoring of the synchronization network and, to some extent, to provide control over the state of the entire digital network. The technical requirements for the SMO and the LMO are defined in ITU-T G.812 [ITU98/2].

The SMO provides the choice of the best synchronization input from a number of sources, while ensuring the necessary processing and filtering of the clock signals, with their subsequent distribution. In the event of failure of all input interfaces, loss of all input signals or degradation of their characteristics to a level below the required values, the LMO should memorize the frequency of switching to the frequency memorization mode in accordance with ETS 300 462-4 and ITU-T Rec. ITU-T G.812 [ITU98/2, ETS98/2]. The secondary master oscillator is synchronized with external synchronization signals of 2048 kHz or 2048 kbit/s. The number of output interfaces of synchronization signals with a frequency of 2048 kHz in the composition of the SMO is usually not less than 12 with the possibility of increasing to 64, and the number of output interfaces of synchronization signals of 2048 kbit/s is at least two.

The SMO also includes a synchronization signal converter, which restores the original clock characteristics in information signals of 2048 Kbit/s that have come to this node using the plesiochronous digital hierarchy (PDH) systems or extracted from the systems of the synchronous digital hierarchy (SDH). This signal, denoted as E1/T, can later be used to synchronize the equipment. For monitoring and for frequency comparison, the SMO consists of 1/5 MHz output interfaces.

Local master generators are used on local network sites as the last reference for synchronization. The local master oscillator is intended for use on synchronization nodes and small-scale network sections that do not contain the SMO, for the reconstruction and multiplication of clock signals arriving via communication lines from PRG or SMO. The structure of the LMO is similar to the structure of the SMO, with the difference that the LMO allows the use of simpler master oscillators, which to a lesser extent suppress phase noise and have a less accurate mode of frequency memorization.

The lowest link in the hierarchy of generator equipment is the generator of the network element (GNE) is a master generator built into the network element (multiplexer), which receives synchronization inputs from a number of external sources, selecting one of them and producing the minimum filtering. In the event of damage to all input reference synchronization signals in the GNE, an internal self-master oscillator shall be used which, in the frequency memorizing mode, will store approximately the frequency of the input clock in accordance with ETS 300 462-5 and Rec. ITU-T G.813 [5, 8].

To the master generator of the network element, according to the recommendation G.813, requirements are imposed for the permissible relative error of the natural frequency, which should not exceed  $4.6 \cdot 10^{-6}$  and the presence of a storage mode, the error of frequency memorization with loss of the synchronization signal should be no more than  $5.0 \cdot 10^{-8}$ , and the daily frequency drift is less than  $1 \cdot 10^{-8}$ .

# **3 Semi-Markov Model Of The Process Of Functioning Of The Generator Equipment**

To assess the stationary reliability characteristics of generating equipment in the TCS, it is necessary to form a model of the process of its functioning. This model is formed (Figure 1) and reflects all the basic modes of operation of generator equipment. Despite the great variety of types and versions of synchronization equipment, clock generators and equipment for their distribution, the model of the process of operation of the generator equipment necessarily includes the following states:

s<sub>1</sub> – power supply of generator equipment;

 $s_2$  – initialization of the software, warming up and self-testing of the generator equipment;

 $s_3$  – setting the mode of free oscillations with the selection of the external synchronization signal;

 $s_4$  – setting the mode of synchronous operation with accumulation of memory for the hold mode;

 $s_5$  – setting the hold mode;

 $s_6$  – failure of generator equipment and subsequent restoration of its operability.

The operation of the generator equipment is as follows. The initial state is the state s1. After turning on the power supply of the generator equipment, the initialization mode starts, software checks with simultaneous heating of the element and self-testing. Further, the generator equipment enters the free oscillation mode with analysis and selection of the external synchronization signal.



Figure 1: Semi-Markov model of the generator equipment operation process

The mode of free oscillations is characterized by the fact that in this mode of operation there is no control over the frequency of the internal oscillator. This mode of operation occurs after the initialization of the software of the generator equipment, when external clock signals are not used, and also when faults occur in the synchronization circuits. In this case, the quality of the external sync signals becomes inadmissible for use, and there is not enough data in the generator to go into hold mode. When the external synchronization signal is selected, the equipment of the synchronization network starts to gradually enter the synchronous operation with external clock monitoring. The synchronous operation mode is the main mode of operation of the generator equipment. In this mode, the signal at the output of the generator equipment is monitored with an adjustable phase-locked loop with an allowable accuracy of the signal change at the input of the external synchronization. At the same time, the necessary data is accumulated to ensure the required accuracy of the signal in the hold mode. If there is an external sync signal and accumulation of memory, the generator equipment is completely switched to the synchronous operation mode. In this mode of operation, if the quality of the sync signals deteriorates or their loss disappears, the generator equipment goes into a hold mode, and it is possible to go back to the synchronous operation mode in case of restoration of the required quality of clock signals and storage of memory. The hold mode occurs after working in synchronization mode as a result of faults in the synchronization circuits. In this mode, the signal at the output of the generator equipment does not depend on the signal at the input, but is determined by the values of the control signal from the device memory, as well as the accuracy of storing the control signals, processing the stored signal values, and frequency deviations of the generator due to the influence of destabilizing factors.

The hold mode only occurs if the generator equipment was in the synchronous operation mode for a long time until the failure, and at the same time enough data was accumulated to ensure the required initial synchronization signal accuracy in the hold mode.

The transition between the operation modes of the generator equipment can be carried out in a directive way by sending the appropriate commands from the operating personnel. It is believed that the occurrence of a refusal can occur at any time.

# 4 Determination Of Reliability Indicators Of Generator Equipment On The Basis Of The Developed Semi-Markov Model Of The Process Of Its Functioning

The main stationary characteristics of the reliability of the generator equipment will be:

Stationary probabilities  $(\pi_i), i = 1, ..., 6; i \in S$  of the generator equipment staying at an arbitrary time in each of the states si;

Mean time between failures T0 and average idle time of generator equipment  $T_{\Pi P}$ .

The initial data for evaluating the reliability of generator equipment are [2]:

- The matrix of transition probabilities  $\Pi = (p_{ij})$ :

- The matrix of the distribution functions of the conditioned random times of the generator equipment in each of the si states  $F_{ij}(t)$ ;

The stationary probability of the generator equipment staying at an arbitrary moment of time in each of the states  $s_i$  can be calculated by the following formula [2]:

$$\pi_{i} = \frac{P_{i}T_{i}}{\sum_{j \in S} P_{j}T_{j}} (i, j = 1, ..., 6; i, j \in S; \sum_{i \in S} \pi_{i} = 1)$$
(1)

where, Pi, Pj is the stationary probability of the stay of the embedded homogeneous Markov chain in the state  $s_i$  and  $s_j$ ,  $T_i$ ,  $T_j$  is the mathematical expectation of the unconditional residence time of the generator equipment in each state, S is the total number of states.

To estimate the mathematical expectation of the unconditional residence time of the generator equipment in each state, we use the following expressions [Shu12] (2, 3):

$$T_{i} = \sum_{j \in S} p_{ij} T_{ij}$$

$$T_{ij}(t) = \int_{0}^{\infty} [1 - F_{ij}(t)] dt$$
(3)

where  $T_{ij}$  is the mathematical expectation of the conventional time of the generator equipment in each state.

To estimate the stationary probability of an embedded homogeneous Markov chain in the state si, we use the following expression [Shu12] (4, 5):

$$P_i = \frac{D_i}{\sum_{j=1}^n D_j}$$

(4)

where Di (Dj) is the minor obtained by deleting i(j) of the row and i(j) of the column of the matrix D.

$$D = \begin{pmatrix} 1 - p_{11} & -p_{12} & \dots & -p_{1n} \\ -p_{21} & 1 - p_{22} & \dots & -p_{2n} \\ \dots & \dots & \dots & \dots \\ -p_{n1} & -p_{n2} & \dots & 1 - p_{nn} \end{pmatrix}$$
(5)

To estimate the mean time between failures  $T_0$  and average downtime  $T_{\Pi P}$ , the final set of states S of the process of generating equipment operation is divided into two disjoint subsets of workable states  $S_P \subset S$  and inoperable states  $\overline{S_P} \subset S$ , where  $S_P \cup \overline{S_P} = 0$ 

The effective states  $S_P$  will be the states  $s_1$ ,  $s_2$ ,  $s_3$ ,  $s_4$ 

and  $s_5$ . An inoperable state  $S_p$  is the state  $s_6$ . In the presence of the specified initial data, the mean time between failures of the system and the average idle time can be found from the following expressions [2]:

$$T_{0} = \frac{\sum_{i \in S_{p}} P_{i}T_{i}}{\sum_{i \in S_{+}} P_{i} \sum_{j \in \overline{S}_{p}} P_{ij}}$$
(6)

$$T_{\Pi P} = \frac{\sum_{i \in S_{p}}^{P_{i}} P_{i} I_{i}}{\sum_{i \in S_{p}} P_{i} \sum_{j \in S_{p}} p_{ij}}$$
(7)

where  $S_+$  and  $S_-$  is the subset of the boundary operable and inoperative states that condition the transition from a subset  $S_P$  to a subset  $S_P$  and vice versa. A subset of the boundary states  $S_+$  is the states  $s_2$ ,

 $s_3$ ,  $s_4$ ,  $s_5$ . The subset of the boundary states  $\begin{array}{c} S \\ - \end{array}$  is the state  $s_6$ .

Estimating the stationary probabilities  $\pi_i$  of the generator equipment at any time in each of the states si, it is possible to determine the availability and idle factors according to the following expressions [Shu12, Sta10]:

$$K_{\Gamma} = \sum_{i \in S_{P}} \pi_{i}$$

$$K_{TP} = \sum \pi_{i} = 1 - K_{\Gamma}$$
(8)

$$\kappa_{\Pi P} = \sum_{i \in \overline{S_P}} n_i = 1 - \kappa_{\Gamma}$$
(9)

# 5 Calculation of numerical values of stationary reliability characteristics of generator equipment

As an example, an estimate of reliability indicators of the generator equipment of the telecommunications operator is given below. The following data are accepted as initial data. The matrix of transition probabilities has the following form (10):

$$\Pi = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.95 & 0 & 0 & 0.05 \\ 0 & 0 & 0.95 & 0 & 0.05 \\ 0 & 0 & 0.45 & 0 & 0.45 & 0.1 \\ 0 & 0 & 0.05 & 0.9 & 0 & 0.05 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

As the distribution of the conditioned random time of the generator equipment in each of the si states  $F_{ij}(t)$ , an exponential distribution is adopted with

the following transition intensities matrix (11):  $\begin{pmatrix} 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 

$$\Lambda = \begin{pmatrix} 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0.01 & 0 & 3 \\ 0 & 0 & 0.01 & 0 & 0.001 & 2 \\ 0 & 0 & 0.01 & 0.001 & 0 & 2 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, h \quad (11)$$

Thus, after the subsequent calculation based on the proposed approach and the developed model of the process of the generator equipment operation, the following results were obtained. The matrix of mathematical expectations of the conventional times of the generator equipment in each state will take the following form (12):

$$T_{ij} = \begin{pmatrix} 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0.25 \\ 0 & 0 & 100 & 0 & 0.33 \\ 0 & 0 & 100 & 0 & 1000 & 0.5 \\ 0 & 0 & 100 & 1000 & 0 & 0.5 \\ 0.2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \text{ h (12)}$$

The mathematical expectations of the unconditional residence times of the generator equipment in each state will take the following form (13):

 $T_i = (0.2 \ 0.25 \ 95.017 \ 495.05 \ 905.025 \ 0.2)$ , h (13) The stationary probabilities of the stay of the embedded homogeneous Markov chain in the states si take the following form (14):

 $P_i = (0.061 \ 0.061 \ 0.235 \ 0.376 \ 0.207 \ 0.061)$  (14) Thus, the following steady-state probabilities of the generator equipment stay at an arbitrary instant of time in each of the states si (15):

$$\pi_{1} = 3.077 \cdot 10^{-5};$$
  

$$\pi_{2} = 3.846 \cdot 10^{-4};$$
  

$$\pi_{3} = 0.057;$$
  

$$\pi_{4} = 0.47;$$
  

$$\pi_{5} = 0.473;$$
  

$$\pi_{6} = 3.077 \cdot 10^{-5}.$$

The probability data  $\pi_i$  show that at any random time the generator equipment is in one of the states  $s_i$ .

Mean time between failures  $T_0$  and average idle time  $T_{\Pi P}$  of generator equipment will be calculated as follows (15, 16):

$$T_{0} = \frac{\sum_{i \in S_{p}} P_{i}I_{i}}{\sum_{i \in S_{p}} P_{i}\sum_{j \in S_{p}} P_{ij}} = \frac{P_{1}T_{1} + P_{2}T_{2} + P_{3}T_{3} + P_{4}T_{4} + P_{5}T_{5}}{(P_{2}P_{26} + P_{3}P_{36} + P_{4}P_{46} + P_{5}P_{56})} = 2418 h$$
<sup>(15)</sup>  
$$T_{IIP} = \frac{\sum_{i \in S_{p}} P_{i}T_{i}}{\sum_{i \in S_{p}} P_{i}\sum_{j \in S_{p}} P_{ij}} = \frac{P_{6}T_{6}}{P_{6}P_{61}} = 0.684 h$$
<sup>(16)</sup>

Accordingly, the availability factor and idle ratio will take the following values (17, 18):

$$K_{\Gamma} = \sum_{i \in S_{P}} \pi_{i} = \pi_{1} + \pi_{2} + \pi_{3} + \pi_{4} + \pi_{5} = 0.99996923 \quad (17)$$

$$K_{\Pi P} = \sum_{i \in S_{P}} \pi_{i} = 1 - K_{\Gamma} = 3.077 \cdot 10^{-5}$$
(18)

Similarly, based on the developed model of the process of generating equipment operation, reliability indicators of any synchronization equipment can be used.

### **6** Conclusion

The synchronization network is an important subsystem of TCS, which directly affects the quality of providing communication services. To timely prevent failures in the synchronization network, a sound evaluation of the reliability of the synchronization equipment is required, with the help of which the maintenance personnel can preventively perform replacement and repair. In this connection, this article presents the results of estimating stationary reliability characteristics of generator equipment using semi-Markov models. The obtained results will allow to estimate the reliability indicators on the basis of the statistics of the technical operation of the synchronization network equipment, and, consequently, to conclude that it is advisable to continue using a separate type of equipment or replace it for further repair. It is assumed that the distribution functions of the conditional random time of the synchronization network element in each of the states will be determined from the practice of operating the synchronization network, which will allow to obtain a model of the process of functioning of the generator equipment with real properties. This model is universal, differs from the completeness of the state registration and can be applied to any kind of generator equipment, and also allows to determine the probabilistic and temporal characteristics of each state of the generator equipment under study, which allows, when imposing regulatory requirements on the probability-time characteristics, to formulate a set of strategies for achieving them through control probability-time characteristics of individual states.

## References

- [Dav04] Clock network synchronization / P.N. Davydkin, M.N. Koltunov, A.V. Ryzhkov - M.: Eko-Trends, 2004. – p. 205.
- [Shu12] Structural reliability of information systems. Methods of analysis / I.B. Shubinsky. -Ulyanovsk: Regional Printing House "Printing Yard", 2012. - p. 216.

- [Ush85] Reliability of technical systems: Reference book. Ed. I.A. Ushakova / Yu. K. Belyaev, V.A. Bogatyrev, V.V. Bolotin and others. - M.: Radio and Communication, 1985. - p. 608.
- [ITU98/1] Time characteristics at the outputs of primary reference reference generators: ITU-T Recommendation G.811. - 1998.
- [ETS98/1] Transmission and multiplexing (TM): General requirements for synchronization networks. Part 5. Time characteristics of slave generators for operation in the equipment of the synchronous digital hierarchy SDH: European Telecommunication Standard ETS 300 462-5. - 1998.
- [ITU98/2] Temporary requirements for slave master generators suitable for use as nodal in synchronization networks: ITU-T Recommendation G.812. - 1998.
- [ETS98/2] Transmission and multiplexing (TM): General requirements for synchronization networks. Part 4. Time response driven generators to ensure synchronization of synchronous digital hierarchy equipment (SDH) and plesiochronous digital hierarchy (PDH): the European telecommunications standard ETS 300 462-4. - 1998.
- [ITU98/3] Timing characteristics of SDH master slave generators: ITU-T Recommendation G.813. - 1998.
- [Sta10] GOST R 53480-2009 reliability in the art. Terms and Definitions. - Moscow: Standartinform, 2010. - p. 33.