### The Controlling of Transmission of Chaotic Signals in Communication Systems Based on Dynamic Models

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Abstract. The article is devoted to the calculation of the characteristics of dynamic chaos based on the traffic of the corporate computer network. An algorithm for the transmission of chaotic information using dynamic chaos based on the model of chaotic masking and nonlinear mixing of the information signal is proposed. The problem of installing chaotic synchronization between two chaotic generators under conditions of phase signal filtration is considered. Based on the characteristics of the phase filter, the channel characteristics of the multichannel information signal filter are determined.

Keywords: dynamic chaos, algorithm, signal, traffic, computer network, transmission.

#### 1 Introduction

At present, the development of telecommunication technologies determines the growth of research in the field of search for new solutions and innovative approaches to the mathematical description of the processes of transmission of information signals. One of the areas in the field of describing traffic in computer networks is the dynamic models that describe the real-time transmission of information in the form of differential or finite difference equations. A significant amount of work on modeling traffic in computer networks is based on the theory of mass service.

The transmission of messages with the help of a modulated chaotic signal has several advantages compared with the air conditioning modulation of the harmonic signal. Indeed, if there are only three controlled characteristics in the case of harmonic signals, then in the case of chaotic oscillations even a small change in the parameter gives a reliably detectable change in the nature of the oscillations. This means that the sources of chaos with changeable parameters have a wide range of information signal input circuits in the chaotic one. In addition, chaotic signals are broadband in principle. In communication systems, a wide frequency band of carrier signals is used both to increase the speed of information transmission and to increase the stability of the systems in the presence of disturbances. The noise similarity and the selfsynchronization of chaos-based systems give them potential advantages over traditional spectrum expansion systems based on pseudo-random sequences.

Currently, it is known that chaotic signals generated by nonlinear deterministic dynamic systems, the so-called dynamic chaos, have a number of properties that facilitate the use of these signals for information transmission. A number of specific information transmission schemes have been proposed that use dynamic chaos, in particular, a scheme for the chaotic masking of an information signal; schemes with nonlinear mixing of the information signal into random messages. The possibilities of creating direct communication systems, in which chaotic oscillations act as a carrier of information generated directly in the frequency domain where information is transmitted, are discussed.

For modern networks characterized by a chaotic distribution of computing resources and a variety of end users, modeling for the creation of control systems for transmission of information signals is a particularly urgent task.

### 2 Formal problem statement

Chaotic synchronization of dynamic systems interconnected through a channel in which filtering of a chaotic signal transmitted from a leading dynamic system to a slave one is carried out is a basic model by which various methods of transmitting information based on chaotic synchronization are tested. Currently, there are a number of approaches aimed at solving the problem of establishing chaotic synchronization in the presence of signal filtering. In this case, as a rule, the filtering of a chaotic signal is carried out by a filter of the first or second order, which does not always adequately reflect the characteristics of a real information transfer channel. There are a number of works in which the influence of filtering on chaotic synchronization is considered in the context of a wireless information transmission channel, which is much closer to possible practical applications of chaotic synchronization.

Achieve synchronization, filters are applied that are inverse to the channel filter, which allows compensating for signal distortion at the input of the slave system, caused by the imposition of several replicas of the same signal on it.

The problem of developing methods and models of chaotic processes in communication systems based on the reflection of a nonlinear dynamic system for encryption and transmission of information has been studied. Conduct a study on this cryptographic algorithm for all required parameters.

The main purpose of the study is the use of chaotic algorithms for the transformation of information signals. The article deals with the characteristics of dynamic chaos for the transmission of messages and it is shown that the value of the highest Lyapunov index does not guarantee the randomness of the dynamics of information signals.

### **3** Literature review

Currently, it is known that chaotic signals generated by nonlinear deterministic dynamic systems, the so-called dynamic chaos, have a number of properties that facilitate the use of these signals for information transfer. The possibilities of creating direct communication systems, in which chaotic oscillations act as a storage medium generated directly in the frequency domain where information is transmitted, are discussed [1].

Fundamentals of the dynamic chaos oscillator operation are disclosed including optical ones, mathematical models are discussed, and principles of its application for information protection are described in [2]. The structure of a communication system is synthesized and its mathematical model that employs the invariant properties of the GCO is proposed. Computer simulation of the communication system is used to estimate the noise immunity of the system that works in the communication channel with distortions and noise in [3].

The experimental simulation shows that using the chaotic pilot signal for synchronization control can help achieve chaotic synchronization between communication transmitting and receiving systems and further enhance security and confidentiality of exchange of information and transfer of data [4]. In order to attain better synchronization, this TS fuzzy modeling is combined with the robust  $H\infty$  observer theory based on linear matrix inequalities [5].

Once the pair is synchronized, the states can be used to secure the communication channel in one of four ways: chaotic modulation schemes, chaotic multicarrier schemes, chaotic multiple access schemes, and chaos-based encryption schemes [6]. The communication channel with a 2N-level logarithmic quantized is described, and the transmission delay of the communication channel is taken into account [7].

In [8] lay out a quantitative cryptanalysis approach for symmetric key encryption schemes that are based on the active/passive decomposition of chaotic dynamics. We end this chapter with a summary and suggestions for future research. The efficiency of data encryption at the transmitter and the recovery performance from an authorized receiver are also presented through diverse fiber transmission experiments. In these experiments, the security discrimination level between authorized and eavesdropping receivers is discussed [9].

The corresponding results demonstrate an ability to achieve initial synchronization. Furthermore, it is shown that in terms of code acquisition the PRBS outperforms the logistic and Bernoulli chaotic maps when used as pilot signals [10]. A smart IoT information transmission and security optimization model based on chaotic neural computing model are proposed. Simulation and analysis show that the proposed algorithm can ensure the availability and confidentiality of data at the same time [11].

Meanwhile, as improved teaching-learning-based optimization algorithm, teaching-learning-feedback-based optimization algorithm is proposed to optimize the parameters more excellently [12]. The subnets in the transmitter and receiver are one-toone correspondence and form a pair of matching subnets, but the node size of each subnet can be inconsistent [13]. Authors in [14] show that our method can stabilize the chaotic oscillations in the user-centric cognitive radio networks. By comparisons with traditional parameter tuning methods, we confirm that our method is more efficient and faster to stabilize the cognitive radio systems. Meanwhile, the transmission error decays to zero exponentially. This implies that the synchronization error converges to zero under a limited communication channel [15].

The high quality of extraction of the hidden information signal is achieved due to the use of digital elements in the scheme, which ensures the identity of the parameters and high stability to noise [16].

Many articles are devoted to the transmission of messages with the help of a modulated chaotic signal. This modulation method has several advantages compared with the air conditioning modulation of the harmonic signal. Indeed, if there are only three controlled amplitudes in the case of harmonic signals, then in the case of chaotic oscillations even a small parameter change gives a reliably detectable change in the nature of oscillations [16]. Noise-like and self-synchronizing systems based on chaos give them potential advantages over traditional spectrum spreading systems based on pseudo-random sequences.

# 4 Analysis of the distribution of information transmission in chaotic traffic

The basis for the analysis was the data on the Internet channel load, obtained in the course of monitoring the work of the university's corporate network, measured during the year. Statistics are obtained when removing information from the router interfaces about the amount of information transmitted and the port loading, using the SNMP-protocol, using the Paessler Router Traffic Grapher package, which forms data tables and download schedules (Fig. 1).



Fig. 1. Fragment of uploaded charts in the data link.

An empirical histogram of channel loading frequencies is shown in Fig. 2. The empirical histogram has a "ponderous tail", indicating the presence of peak network load moments, in which there is a strong increase in delays and packet loss.



Fig. 2. The histogram of empirical in a network of output channel loading.

Information on the loading of communication channels was also obtained by monitoring the external communication channels of one of the provider companies and the site optimization company. The obtained histograms also have heavy tails (Fig. 3), indicating the presence of peak network load points, in which there is a strong increase in delays and loss of information.



Fig. 3. The empirical distributions function of loading channels of the transmission information signal.

Due to the fact that the distribution function has a heavy tail and is not consistent with the Poisson distribution, queuing theory for the networks in question cannot provide an adequate mathematical description. For TCP/IP, the distribution with heavy tails makes the main contribution to the self-similar nature of traffic and, therefore, the chaotic nature of the dynamics.

A number of studies are devoted to the study of traffic randomness. The values of the Lyapunov senior indicator were estimated based on the traffic generated on the experimental bench, Internet traffic was for calculating various characteristics. The dynamic properties of chaos were used to solve telecommunication data exchange problems, but the study of chaotic properties remained outside the scope of publications. It is assumed that the time series is generated by a discrete:

$$x_{k+1} = f(x_k x_0), (1)$$

or continuous system:

$$\frac{dx(t)}{dt} = F(x(t), x(0)).$$
<sup>(2)</sup>

Where  $x=(x_1(t), ..., x_n(t))$ ; *n* – the dimension of the phase space; t is time; k - discrete time (number); F, f - function vector. The phase trajectory of a continuous system is an n-dimensional curve, which is the solution of the system in the coordinates of the state space under given initial conditions x0. For discrete systems, states are connected by lines in accordance with the sequence of samples k=1, 2, ....

An important concept of dynamical systems is an attractor. For systems in the equilibrium position, the attractor is a point, for oscillatory systems, closed cycles. For chaotic systems, there is an attractor, which is called strange, in this case, the trajectories are tightened, but not to a point, a curve, a torus, but to some subset of the phase space. An attractor is an invariant characteristic of the system; it is preserved under the actions of transformations.

The unambiguous characteristics of the randomness of the signal are the spectrum of Lyapunov indicators. A positive maximum Lyapunov exponent is an indicator of chaotic dynamics, a zero maximum Lyapunov exponent indicates a limiting cycle or a quasiperiodic orbit, and a negative maximum Lyapunov exponent represents a fixed point. The system of dimension n has n Lyapunov exponents:  $\lambda 1$ ,  $\lambda 2$ , ...,  $\lambda n$ , ordered in descending order. By definition, introduced by Lyapunov:

$$\lambda_i(x_0) = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\left| \delta_1(t) \right|}{\left| \delta_i(0) \right|} \tag{3}$$

where  $|\delta_1(t)|$  - the fundamental solutions of the system, linearized in the neighborhood of x0.

Dynamical systems for which the n-dimensional phase volume decreases are called dissipative. If the phase volume is conserved, then such systems are called conservative. Conservative systems always have at least one conservation law. The presence of a conservation law often entails the existence of a Lyapunov zero exponent corresponding to it. For dissipative dynamic systems, the sum of Lyapunov indices is always negative. In dissipative systems, the Lyapunov exponents are invariant with respect to all initial conditions.

The positives of the highest Lyapunov exponent cannot be a necessary condition for the existence of chaos. Even in the Lorentz system, with the positive index being positive, as is known, under a number of conditions, a limit cycle takes place.

As an additional criterion, it is proposed to use the property of the absence of trivial conservation laws. Note that compression of the phase volume does not mean compression conversion.

To check for the presence of transformations of trajectory fragments, a genetic algorithm was developed and a program for MATLAB. At the same time, the following assumption has checked the system allows transformations, under conditions of weak symmetry breaking; there is some small value that slightly deviates from the symmetric mapping. Clearly, geometrically, this can be seen with almost similar loops on the attractor. Obviously, under such a test, under different initial conditions, for systems with regular dynamics, the presence of identical symmetry will be revealed, for a more complex, but not chaotic, translation, for systems seeking a stable equilibrium position, compression, for chaos - almost repetitive areas of phase trajectories.

The attractor reconstructed according to the traffic load is shown in Fig. 4.



Fig. 4. Attractor, built on the basis of network traffic.

Confirmation of chaos can be the basis for building dynamic models: in the form of an ensemble of pendulums, affinity systems with control or in the form of series.

# 5 The construction of the modulation algorithm of the chaotic signal information

In most modern communication systems, harmonic oscillations are used as the information carrier. The information signal from the transmitter modulates these oscillations in amplitude, frequency, or phase; in the receiver, information is extracted by means of the reverse operation demodulation. The modulation of the carrier can be carried out either by modulating the already formed harmonic oscillations or by controlling the parameters of the generator in the process of generating oscillations.

Similarly, it is possible to produce a modulation of a chaotic signal with an information signal. However, the possibilities here are much wider. Indeed, if there are only three in the case of harmonic signals of controlled characteristics, then in the case of chaotic oscillations even a small parameter change gives a reliably detectable change in the nature of oscillations. This means that the sources of chaos with changeable parameters, there is a wide range of information signal input schemes in the chaotic one. In addition, chaotic signals are broadband in principle, the interest in which is associated with higher information capacity. In communication systems, a wide frequency band of carrier signals is used both to increase the speed of information transmission and to increase the stability of the systems in the presence of disturbances.

In Fig. 5 shows the simplest communication scheme using chaos. Re-sensor and receiver include all the same non-linear or linear systems as the source. In addition, the transmitter is switched on the adder and the receiver and subtracted. In the accumulator, the chaotic signal of the source of the information signal is added; the receiver is designed to extract the information signal. The signal in the channel is chaos-like and does not contain visible signs of the transmitted information, which allows transmitting confidential information. The signals at points A and A', B and B' are pairwise equal. Therefore, if there is an input information signal S at the input of the transmitter adder, the same signal will be allocated at the output of the receiver's subtracted.



Fig. 5. The simplest communication scheme using the chaos of transformation information signal.

The scope of application of chaotic signals is not limited to systems with the expansion of the spectrum. They can be used for masking the transmitted information and without spreading the spectrum if the frequency band of the information and transmitted signals coincide.

All this stimulated active research on chaotic communication systems. At present, on the basis of chaos, several approaches have been proposed for expanding the spectrum of information signals, building self-synchronizing receivers, and developing simple transmitter architectures and receivers (Fig. 5).

The noise similarity and the self-synchronization of chaos-based systems give them potential advantages over traditional spectrum expansion systems based on pseudorandom sequences. In addition, they allow for a simpler hardware implementation with higher energy efficiency and higher speed of operations.

#### 6 Chaotic synchronization of dynamic systems

Filtering a chaotic signal with a phase filter is equivalent to passing a signal through a multipath channel, which passes all frequencies of the signal with equal amplification, but changes the signal phase. Under these conditions, to achieve chaotic synchronization, a method is needed to establish chaotic synchronization between the master and slave systems, regardless of the number of signal replicas in the channel and the magnitude of their delays, regardless of the channel filter size and its characteristics.



Fig. 6. Synchronization model of a dynamic system with chaotic data transfer filter on communication channels.

We simulated the dynamic system with chaotic data transfer filter on communication channels for a) I is the leading system, II is the slave system, III is the channel filter, 1, 2 are linear subsystems, 3 is the phase a filter equivalent to channel III filter; 4 — non-linear subsystem; b) implementation of the response function of the phase filter 3 and channel III filter; c) bifurcation diagram for the parameter M of the leading system I

The leading system consists of a nonlinear subsystem 4 and linear subsystems: low-pass filters of the first and second orders and filter 3, which is equivalent to the filter of channel III. The slave system consists of the same elements. If the subsystem and channel III filter are excluded from the model, then the proposed scheme is reduced to the traditional scheme for obtaining a chaotic synchronous response.

In the presence of a chaotic synchronous response, the signal  $z_2$  at the output of the slave system is identical to the signal  $z_1$  of the leading system. Indeed, the signal at the output of the slave system  $z_2$  is a copy of the signal  $y_1$ , passed through filter 3.

Let the transient response of the phase filter 3 be described by the function:

$$h(t) = \sum_{k=0}^{N} a_k \delta(t - \tau_k), \qquad (4)$$

where h(t) is the impulse response of the phase filter to the  $\delta$  pulse,  $a_k$  is the amplitude of the k-th replica,  $\tau k$  is the delay in the transmission through the filter of the kth replica relative to the first replica of the input signal, for which, by definition,  $\tau 0=0$ . Physically, this is equivalent to forming at the output of the filter a sum of replicas of a signal having different delays. The implementation of the signal s(t), passed the filter with the characteristic (1), is described by the expression:

$$\Phi(s(t)) = \int_{-\infty}^{t} h(t)s(t-\tau)d\tau,$$
(5)

in the form of a response function (5) with the signal s (t) at the input of the filter.

As a result, the model shown in Fig. 6, described by the following system of equations:

$$Tx_{1} + x_{1} = F(z_{1}(t)),$$

$$\ddot{y}_{1} + \alpha_{1}\dot{y}_{1} + y_{1} = x_{1},$$
(6)
$$z_{1}(t) = \int_{-\infty}^{t} h(\tau)y_{1}(t-\tau)d\tau,$$

$$U(t) = \int_{-\infty}^{t} h(\tau)y_{1}(t-\tau)d\tau,$$

$$T\dot{x}_{2} + x_{2} = F(U(t)),$$
(7)
$$\ddot{y}_{2} + \alpha_{1}\dot{y}_{2} + y_{2} = x_{2},$$

$$z_2(t) = \int_{-\infty}^{t} h(\tau) y_2(t-\tau) d\tau.$$
(8)

The first two equations in (8) describe the filters of the first and second orders, respectively, the third equation (8) - the channel filter. Thus, the leading system in Fig. 6, a is an oscillatory system with a first-order low-pass filter, a second-order low-pass filter, and an N-th order phase filter that implements N different delays with respect to the signal at its input. The slave system is passive, so there is a reason to believe that in the system as a whole, it will be possible to obtain a stable chaotic synchronous response in the presence of phase filtering. To prove this possibility, a numerical simulation of the synchronization process was carried out. Numerical simulation was carried out in the framework of the discrete representation of signals and equations.

The discrete representation eliminates the need to transform the discrete responsefunction of the phase filter into a dynamic system with continuous time, which upon such transformation will have a high dimension with the number of phase variables equal to the number of discrete samples, the amplitudes of which are approximated by numerical modeling.

When switching to the discrete representation of the response function (6), the delays  $\tau k$  in the arrival of the replicas of the signal, taking a continuous spectrum of values in the real channel, in the discrete representation will be defined as  $\tau_k = n_i \Delta t$ , where ni is the number of the reference on which i - replica with respect

 $\Delta t = \frac{1}{f_s}$  is the sampling time step,  $f_s$ 

to the first replica (for the first replica n0=0),  $J_s$  is the sampling time step,  $J_s$  is the sampling frequency. Thus, the channel response function (8) takes the form:

$$h(t_k) = \sum_{k=0}^{N} a_k \delta(t_k - n_k \Delta t).$$
<sup>(9)</sup>

Transformation (9) in a discrete form with a response function (11) takes the form:

$$\Phi(s(t_k)) = \alpha_0 s(t_k) + \alpha_1 s(t_k - n_1 \Delta t) + \alpha_2 s(t_k - n_2 \Delta t) + \dots + \alpha_N s(t_k - n_N \Delta t)$$
(10)

In fact, the conversion is a non-recursive N-th order digital filter with the impulse response (10).

Discrete analogs of first and second order filters included in equation (8) can also be represented as recurrence relations by replacing the derivative operator  $p = \frac{d}{dt}$ with a delay operator by one sample  $q^{-1} = s(t_k - \Delta t)$  using bilinear transform:

$$p = \gamma \frac{1 - q^{-1}}{1 + q^{-1}} \tag{11}$$

where  $\gamma = \frac{2}{\Delta t}$ .

After substitution (11) in (8), the first equation takes the form:

$$x_1(t_k) = b_0^{(1)} F(z_1(t_k)) + b_1^{(1)} F(z_1(t_k - \Delta t)) + a_1^{(1)} x_2(t_k - \Delta t)$$
(12)

The second equation (9) - the filter of the second order - takes the form:

$$x_{1}(t_{k}) = b_{0}^{(2)}x_{1}(t_{k}) + b_{1}^{(2)}x_{1}(t_{k}-\Delta t) + b_{2}^{(2)}x_{1}(t_{k}-2\Delta t) - \cdots - a_{1}^{(2)}y_{1}(t_{k}-\Delta t) - a_{2}^{(2)}y_{1}(t_{k}-2\Delta t)$$
(13)

The coefficients in (14) and (15) are defined as:

$$b_{0}^{(1)} = b_{1}^{(1)} = \frac{1}{1 + \gamma T}, a^{(1)} = (1 - \gamma t) b_{0}^{(1)}, b_{0}^{(2)} = b_{2}^{(2)} = \frac{1}{\gamma^{2} + \alpha_{1}\gamma + 1},$$

$$b_{1}^{(2)} = 2b_{0}^{(2)}, \alpha_{1}^{(2)} = 2(1 - \gamma^{2})b_{0}^{(2)}, \alpha_{2}^{(2)} = (1 - \alpha_{1}\gamma + \gamma^{2})b_{0}^{(2)}.$$
(14)

The transformation to the discrete representation in equations (10) is carried out similarly.

The first three relations describe the leading system, the fourth the phase filter of the Nth order channel, the last three the driven system. The first equation describes a sequentially connected inertialess nonlinear and a first order low pass filter.

The inertialess nonlinear transformation is given by an expression of the same kind:

$$F(z) = M\left[ \left| z + E_1 \right| - \left| z - E_1 \right| + \frac{1}{2} \left( \left| z - E_2 \right| - \left| z + E_2 \right| \right) \right]$$
(15)

where E1, E2 — constants, M — the gain. The specific choice of E1, E2, and M uniquely determine the characteristic.

The result of the numerical solution of system is the implementation of the phase variables  $x_1(t_k)$ ,  $y_1(t_k)$ ,  $z_1(t_k)$ ,  $U(t_k)$ ,  $x_2(t_k)$ ,  $y_2(t_k)$ ,  $z_2(t_k)$ , calculated for the moments of time  $t_k = \frac{k}{t}$ , where k is the reference number. This implementation is equivalent

to the implementation of the phase trajectory obtained by numerically solving the equations (9).

During the simulation, the parameter values were M=7.3, T=3,  $\alpha_1=0.1$ ,  $E_1=0.5$ ,  $E_2=1$ ,  $\Delta t=0.05$ .. The channel phase filter was simulated using the impulse response (4), including N = 10 replicas (Fig. 6,b). The delays in the arrival of the replicas of the

signal are in the range from one to ~20 quasi-periods of chaotic oscillations. Model

(8) is dimensionless, the quasi-period  $T_0$  is equal to the reciprocal of the resonant frequency of the second-order filter (8), that is T0=1/f0, f0=1-( $\alpha$ 1/2)2/2 $\pi$ . Since  $\alpha$ 1<<1, the value of  $f_0 \approx 1/(2\pi)$ .

The bifurcation diagram of the system with respect to the parameter M is shown in Fig. 6, c. The leading system retains the property to generate chaotic oscillations, the ring generator remains a source of chaos with the introduction of an additional filter 3 (Fig. 6, a).

In fig. 7 shows the results of numerical simulation of chaotic synchronization in the system (19). In fig. 7 shows a fragment of the implementation of the signal before, and fig. 7, b - after the phase filter.



Fig. 7. The algorithm of transmitting information signals at different points of the chaotically synchronized dynamic system.

In Fig. 7 is presented the signal  $y_1$  at the input of the channel filter, b is the signals U and  $z_2$  at the output of the channel III filter and the output of the slave system II, respectively, c is the difference signal  $z_1(t)-z_2(t)$  between the signal  $z_1(t)$  of the master system and the output signal of the slave system  $z_2(t)$ .

Since the system is passive and the signal  $z_2$  at its output undergoes the same transformations as the signal  $z_1$  in the master system, the output signal  $z_2$  of the slave system asymptotically tends to the signal  $z_1$  of the master system (Figure 7, c); thus, a chaotic synchronous response in the system can be achieved.

The signal amplitude after the phase filter (Fig. 7, b) increases since the signal

passing through the phase filter is equivalent to forming at its output an incoherent sum N=10 replicas of a chaotic signal with delays determined by the response function.

Thus, it is shown in the paper that the elimination of the influence of the channel filter on the synchronization of the master and slave dynamic systems is achieved by introducing into their structure a phase filter equivalent to the channel filter. Due to this, the leading dynamic system forms a signal whose structure is not disturbed after passing through the channel filter.

The advantage of the considered method is the possibility of establishing chaotic synchronization for an arbitrary number of signal replicas entering the slave system, there are no restrictions on the order of the channel filter.

### 7 Conclusion

The advantage of the considered method is the possibility of establishing chaotic synchronization for an arbitrary number of signal replicas entering the slave system, there are no restrictions on the order of the channel filter.

The paper deals with chaotic phenomena in computer data networks. Based on the chaotic properties, mathematical models of the dynamic behavior of traffic can be constructed. Models can be used to provide guaranteed quality of service, analyze bottlenecks in the structure of a corporate network, and exchange data in cloud infrastructures.

At the same time, the indicators of chaos themselves, the structure of the attractor may have practical value. The change in the values of the Lyapunov senior indicator, the change in the topology of the attractor, is an indicator of the change in network activity. Computer attacks, failure of corporate data exchange systems or being the basis for a change in the administration policy - expansion of communication channels or replenishment of the list of prohibited network resources. The latter was observed with the growth of the popularity of social networks and video sharing resources.

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