

# Simulation of ACS for magnetic susceptibility measurements in ECAD based on time domain functions

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**Abstract.** The task of ACS analyzing in ECAD in the large-scale signal mode was considered. New models based on continuous time domain functions allow us to solve the problem of transient analysis in non-stationary nonlinear ACS. This approach does not require a transition to the frequency domain by calculating the transfer functions for each block, while the model retains its nonlinearity. It was found that such modeling in ACS in ESAV is quasi-causal indeed. According to the chosen approach, dynamic nonlinear macromodels for subsystems of ACS for determine magnetic susceptibility were developed: for the actuator/stepper motor, PWM-controller, Choke with movable core, etc.

**Keywords:** Magnetic Susceptibility Measurement System, ACS, CAE, ECAD, quasi-causal modeling, macromodels, time-domain simulation

## 1 Introduction

For measure of magnetic susceptibility, which has a high informative value when conducting research on the structural state of austenitic materials, various installations are used, for example, magnetometric scales. The block diagram of the automated system for magnetic susceptibility measuring is given in [1]. This information-measuring system with microcontroller control is a multi-domain Automated Control System (ACS).

Traditionally, for the analysis of such ACS is used approach, in which for the each block it was necessary to obtain transfer functions by converting the equations of the time domain to the equations of the frequency domain [2]. Such transition is possible when the several simplifications and initial conditions are met: the system must be stabilized at a certain operating point (the transition process is neglected), the static characteristics are linearized, which is possible for the Small Signal Mode. Thus, the ranges of adequacy of such models are limited to the calculation of frequency characteristics, for example, hodographs, which are often used to analyze the stability of systems, etc.

However, it is quite often necessary to calculate the time of transients, response delays, it is also necessary to take into account the nonlinearity of characteristics to predict the behavior of systems in real time and to write and debug programs for generating control signals by microcontroller.

## **2 Formal problem statement**

In ACS the microcontroller performs measurements, calculations and control, so, the main requirements for the designed model are data representation in the time domain and the ability to determine the nonlinearity of parameters in full scale variation of variables. This approach does not require calculation the transfer functions for each block. The model must retain its nonlinearity.

Consider an example of building a dynamic non-linear model of an ACS adapted to the capabilities of the Electronic Computer Aided Design (ECAD) mathematical processor Micro Cap v.11 (MC 11) [3]. Based on the new developed models, it is necessary to obtain the optimal variant of the automated system for measuring magnetic susceptibility.

## **3 Literature review**

Simulation of ACS as multi-domain dynamical systems, for example, mechatronic, is one of the most powerful tools of system research [4]. Simulation is an extremely important tool for every control engineer who is doing practical control system design in industry. For arbitrarily nonlinear plants, there is often no alternative to designing controllers by means of trial and error, using computer simulation [5]. Once the dynamic behavior is understood, changes to the system can be made or control systems can be designed to make the system perform as desired.

Simulation of ACS is used in the design of microcontroller systems at the stage of Control Law Design [6,7]. The result of the simulation is the evaluation of integral criteria: accuracy, stability, performance. There are two main approaches to quality assessment. The first uses information about the time parameters of the system (the transition function). The second uses information about some frequency properties of the system: bandwidth; relative height of the resonant peak, etc. Frequency quality criteria are used when known or it is possible to determine experimentally the frequency characteristics of the ACS. The type of transitional process is not considered at the same time. Frequency criteria can be used to assess the stability and performance of control systems. Some Computer-aided Engineering (CAE) programs allow automatically determining the quality parameters of a designed ACS, using its various characteristics.

There are different approaches to the study of ACS and various levels of model abstraction, and, accordingly, different software (CAD/CAE). There are many tools from CAE and CAD market for automated mechatronic systems simulation [8,9]. Some of them are well-known and popular (MATLAB \ Simulink [10,11], Maple \ MapleSim [12], VisSim [13]), others have not been widely distributed (SimApp [14],

20-sim [15] etc). Some programs are universal and can be used to simulate any technical systems (20-sim, Dymola [17]). Others specializing in a particular subject area (Modelica [18], Powersim [19], CVODES [20]). Simulink is integrated with MATLAB and data can be easily transferred between the programs [21].

The capabilities of many packages are largely overlapping and approaches to solving the same tasks are roughly the same.

The computer simulation model is built visually through block diagrams. Usually for the study of ACS, it is necessary to obtain transfer functions for its subsystems by converting the functions of time and transition to the frequency domain. Conversion (e.g., Laplace transform) allows to obtain simple model as algebraic equations with complex coefficients. However, the area of the adequacy of such models is small, because for an adequate transition to the frequency domain, it is necessary to linearize the models and make an assumption that the processes are stationary. Transients in nonlinear systems with this approach cannot be modeled. As a rule, in CAE programs, blocks are built on the basis of transfer functions, however, SimApp [14] allows for dynamic modeling in the time and frequency domains.

Analysis of ECAD programs [8] showed that they also provide the ability to simulate ACS using the flowchart method. Despite the similarity of models at the stage of modeling in ECAD and CAE, the simulation process is fundamentally different. In CAE, a causal algorithm is used, in ECAD – an a-causal algorithm [4]. The a-causal algorithm makes it possible to simulate systems, composed of blocks with complex frequency functions, blocks with analytical functions of time (programmable, including), elements of circuit diagrams, both analog and digital. Since most of the designed automated system for measuring magnetic susceptibility is an electronic system, the choice of the ECAD for its preliminary analysis is quite logical.

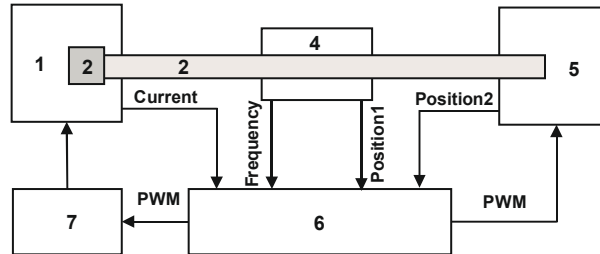
MC 11 contains many features that help you grow the model sophistication to the level needed for realistic results [3]. Non-linear elements and custom inputs bring reality to the models. The library has a standard PID controller model. The graphemes used in the classical theory of automatic control (adders, integrators, and so on) are also used in the construction of the ACS model. This makes it possible to investigate the behavior of multi-domain systems, in which blocks of continuous and discrete physical nature are interconnected by summers, dividers, multipliers etc. Blocks with Transfer Functions may also be part of the model.

## **4 Conceptual model of information measuring system**

The optimality of the designed system was checked by mathematical modeling, which was carried out in two stages: the construction of the model according to the selected criteria (modeling) and the implementation of the model experiment (simulation). The purpose of the simulation is to determine the quality of the system (stability, dynamic characteristics, sensitivity) and choose the directions of its parametric (structural) optimization.

Information-measuring system is an ACS (magnetometric scales with the registration of the zero position on the basis of determining the frequency changes of LC

generator). ACS is designed to compensate for the displacement of the rod caused by the magnet's impact on the magnetic material. The resulting scheme of ACS is shown in fig. 1.



**Fig. 1.** Structural diagram of the ACS model

The total model of ACS is built visually through block diagrams and consists of subsystems: 1 – Experimental subsystem / Power Magnet; 2 – tested sample; 3 – the movable rod; 4 – Measurement subsystem on Colpitts variable frequency oscillator; 5 – Compensating Subsystem (implemented on an inductive choke); 6 – Microprocessor Control System; 7 – SMPS (Switch Mode Power Supply) with DAC (Digital – to Analog Converter).

Microcontroller allowed increasing the accuracy and stability of automated measurement of magnetic susceptibility. ACS measuring and fixing the current of a power magnet (*CURRENT*), resulting displacement of the rod (*POSITION1* – displacement of the rod (ejection/retraction of the tested sample from the Power Magnet); *POSITION2* – result displacement (ejection/retraction from Compensating Subsystem)), frequency difference (*FREQUENCY*), duty circle of pulse width modulation signal (*PWM*-signal) for the compensating choke (*PWM*) and for SPMS, which generated signal for Power Magnet.

The rod, to which the ampule with the test materials attached, is pushed out or retracted by the magnetic field in the Experimental Subsystem, accordingly, its position changes relative to the coil in Measuring Subsystem. The subsystem for measuring the displacement of the rod is realized on the composition of the alternator generator and the electronic frequency meter, which transmits the data about frequency of oscillations changing relative to the base frequency  $f_0$  to the microprocessor system, which, based on these data, determines the Duty Cycle (D.C.) of PWM output voltage for the Compensation System to return the rod in start position.

Operations of the measuring and compensating subsystems are based on the variation of the chokes' inductance, for which a specific choke model was developed. During the development of the system, the simulation based on a-causal block modeling principle was applied.

## 5 Measurement Subsystem's modeling

The biasing sensor is actually implemented on the Colpitts oscillator with an inductance, which changes its value by changing the core's depth of penetration into the

coil [23,24]. Parameters of the Colpitts oscillator are calculated for the reference frequency  $f_0=300$  kHz based on the formula:

$$f_0 = \frac{1}{2\pi \sqrt{L_0 \cdot \frac{C_1 \cdot C_2}{C_1 + C_2}}} \quad (1)$$

where  $L_0$  – initial inductance (solenoid without core);  $C_1, C_2$  – capacitance of  $C_1, C_2$  capacitors from circuit.

All parameters have been optimized by the Powell algorithm in MC11 [22]. The Goal Function is the maximum of amplitude of output signal.

The inductance of the generator's coil depends on the current position of the core ( $\Delta l$ ) and the magnetic field parameters ( $\mu_{VAR}$ ):

$$L_{VAR} = L_0 + \mu_0 \cdot \mu_{VAR} \cdot k^2 \cdot \Delta l \cdot S, \quad (2)$$

$$k = \frac{N}{l},$$

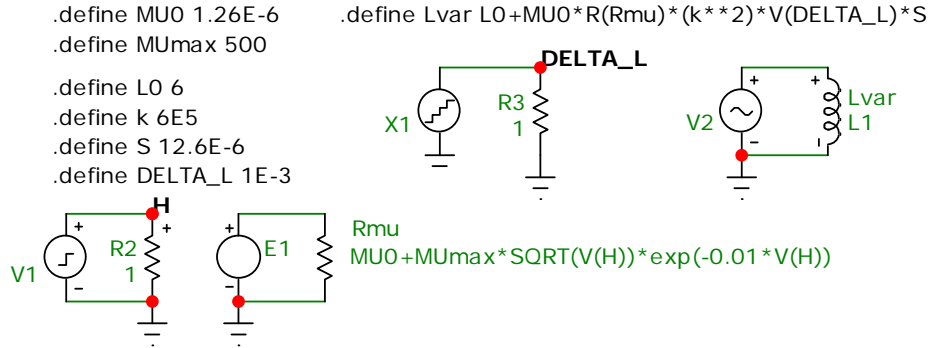
where  $\mu_0$  – absolute magnetic permeability;  $\mu_{VAR}$  – varying magnetic permeability;  $L_0$  – initial value, calculated by (1) on 300 kHz;  $N$  – number of turns;  $l, S$  – length and area of the solenoid cross, respectively;  $\Delta l$  – variation of the rod's penetration depth.

To take into account the change in the magnetic permeability  $\mu_{VAR}$  from the intensity of the magnetic field  $H$  [23,24], the following approximation of the Stoletov's curve is used:

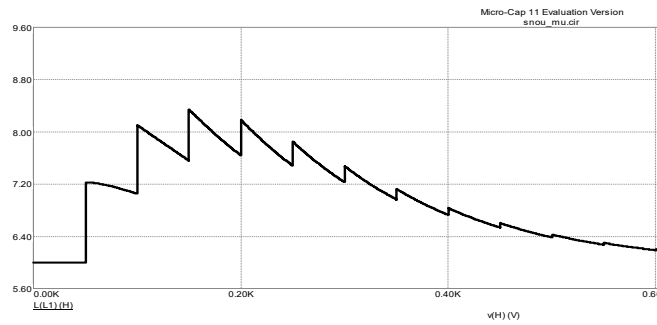
$$\mu_{VAR} = \mu_0 + \mu_{max} \cdot \sqrt{H} \cdot e^{-0.01 \cdot H}, \quad (3)$$

where  $\mu_{max}$  – maximum magnetic permeability on the Stoletov's curve for the selected material.

The scheme for modeling the Stoletov's curve and nonlinear inductance is shown in fig. 2. Basic parameters of approximation predefined. The field intensity changes (node with the name  $H$ ) is emulated by the generator  $V1$ . The subsystem of  $L_{VAR}$  (determining the inductance of the measuring oscillator by the formula (2)) is consist of: generator  $X1$ , which defines a step change in the depth of penetration of the rod into the coil (node with the name  $DELTA\_L$ ); resistors  $R2, R3$ ; inductance  $L_{VAR}$ , through which the pulsating current flows from the generator  $V2$  and resistor  $R_{mu}$ , which variate its resistivity by the formula (3). The results of  $L_{VAR}$  simulation in the full range of field intensity variation are shown in fig. 3.

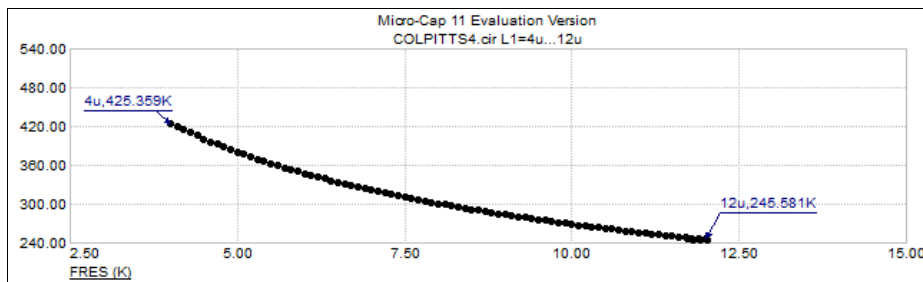


**Fig. 2.** Nonlinear inductance Modeling



**Fig. 3.** Simulation of nonlinear inductance

But we must note that with increasing frequency inductance sensor sensitivity nonlinearly decreases (fig.4).



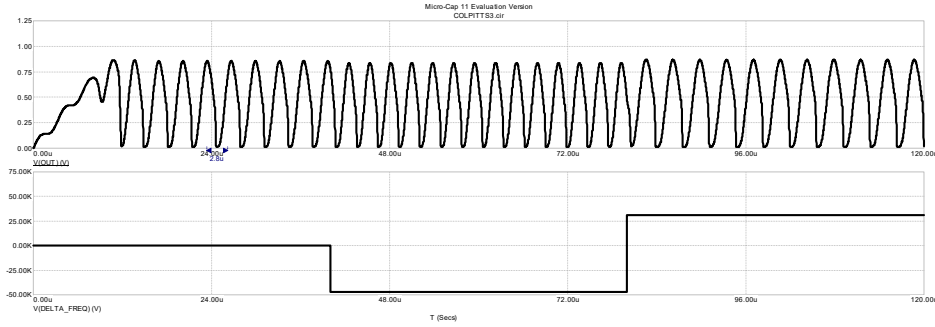
**Fig. 4.** Frequency vs. Inductance Curve

The dependence of the resonant frequency FRES on the value of the inductance is approximated by the results of a multivariate analysis in the following way:

$$FRES = 465 \cdot L(LCOIL)^{-0.2}, \quad (4)$$

where  $L(LCOIL)$  – variable choke inductance.

The characteristics obtained by simulation are shown in fig. 5.



**Fig. 5.** Transient Analysis of Measurement subsystem

The upper graph shows the output voltage oscillogram, the frequency of which varied by the variation of the inductance: from the base frequency at 300 kHz initially increased by 48kHz, then decreased by 28kHz, as illustrated by the lower graph  $DELTA\_FREQ$  vs T (Time). The initial frequency is set after the full charge of resonance circuit capacitance.

Inductance's changing leads to the frequency change in the oscillation circuit. Current value of measured frequency is compared with the reference frequency 300 kHz. The difference between the measured and the base frequency  $DELTA\_FREQ$  is transmitted to the voltage source and then sent to the subsystem, which generates certain pulse duration of the PWM-signal.

According to the simulation results, the program for the microcontroller has been adjusted.

## 6 Compensation Subsystem modeling

Model of the Compensating Subsystem (nonlinear solenoid with moving core) is based on the relay model [3] with several modifications, includes a flux circuit and derives a magnetizing force from the flux. Choke current depends on the initial inductance and the core's position (variable  $POSITION2$ ). Reactive component given in the functional current source by the empirical formula:

$$I_L = \frac{1}{LCOIL} \cdot (V(FLUX)) \cdot (1.1 - V(POSITION2)) + (1.1 - V(POSITION2))^2 \quad (5)$$

where  $LCOIL$  – initial choke inductance;

The FLUX is determined by the magnetizing  $FORCE$ , by the formula (given in functional source E3):

$$FORCE = K_{FORCE} \cdot \left( K_{FLUX} \cdot \frac{V(FLUX)^2}{AREA} \right), \quad (6)$$

where  $K_{FORCE}$ ,  $K_{FLUX}$  – coefficients are given by operators .define;  $V(FLUX)$  – flux emulation, received as the voltage of the “FLUX” node;  $AREA$  – the cross-sectional area of the rod given by the operator .define.

By Faraday's law of induction, any change in flux through a circuit induces an electromotive force (EMF) or voltage in the circuit, proportional to the rate of change of flux. Any alteration to a circuit which increases the flux (total magnetic field) through the circuit produced by a given current increases the inductance. The force produced by a magnetic field [25], so for there to be a force on a piece of iron (core) then a displacement of the rod must result in an alteration to the field energy:

$$FORCE = \frac{i}{2} \cdot \left( \frac{dL}{dx} \right), \quad (7)$$

where  $i$  is the coil current and  $x$  is displacement of the core.

This result is proved in textbooks such as Hammond, and also Smith [23,24]. Unfortunately, it might be tricky to calculate how the inductance changes unless the system you have is particularly simple to analyze.

There is another way to determine this FORCE. Force affects to the rod by pushing it, or pulling it at the speed  $VELOCITY$ , which is determine through the second law of Newton. The current position of the rod (variable  $POSITION2$ ) is determined by the integration of  $VELOCITY$ .

## 7 Total ACS modelling and simulation

The simplified model of ACS is shown on fig.6.

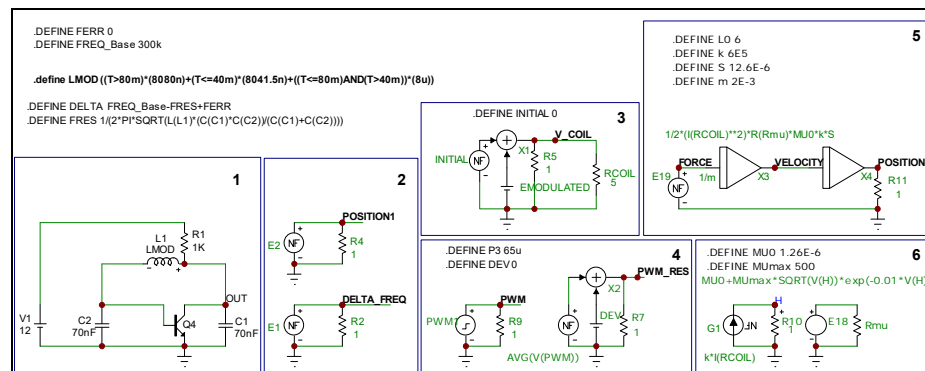


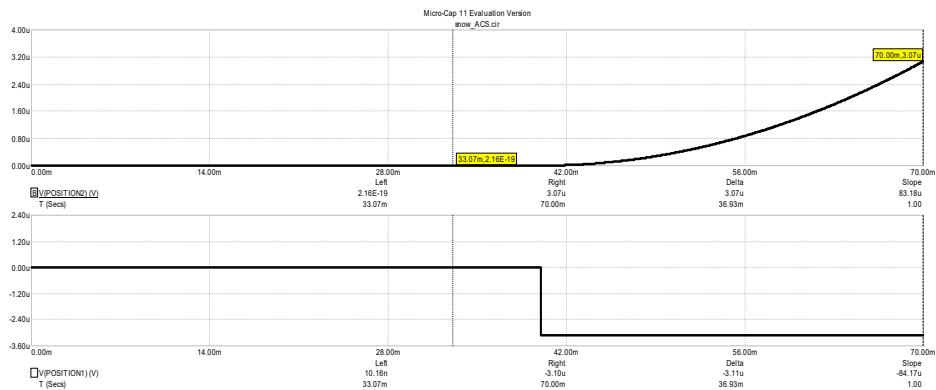
Fig. 6. Model of automated measurement system for Micro-Cap 11



Total model consist of macros: 1. Measurement subsystem on Colpitts oscillator; 2. Finding the frequency difference – *DELTA\_FREQ*; displacement of the rod – *POSITION1*; 3. Calculation of Current/voltage for compensating system – *V\_COIL*; 4. Emulation of PWM signal – *PWM\_RES*; 5. Finding the result displacement in compensating system – *POSITION2*; 6. Stoletov’s curve fitting.

PWM controller model [26] in which compensating signal produced as a constant continuous signal with variation of its amplitude (staircase signal). This assumption is acceptable provided if the output signal is smoothing effectively by filter (Choke’s inductance). Since the PWM signal regulates the average value of the voltage by changing the duty cycle, it is possible to directly use the variation of the mean/average value of PWM signal. The output PWM signal is control the current of the choke with inductance 10 mH. The compensation signal stops changing when the values of *POSITION1* and *POSITION2* become equal.

Verification during the simulation was carried out by comparing the founded values of the rod movement (*POSITION1*), which should be the same in the measuring and compensating subsystems. The displacement of the rod in the measuring system (*POSITION1* – lower graph) and its response in the compensation system (*POSITION2* – upper graph) is shown on fig.7.



**Fig. 7.** Displacement of the rod in the measuring system (*POSITION1*) and response in the compensation system (*POSITION2*)

The variation of the inductance in the range  $8040 \pm 40$  nH has changes the frequency ( $\pm 800$  Hz), the displacement of the rod ( $\pm 3.2$   $\mu$ m), and the voltage on the compensation circuit ( $\pm 3.2$  mV).

A-causal models work together with causal models very well due to the development of the simulation SPICE algorithm [3,27]. This makes it possible to analyze the system composed of models of different levels of abstraction. For example, the a-causal model of the Colpitts generator and the causal macromodel of a PWM-controller.

## 8 ACS implementation

This system is implemented on the ATmega8 microcontroller family. It has 23 inputs, six of which can receive an analog signal through the built-in ten-bit analog-to-digital converter (ADC). This controller is capable of generating a PWM signal by hardware, as it has three built-in PWM controllers. One PWM signal is used for stepped pumping of a Power Magnet, and the other is for regulating the current in a Compensating Subsystem.

The PWM controller is tuned by setting the control bits in the corresponding registers. The PWM controller operates in Fast PWM mode, in this mode the counter counts from zero to 255, after an overflow is reached, it is reset to zero and the counting starts again. The timer has a special OCR comparison register, the reset also occurs when the value in the counting register is compared with the value written to the comparison register. The D.C. factor of the signal is determined with an accuracy of 2-8 by writing a number from 0 to 255 in the OCR register. With a clock frequency of 8 MHz, PWM frequency will be equal to 31.25kHz. If frequency dividers from 8 to 1024 is used, the frequency will decrease proportionally, but the accuracy of the interval setting will not improve.

The current measurement of the Power Magnet is made using a Hall sensor. At the output, it produces a voltage proportional to the current flowing. The signal enters the controller via an ADC. In the Atmega8 controller, the ADC has the following characteristics: resolution of 10 bits; the time of conversion of one indication from 13 to 250  $\mu$ s depending on the measurement bit depth, as well as on the clock frequency of the controller generator; start support by interrupts, which allows you to automatically process data and save them to the correct register, for processing them in the main cycle.

The change in the state of the system and the processing of information occur after system initialization and after calculating the frequency of the Colpitts generator. The signal from the generator after converting it to a rectangular view is fed to the input of the controller. To determine the frequency, two interrupt vectors are used: the timer overflow interrupt and the external pulse interrupt. An interrupt from an external pulse increases the register value by one. The higher the timer's interrupt frequency, the faster the controller response, but the frequency measurement accuracy is lower. However, there is a limit value of the frequency at which the controller does not have time to make calculations. The minimum frequency of the timer 244.14 Hz, which is quite suitable for this ACS, has been determined.

The speed of the measurements is determined mainly by the transition process in the compensating coil, which is about ten times longer than the period of the microcontroller timer (46 ms vs 4.1 ms). Measurement delay is also associated with transitions to a stationary state in a power coil. The system is stable, oscillations of transient processes are not revealed either during modeling, or in «field» experiments.

## 9 Conclusion

A new way of modeling ACS is based on dynamic models for its subcircuits. This will allow you to remain inside the time domain for a full study of the dynamics even for non-stationary systems. Different levels of abstraction Macromodels for all subsystems of the Automated Magnetic Susceptibility Measurement System have been developed. Analysis of static and dynamic characteristics in the environment of the ECAD program Micro-Cap 11 has been conducted.

The resulting model is dynamic, nonlinear, quasi-causal [4,8]. Causality eliminates unwanted feedback and sets a certain sequence of calculations. In fact, this model is composition of proportional-integration blocks and analog schemes. Its time-behave and nonlinearities have been evaluated during the Transient Analysis. The delay of operation of the compensation system is found ( $4 \mu\text{s} \pm 10\%$ ). Directions of its parametric optimization and improvement on criteria of accuracy and versatility are found.

The relative error of the model is not more than 20%, so it can be recommended for the study of such systems.

After the model experiment, in which the parameters of the system has been optimized and refined on the basis of physical features of the real hardware, a prototype of compensation system was designed [1]. So, the adequacy of the models has been confirmed not only by comparison of the simulation results (*POSITIONS1* vs *POSITIONS2*), but with the results of experiments on real ACS [11].

New approach, chosen for measurement system modeling lies within the multidomain modeling paradigm, which allows us to investigate a wider spectrum of characteristics of nonlinear dynamic systems, in contrast to the standard approach adopted in the theory of automatic regulation, which focuses on the analysis of stationary processes in linearized systems. The developed macromodels are distinguished by minimalism and simplicity and therefore, may be recommended for the study of ACS on the different levels of abstraction in ECAD and in other similar mathematical processors.

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