Computer Modeling of Viscous Fluid Flow Based on the Regularized Lattice Boltzmann Model

Artem Ostapenko [0000-0003-3368-8203]

Pryazovskyi State Technical University, Universitetska str., 7, Mariupol, 87555, Ukraine

ostapt5@gmail.com

Abstract. The problem of the laminar fluid flow simulations with the lattice Boltzmann method at moderate and at large Reynolds numbers up to 20000 is considered. To avoid the fluctuations that appear in the numerical solutions with the LBM the regularization method that based on the median filtration is proposed. The regularization method was tested on the classical problem of the flow over the circular cylinder. The fluid flow modeling over the Nasa 0012 profile at different angles of attack is carried out at the Reynolds number 1000. The flow patterns, the drag and lift coefficients of the NASA profile studied in detail.

Keywords: viscous fluid, Boltzmann equation, regularization, circular cylinder, Nasa 0012 profile, Reynolds numbers

1 Introduction

Methods of the computational fluid dynamics (CFD) are classified according to the different approaches in continuous medium describing. These approaches are the Euler and the Lagrange ones [1]. Besides them there are hybrid methods that combine the advantages of the Euler and Lagrange approaches. The first hybrid CFD method for the non-stationary flow calculating was developed in 1955 in the USA by Harlow on the basis of the Euler equation. It was called Particles in Cell method (PiC) [2]. The computational domain divides by a stationary Euler grid into cells in which particles are located. Fluid describes by a discrete model, as a collection of particles of fixed mass, moving along the cells of a grid. The method of particles in cells is convenient to study the dynamics of multicomponent flows and flows with free boundaries. However, it has two significant disadvantages. The first one follows from the fluid discrete model: the fluctuations appearing which is the irregularity of the movement of particles. The second disadvantage is the high memory demands and the low speed of calculations.

In 1965 as a development of the PiC method was proposed the method of large particles, developed by Davidov U. M. and Bilatserkovskiy O. M [1]. The computational domain divides by an Eulerian grid which cells are considered as large particles. The motion of such large particles is modeled on the basis of the non-stationary

Euler model. Thus, the method occupies an intermediate position between the Harlow`s PiC method and the classical finite-difference methods.

In 1973 was appeared another approach in the fluid flow modeling based on the cellular automata - Lattice Gas Cellular Automata (LGA) [3]. In this method particles can move to the next cells and collide in them according to the laws of gas dynamics (Boltzmann equation). The disadvantage of this method is the stochastic noise that appears when densities, impulse and velocities are calculating. As a means of sealing such noise the integer number of particles replaces by the particles distribution function. The evolution of the distribution function is described by the Boltzmann kinetic equation. This approach was called the lattice Boltzmann method (LBM) [3-6].

The LB method becomes widespread. A lot of scientists in Europe and America are engaged in its development [7-10]. The popularity of the method is due to its advantages:

• Simplicity in description and programming, since all stages of the method are the linear equations that solves by the explicit schemes [3, 6];

• the simplicity in the boundary conditions settings, due to the fact that the method is based on the kinetic approach in the continuous medium describing [6, 11];

• the ability to simulate flows in different difficult domains;

• ease in external forces adding [12];

• the possibility of using parallel computing technologies [13, 14];

• solving a wide class of tasks.

However, despite the growing popularity, there are still the following problems:

1.Significant calculation time, which increases with Reynolds number increasing [15];

2.Conditional stability [3, 15, 16].

These problems make it difficult to obtain numerical solutions for the flows at moderate Reynolds numbers Re~100 and prevent simulations at large numbers Re>100. For a partial solution of these problems at small Reynolds numbers Re<100 schemes with several relaxation parameters [17, 18] or implicit schemes [19] are used. However, the above problems are still not solved to the end and determine the relevance of the article as well as its scientific and practical significance.

2 Regularized lattice Boltzmann model

According to the lattice Boltzmann method, the computational domain divides into a grid which cells are treated as large particles [3-6]. These large particles can move to one of the neighbors cells and collide in them according to the Boltzmann's kinetic equation. Such large particles are describing statistically using the particle distribution function [20].

The mathematical formulation of the problem consists of the Boltzmann equation (1), in which the particles collision integral is replaced by the Bhatnagar-Gross-Kruck approximation (2) [21]:

$$\partial_t f + \vec{\upsilon}\partial_x f + \frac{\vec{F}}{m}\partial_\upsilon f = \int \left|\vec{\upsilon}1 - \vec{\upsilon}2\right| \left(f_1^{'} f_2^{'} - f_1 f_2 \right) d\sigma d\vec{\upsilon}2 \quad (1)$$

$$I_{coll} = \int \left|\vec{\upsilon}_1 - \vec{\upsilon}_2\right| \left(f_1^{'} f_2^{'} - f_1 f_2 \right) d\sigma d\vec{\upsilon}_2 \approx \frac{f^{eq} - f}{\tau} \quad (2)$$

Numerical model is a system of the lattice equations (3), obtained by sampling the mathematical model (1) - (2) with the absence of the external forces $\vec{F} \equiv 0$.

$$\underbrace{f_k\left(\vec{r} + \vec{V}_k \Delta t, t + \Delta t\right) = f_k\left(\vec{r}, t\right)}_{stream} - \underbrace{\frac{1}{\tau}\left[f_k\left(\vec{r}, t\right) - f_k^{eq}\left(\vec{r}, t\right)\right]}_{collision} \quad (3)$$

where $f_k\left(\vec{r}, t\right)$ – discrete particle distribution function;
$\vec{V}_k$ – discrete series of particles possible velocities;
Δt – time step;
τ – dimensionless relaxation parameter;
$f_k^{eq}\left(\vec{r}, t\right)$ – discrete approximation of the Maxwell-Boltzmann local equilibrium distribution function.

The justification of using the lattice Boltzmann method to model viscous flows is given in [22]. It was shown that using the Chapman-Enskog expansion on the equations (3) the Navier-Stokes equation and the continuity equation for a uncompression isothermal fluid (4) can be obtained as well as the formulas that associate macroscopic and lattice parameters (5) - (8).

$$\begin{aligned} &\frac{\partial u}{\partial t} + \left(\vec{u} \cdot \nabla\right)\vec{u} = -\nabla p + v\Delta\vec{u} + o(\Delta t); \\ &\nabla \cdot \vec{u} = 0 + o(\Delta t) + o\left(M_p\right). \end{aligned} \quad (4)$$

Kinematic viscosity of the liquid

$$v = c_s^2 \Delta t(\tau - 0.5) \quad (5)$$

Lattice speed of the sound

$$c_s = \frac{c}{\sqrt{3}} \quad (6)$$

Lattice speed

$$c = \frac{d}{\Delta t} \quad (7)$$

where d – cell`s size

Lattice Mach number

$$M_p = \frac{U_{\max}}{c_s} \tag{8}$$

where $U_{\max}$ – maximum velocity value in the domain

The stability of the LBM numerical scheme was investigated analytically in [3, 5, 16, 22]. The instability of numerical solutions may be caused by one of the following factors:

– lattice Mach number increasing. The Boltzmann kinetic equation approximates the Navier-Stokes equation only at small Mach numbers $M_p < 0.3$ [22].

– relaxation parameter decreasing. The "safe" value (according to [5]) is $\tau = 1$. The relaxation parameter decrease τ causes instability: the particles accumulate in some cells, which results in pulsations in the velocity field (fig. 1);

– speed increasing. The numerical model assumes modeling flows only at low speeds $U_{\max} < c_s$.

– Reynolds number increasing. As the Reynolds number increases, the flow becomes turbulent (according to [23-24]).

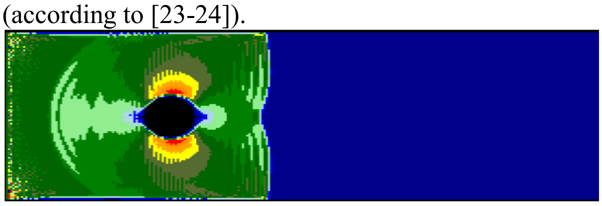

Fig. 1. Fluctuations in the velocity field

Macroscopic fluid parameters: density, velocity and pressure are defined as moments of the distribution function according to the formulas:

$$\rho\left(\vec{r},t\right) = \sum_{k=0}^{8} f_k\left(\vec{r},t\right), \vec{u}\left(\vec{r},t\right) = \frac{1}{\rho\left(\vec{r},t\right)} \sum_{k=0}^{8} \overrightarrow{V_k} f_k\left(\vec{r},t\right), p\left(\vec{r},t\right) = c_s^2 \rho\left(\vec{r},t\right) \tag{9}$$

The possible particles moving directions $\left\{\vec{e}_k\right\}_{k=1}^{n}$ determines the lattice of the model. The lattice is defined in such a way as to ensure the isotropy of the corresponding tensors of the 4th and 6th orders and to provide the momentum moment conservation law. Numerous scheme for solving two-dimensional tasks on the basis of lattice model with nine possible directions of particle movement (D2Q9) was used.

To simulate viscous fluid flow at large Reynolds numbers with the LBM a smoothing scheme based on the A. L. Chudov [25] idea was constructed. The smoothing scheme is based on the function value correction at a space point in accordance with the neighboring values. The basis of such correction is the median filtration (10) with the smoothing condition (11):

$$u_x(i,j) = med\big(u_x(i,j-2), u_x(i,j-1), u_x(i,j), u_x(i,j+1), u_x(i,j+2)\big) \tag{10}$$

$$u_x(i,j) > u_x(i,j \pm 1) \vee u_x(i,j) < u_x(i,j \pm 1) \tag{11}$$

The advantage of such filter is the nonlinearity with the following properties [26]:
– after smoothing, the sharp boundaries of the solution areas are preserving;
– non-correlated or weakly corrected obstacles are suppressing;
– fluctuations are reducing.
Smoothing using (10-11) equations schematically depicted in fig. 2.

Fig. 2. Velocity smoothing

3 Verification

To verify the proposed method a series of numerical experiments of numerical solutions regularizing was conducted. Considered the flow pattern at Re=500. Fig. 3a illustrates velocity diagram got with the LBM algorithm at the moment t=2.5. The fig. 3a shows pulsations, which arise mainly on the boundary of the region. Further solution is unstable. Figure 3b illustrates velocity diagram of smoothed solution.

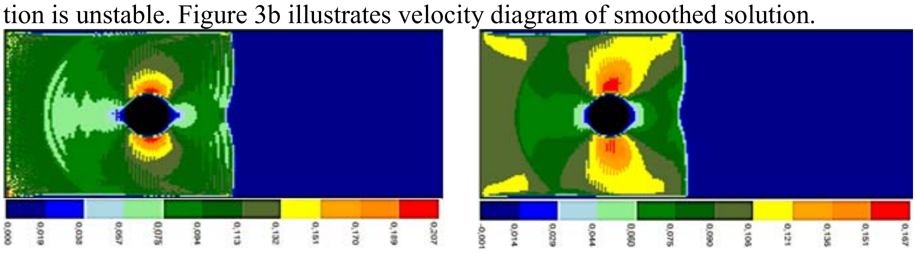

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Fig. 3. Velocity diagram at Re = 500 at time t = 2.5 a) without smoothing b) smoothed solution

The regularization method has allowed to smooth the fluctuations and obtain a stable solution over large time intervals, such as the velocity diagram (fig. 4 a) and the streamlines (fig. 4 b).

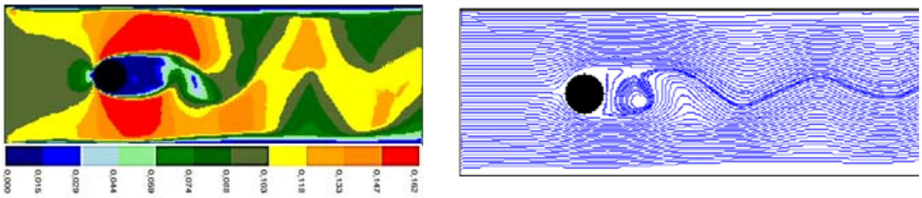

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Fig. 4. Simulation with the LBM with smoothing at Re = 500 at time t = 50 a) velocity diagram b) streamlines

The resulting solution was compared with the solution of a similar problem that was obtained in the Comsol Multiphysics package with the finite element method (fig. 5). Figures 4 and 5 shows the similarity of the obtained patterns and the streamlines.

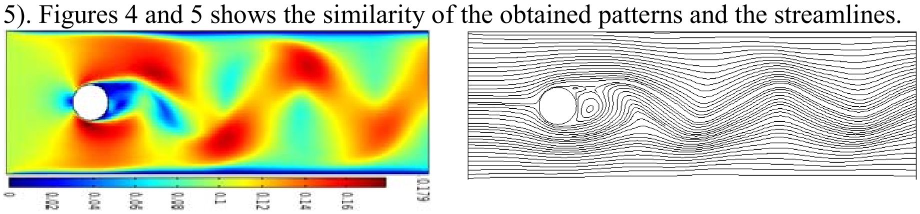

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Fig. 5. Simulation in the Comsol Multiphysics package at Re = 500 at time t = 50 a) velocity diagram b) streamlines

A more detailed comparison was made in vertical sections near the streamlined body x=0.8, 1.0 (fig. 6). The graphs of the velocity distribution, obtained by the LBM method with smoothing and the finite elements method (FEM), were constructed.

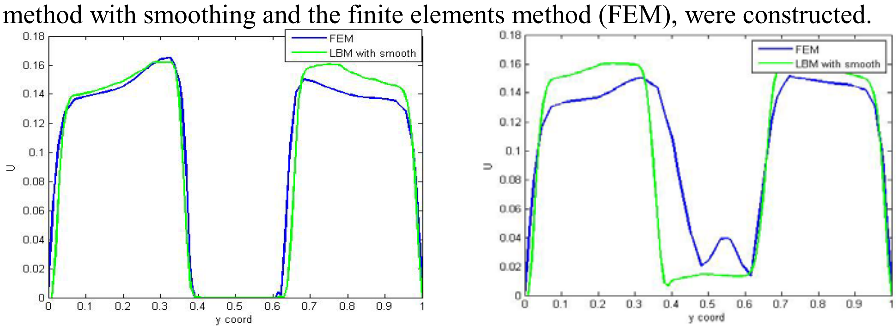

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Fig. 6. Velocity distribution at the section a) x=0.8 b) x=1.0

Consequently, the solutions of the test problem obtained by the lattice Boltzmann method with smoothing showed a good correspondence with the results of other numerical experiments at the Reynolds number Re = 500.

Increase the Reynolds number up to Re = 10,000. The structuring of such flow disappears and the turbulent is appearing [23, 24]. A photo of such flow is shown in fig. 7 [27].

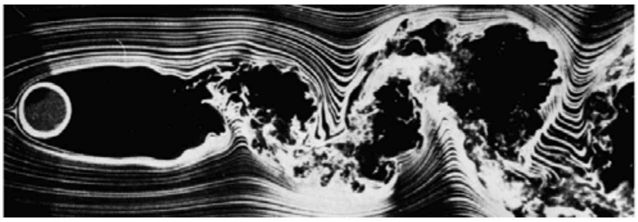

Fig. 7. Photo of the fluid flow at Re = 10 000

The conducted numerical experiments give the following results. Fig. 8 illustrates the simulation results got with the LBM with smoothing.

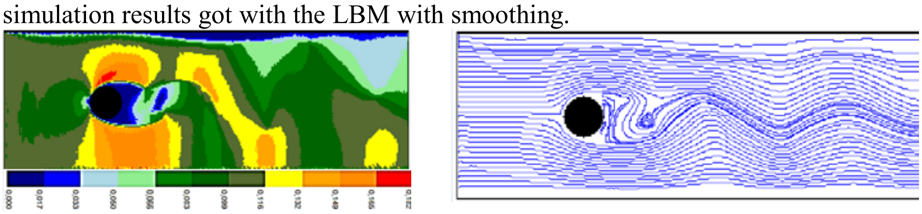

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Fig. 8. Simulation with the LBM with smoothing at Re = 10000 at time t = 100 a) velocity diagram b) streamlines

Fig. 9 illustrates the results of flow simulation with the FEM method.

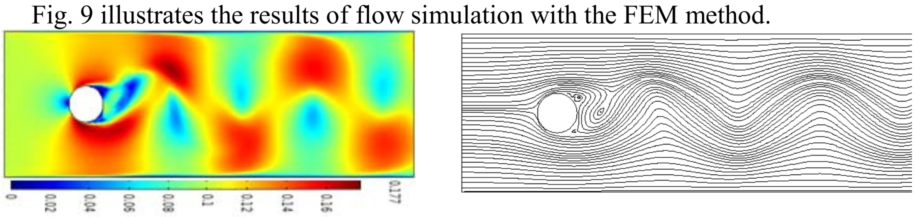

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Fig. 9. Simulation in the Comsol Multiphysics package at Re = 10000 at time t = 96 a) velocity diagram b) streamlines

As the Reynolds number increases, the periodicity of vortex formation increases: vortices descend from the edges of the cylinder at about t = 2. Such a rapid formation of vortices led to a divergence of numerical solutions by the FEM and LBM methods at about t = 2 (figures 8 and 9).

The results of the fluid flow modeling at Reynolds number Re = 20000, are depicted in fig. 10 with LBM method and fig.11 with the FEM method.

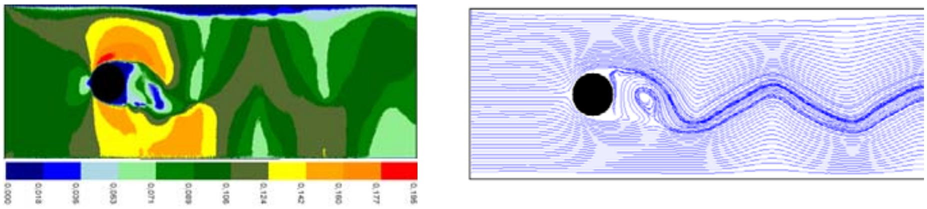

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Fig. 10. Simulation with the LBM with smoothing at Re = 20000 at time t = 50 a) velocity diagram b) streamlines

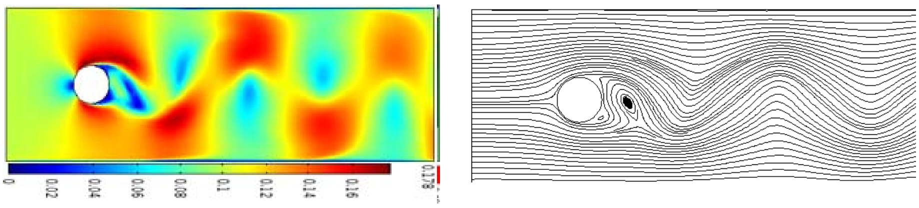

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Fig. 11. Simulation in the Comsol Multiphysics package at Re = 20000 at time t = 50 a) velocity diagram b) streamlines

The results of numerical experiments for the viscous fluid flow modeling with the lattice Boltzmann method and the finite element method in the Comsol package at Reynolds numbers 500<Re<20000 have been studied in detail. The comparison shows the good correspondence of the results, namely the process of the boundary layer separation and the formation of vortices over the cylinder. The structure of the streamlines also coincides.

4 Experiments and results

The fluid flow modeling over the Nasa 0012 profile at different attack angles was carried out at the Reynolds number Re=1000. Figure 12 illustrates the flow scheme: the velocity vector V_{in}, angle of attack α, drag and lift coefficients $\vec{F}_{drag}, \vec{F}_{lift}$.

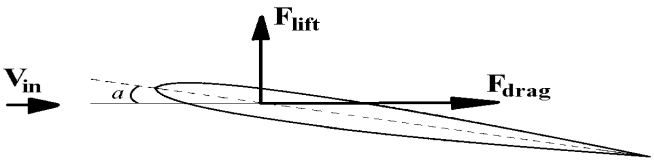

Fig. 12. Scheme of the flow over the NASA 0012 profile

The influence of the angle of attack on the flow pattern and on the drag and lift coefficients was studied. The flow pattern with the zero of the angle of attack is depicted in fig. 13.

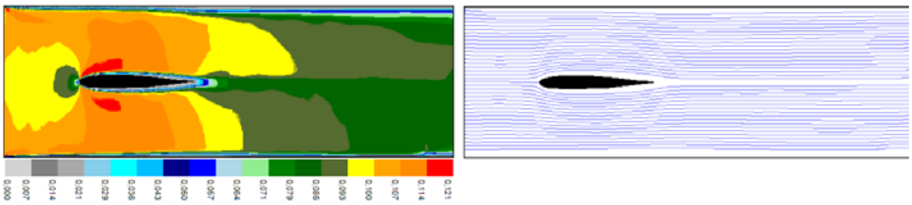

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Fig. 13. Flow over the Nasa 0012 profile with the absent angle *a*) velocity diagram b) streamlines

As can be seen from the fig. 12, the flow around the profile is symmetric. Was calculated the average values of the drag *Cd* and lift *Cl* coefficients at different times up to *t*=100. The appropriate graphs are depicted in fig. 14.

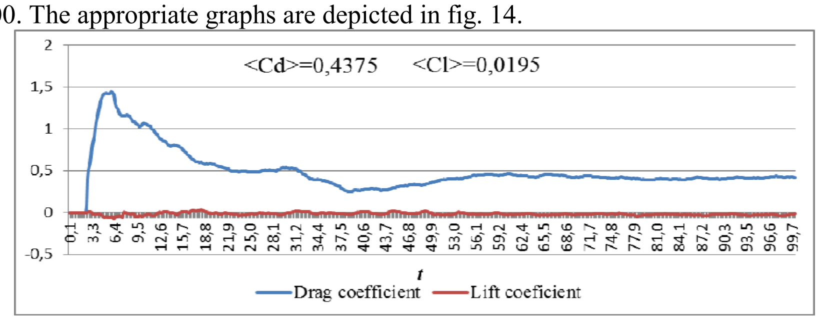

Fig. 14. Graphs of the drag and lift coefficients average values at different times

The change of the flow pattern by the angle of attack increasing α=10°, 20°, 30° was considered. The average values of the hydrodynamic coefficients shown in fig. 15.

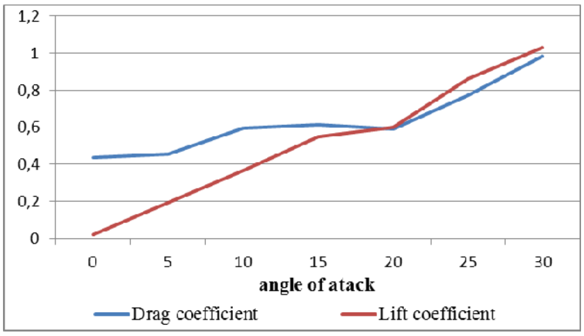

Fig. 15. Graphs of the dependence of the hydrodynamic coefficients on the angle of attack

The angle of attack increasing cause the drag and lift coefficients increasing too as can be seen from the fig. 15. The numerical simulations with the angles of the Nasa 0012 profile more than 30° are unstable with the regularized lattice Boltzmann model. Fig. 16 illustrates the velocity diagrams and streamlines of the flow over the profile with the different angles of attack α=10°, 20°, 30°.

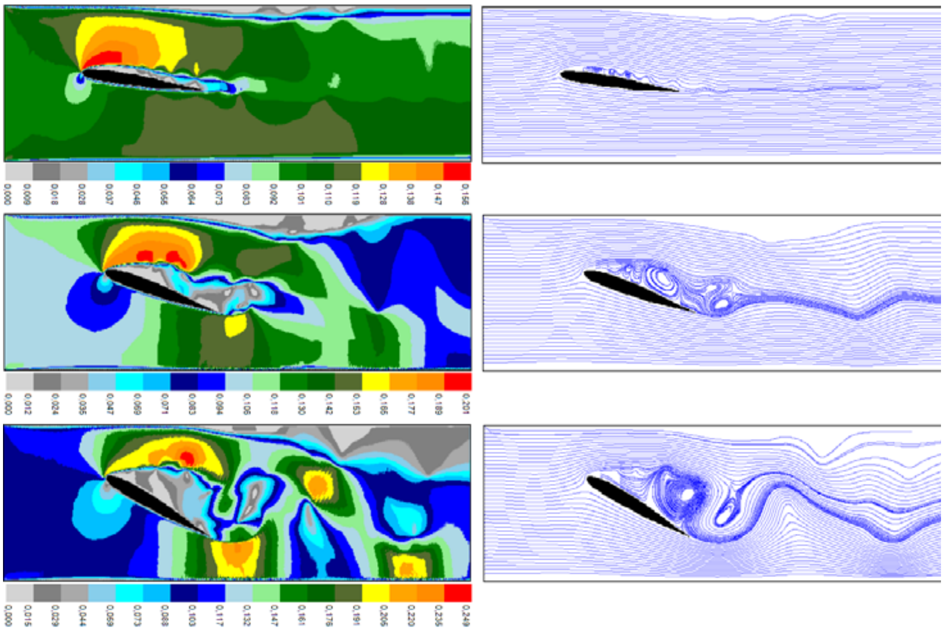

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Fig. 15. Flow pattern over the Nasa 0012 profile with the different angels of attack α=10°, 20°, 30° *a*) velocity diagram *b*) streamlines

The conducted numerical experiments indicate the growth of the hydrodynamic coefficients and the formation of vortices over the profile with the angle of attack increasing.

5 Conclusion

The problem of the laminar fluid flow simulations with the lattice Boltzmann method at moderate and at large Reynolds numbers up to 20000 was considered. Noticed, that LBM is a new approach in CFD that used the Boltzmann equation to simulate fluid behavior. Although this method becomes more popular there are some problems that make fluid flow modeling at moderate and large Reynolds numbers impossible. They are the significant calculation time, which increases with Reynolds number increasing and conditional stability. The last one may causes fluctuations which are the irregularity of the movement of particles. To avoid the fluctuations the regularization method that based on the median filtration was proposed.

The regularization method was tested on the classical problem of the flow over the circular cylinder. The comparison of the numerical experiments with the lattice Boltzmann method and the finite element method in Comsol Multiphysics package showed the good correspondence of the results, namely the process of the boundary layer separation and the formation of vortices over the cylinder. The structure of the streamlines also coincides. So, the regularization method has allowed to smooth the fluctuations and to obtain a stable solution. This allows to use the LBM for studying the laminar patterns at large Reynolds numbers.

The fluid flow modeling over the Nasa 0012 profile at different angles of attack was carried out at the Reynolds number Re=1000. Declared the vortex formation at the angles more than 7°. Size of the vortex is increasing with the angle increasing as well as the hydrodynamics coefficients. The numerical simulations with the angles of the Nasa 0012 profile more than 30° are unstable even with the regularized lattice Boltzmann model so demands more detail study.

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