Building a Spatial Model of Destructive Processes Based on Fuzzy Rough Soft Topology

Maryna Zharikova[0000-0001-6144-480X], Volodymyr Sherstjuk[0000-0002-9096-2582]

Kherson National Technical University, 24 Berislav Road, Kherson, 73008, Ukraine
marina.jarikova@gmail.com, vgsherstyuk@gmail.com

Abstract. This work presents a spatial model for the real-time GIS-based decision support systems based on dynamic fuzzy rough soft topology, which represents a spatial structure that contains a multitude of interacting processes, which evolve in space and time. The dynamics of destructive processes are modeled using the spread model. The area of interest is represented as an approximation by a grid of cubic cells. This allows taking into account the peculiarities of the initial information obtained using remote sensing techniques and having a significant uncertainty. As a result, boundaries of contours of spreading destructive processes are blurred using fuzzy rough soft topology. The proposed model reduces the computational complexity and provides the acceptable performance.

Keywords: destructive processes, spatial model, fuzzy rough soft set, fuzzy-rough soft topology, grid of cells, blurred boundaries

1 Introduction

Nowadays, the society faces the problem of increasing loss of lives and damages to properties caused by natural disasters that rise steadily due to population growth, urbanization, deforestation, environmental degradation, and global climate change. An effective way to overcome this problem is a proper risk management strategy that calls for disaster analysis consisting of spatiotemporal modeling of disaster in the area of interest (AOI).

The authors are concerned with the areas containing natural and artificial objects among which are valuable objects requiring disaster protection. AOI with a multitude of interacting disasters, which evolve in space and time giving rise to danger and risk to some valuable objects is considered a dynamic system. The paper deals with real-time disaster spatial modeling.

However, the most of the destructive processes are poorly observed and their spreading within the AOI is weakly modeled, so real-time disaster modeling is a complex and non-trivial task, which becomes more complicated due to uncertainty of information, a wide geographically distribution of events and, as usual, a lack of time [1]. The efficiency of decision-making strongly depends on the availability of online disaster monitoring tools aimed at the real-time computation of the most important parameters related to the spreading of the destructive processes.

Today, a suite of the most advanced methods and techniques, such as remote sensing, GIS, geospatial analysis, unmanned aerial vehicles (UAV), can be synergistically used for GIS-based disaster modeling. Remote sensing techniques play a crucial role, as they provide powerful tools for the rapid acquisition of relevant data for disaster monitoring [2] in a form of streams of great volumes that come from sensors on a continuous basis at a high rate and should be analyzed in a real-time [3]. These data can be used for representing spatial distribution and properties of disaster, and for supporting forecasting disaster models.

This paper presents a spatiotemporal disaster model in the context of the most common types of disasters such as wildfires. The authors consider fire monitoring as a continuous or discrete process of observing a status and changes in an active fire directly or indirectly and determining some fire parameters such as intensity, size, the rate of spread, and others relevant to respond operation and important for the decision maker. UAVs can effectively perform long-time missions to obtain remote sensing data [4]. However, due to the instrumental inaccuracy and distortions caused by vibrations, remote sensing information obtained from UAVs is incomplete, imprecise, vague, and often blurred [5]. The dynamics of wildfire spreading depends on the accuracy of determining the boundaries of its dynamic contour. However, the uncertainty of observations significantly reduces the accuracy of determining the boundaries of such contours [6]. Obtained remote sensing data should be correctly transferred to wildfire spread model, geolocated and mapped to the AOI.

Numerous methods have been developed to model disasters based on a huge array of remote sensing and other data gathering techniques. Using the well-established traditional approaches for spatial modeling such as statistical methods do not provide the required performance and acceptable efficiency of GIS-based real-time wildfire modeling [7]. A key aspect to achieve the desired performance is to build an approximate spatial model of wildfire spreading, taking into account partial observability and uncertainty of observations. Thus, we need to soften the requirements for the accuracy of remote sensing data representation, which will give us the opportunity to improve modeling performance. In this case, the boundaries of the dynamic contours of the spreading processes can be vague and blurred.

There are several well-known approaches to deal with the uncertainty and vagueness in the spatial models, such as fuzzy set theory [8], rough set theory [9] and soft set theory [10]. Each of these approaches has its inherent difficulties as pointed out in [10]. It should be noted that due to the absence of some important information a priori, such as membership functions for fuzzy sets, equivalence relations for rough sets, or parameterizations for soft sets, these approaches cannot ensure the adequacy of the spatial model of the destructive process independently. Therefore, many researchers combine some of these approaches. Some authors proposed to use for spatial modeling the combinations of rough and fuzzy sets [1], rough and soft sets [11], fuzzy and rough sets [12]. In [13], the authors proposed the concepts of rough fuzzy soft sets and fuzzy rough soft sets, which have a number of advantages to build a blurred spatial model. Based on this, we can use soft topological spaces to build a spatial model of the destructive process, as well as the fuzzy rough method for its blurring.

The aim of this work is to develop the approximate spatiotemporal disaster model within AOI [14] in the context of forest fires. To overcome the computational complexity problem, we build a topological spatial model and soften the effects of discretization using the fuzzy rough sets. The developed model allows analyzing big data streams coming from remote sensors and representing them in a user-friendly style.

2 Modeling dynamics of destructive processes

Let us consider the AOI as an open connected subspace X of three-dimensional Euclidean space endowed with the topological properties [15, 16]. Firstly, the considered AOI is divided into a finite set of disjoint spatial objects represented as geometric shapes, which outline boundaries of certain areas. Such objects are named as geotaxons and represent geo-referenced natural parts of the terrain with the same characteristics. GIS can contain an unlimited number of geotaxons' layers. To build a topological space on X we use an equivalence relation $\Re_X \subseteq X \times X \times X$ (reflexive, symmetric, and transitive) [1]. Then the pair $apr_X = (X, \Re_X)$ is called the approximation space. The family of all composite sets is denoted by $Def(apr_X)$ and uniquely determines the topological space $T = (X, Def(apr_X))$.

Suppose that each point $x \in X$ has a non-empty finite set of attributes A, V_a is a domain of $a \in A$ and f is a function such that $f : X \times A \to V$. Let's impose a metrical grid of coordinate lines with $\delta = \Delta\alpha_1 = \Delta\alpha_2 = \Delta\alpha_3$ within X, which form a set C of cubic cells with the size being $\delta \times \delta \times \delta$. Thus, space X is discretized by a grid C of isometric cubic cells $c \in C$. Assume that a cell $c \in C$ is a spatial homogeneous object of minimal size. The grid C approximates AOI and constitutes a certain GIS layer.

Each cell $c \in C$ is associated with a set of attribute values, which is called the cell state, via the value function $f(\mathrm{c}, A)$. The proposed discretization assigns equal values of the attributes to each point belonging to a certain cell c, therefore each cell $c \in C$ represents a homogeneous area of the AOI in terms of attribute values A, so it can be reduced to a point of X. It's suggested to model disaster dynamics by means of a change of states of the cells covered by the disaster.

Suppose the set of attributes A can be divided into subsets [16]: not changing over time (static) attributes A_S, time-varying (dynamic) attributes A_D, slowly changing (environmental) attributes A_E, $A = A_S \cup A_D \cup A_E$. Suppose $W = \{w_0, \ldots w_i, \ldots w_F\}$ is an ordered set of the cell states (phases), where w_0 is the initial phase, w_F is the final phase, and each w_i is the transitional phase. We consider each significant change of the cell attribute's value, which forces the cell to change its state, as an event. Assume, during the destructive process, the cell moves through a sequence of qualitatively different categories of states, which should be evaluated during continuous remote sensing. It is clear that the model of the destructive process can be represented as a model of dynamic change of states of a subset of cells covered by the process within the spatial model. Thus, the spatiotemporal structure of AOI can be represented as a topological space, which includes subspaces of cells of the same phase and makes it possible to assess the position and boundaries of the dynamic contour of the process. Since the belonging of each cell to a certain phase is determined approximately due to the uncertainty of remote sensing, the topological space describing the structure of the spatial model as well as the boundaries of the contour of the spreading destructive process is blurred.

3 Soft sets and soft topologies

Dynamics of destructive processes can be described by a blurred structure of AOI containing the sets of cells, which belong to a certain phase at any given time t. It is proposed to represent such a structure as a soft set, which blurring in different ways makes it possible to obtain blurred structures. The most common way of blurring the boundaries between subsets of cells belonging to different phases is to represent them in the form of a fuzzy set, but in practice, this method is impossible to implement. The soft set allows us to represent the AOI as a blurred topological space, which can be created by blurring the boundaries between the sets of cells corresponding to different phases.

Suppose destructive processes at each time gives rise to a certain state of AOI represented by a blurred structure, which consists of plausible sets of cells that belong to a certain phase. Such a structure is proposed to be presented as a soft set.

Consider the concept of soft sets in general. Let $W = W_S \cup W_D$ is a union of the sets of environmental conditions and the set of phases of the cells, $W_i \subseteq W$ is any of its subsets, and 2^W is the set of all subsets of W, $W_i \subseteq 2^W$ [16].

A couple $\Upsilon_{W_i} = (\Upsilon, W_i)$ is called a soft set on the set of cells C if Υ is a mapping $\Upsilon : W_i \to 2^C$, where 2^C is the set of all subsets of C [16]. In other words, the soft set is a parameterized family of subsets of cells C. Each set $\Upsilon(w)$, $w \in W_i$ of this family can be considered as a set of w-elements of soft sets (Υ, W_i).

A soft set can be defined by a plurality of pairs $\Upsilon_{W_i} = \left\{ \left(w, \Upsilon_{W_i}(w) \right) : w \in 2^W, \Upsilon_{W_i}(w) \in 2^C \right\}$. A set $\Upsilon_{W_i}(w)$ is called a w-element of the soft set and is determined for each $w \in W_i$. The soft set is associated with a set of equivalence classes generated by the Pawlak's indiscernibility relation. As mentioned above, we can identify an A_i -indiscernibility relation in the set of cells.

If a certain set of parameters A_i determines the class $w \in W_i \subseteq W$, then A_i-indiscernibility relation can be substituted by w- indiscernibility relation. Thus, we assume that the soft set Υ_{W_i} splits the cell set C into equivalence classes generated by w-indiscernibility relation $\Re_C^w$, where $w \in W_i$. In other words, a parameterized family of subsets of cells C, which forms the soft set Υ_{W_i}, is a factor set $C / \Re_C^w$ consisting of all equivalence classes of the set C generated by the relation $\Re_C^w$.

Thus, the soft set Υ_{W_i} can be used to generate equivalence classes in the set C instead of the equivalence relation $\Re_C^w$, $w \in W_i$. $\Re_C^w$ can be generalized and represented as a similarity or a tolerance relation such that the soft set Υ_{W_i} splits the plurality of cells C onto the vague sets (fuzzy or rough). Of particular interest for building the structure of AOI is the A_S - indiscernibility relation, which can be replaced by the w_S -indiscernibility, as well as A_D -indiscernibility relations, which can be replaced by the w_D - indiscernibility relation. The approximation spaces generated by these relations can be represented as soft sets Υ_{W_S} and Υ_{W_D} respectively.

Consider the w_S -indiscernibility relation in a plurality of cells and the corresponding soft set Υ_{W_S}. The set of cells defining the AOI can be represented as the soft set, which divides the set of cell onto classes with respect to terrain conditions as $\Upsilon_{W_S} = \left\{\left(w, \Upsilon_{W_S}(w)\right) : w \in 2^W, \Upsilon_{W_S}(w) \in 2^{W_C}\right\}$, where $\Upsilon_{W_S}(w)$ is a set of cells, which corresponds to a class $w \in W_S$, maps it into a set of cells, and describes the static component of AOI as $\Xi = \Upsilon_{W_S} = \left\{\left(w, \Upsilon_{W_S}(w)\right) : w \in 2^W, \Upsilon_{W_S}(w) \in 2^C\right\}$ [16]. While A_S - indiscernibility relation generates static equivalence classes in a set of points $x \in X$, which constitute a topological space $T_X^{A_S}$, and geotaxons are their connection components, W_S -indiscernibility relation defined on the set of cells also generates static equivalence classes. Their connection components are subsets of cells, which approximate geotaxons, and they constitute a topological space $T_C^{w_S} = \left(C, Def\left(apr_C^{w_S}\right)\right)$.

The decomposition of the subspace C of approximation subspace X using geotaxons, approximated by cells, is a topological space that represents the static component of the spatial model Ξ as $T_{G_C}^{w_S} = \left(C, Def\left(G_C^{w_S}\right)\right)$. Each i - class of equivalence $\left(apr_C^{w_S}\right)_i$ of approximation space $apr_C^{w_S}$ can be represented as the value of the soft set $\Upsilon_{W_S}(w_i)$, $w_i \in W_S$, i.e. $\left(apr_C^{w_S}\right)_i = \Upsilon_{W_S}(w_i)$. Let $Def\left(\Upsilon_{W_S}\right)$ is a family of all composite sets of the soft set Υ_{W_S}. Obviously, $Def\left(\Upsilon_{W_S}\right) = Def(apr_C^{w_S})$. Thus, the topological space can be represented as the soft set $T_C^{w_S} = \left(C, Def\left(apr_C^{w_S}\right)\right) = \left(C, Def\left(\Upsilon_{W_S}\right)\right)$.

The special role is related to W_D -indiscernibility relation, which splits the set of cells into phases and generates dynamic equivalence classes. A dynamic topological space $T_C^{w_D}$ is built upon dynamic equivalence classes and determines the dynamic behavior of the destructive processes. As well, each i -class of equivalence $\left(apr_C^{w_D}\right)_i$ can be represented as the value of the soft set $\Upsilon_{W_D}(w_i)$, $w_i \in W_D$. At any time t, the set of cells can be represented as a dynamic soft set that splits the set of cells into phases (Fig. 1) $\Upsilon_{W_D}(t) = \left\{\left(w, \Upsilon_{W_D}(w,t)\right) : w \in 2^W, \Upsilon_{W_D}(w,t) \in 2^C\right\}$, where $\Upsilon_{W_D}(w,t)$ is a set of cells, which belong to the phase $w \in W_D$ at the time t. This set describes the state of the process F : $State_F^t = \Upsilon_{W_D}(t) = \left\{\left(w, \Upsilon_{W_D}(w,t)\right) : w \in 2^W, \Upsilon_{W_D}(w,t) \in 2^C\right\}$.

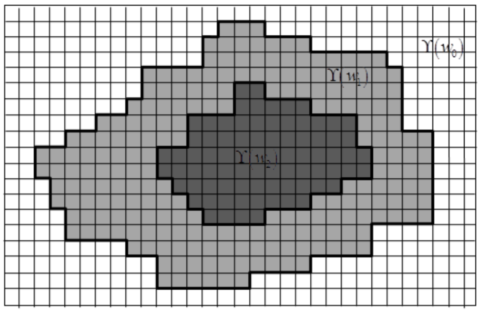

Fig. 1. State of the destructive process in the form of a soft set

Fig. 1 reflected the state of destructive processes in the form of soft sets, which divides the set of cells into three subsets: w_0-elements (in white), w_1-elements (in dark gray), and w_2-elements (in light gray). If w_i and w_j elements related by achievability relation, then the areas approximated by cells, which are w_i- and w_j- elements of the soft set, must be adjacent to each other. The decomposition of the subspace C of approximation subspace X using W_D-indiscernibility relation is a topological space $T_C^{w_D} = \left(C, Def\left(apr_C^{w_D}\right)\right)$ superimposed on topological space $T_C^{w_S}$. Each i-class of equivalence $\left(apr_C^{w_D}\right)_i$ can be represented as the value of the soft set $F_{W_D}(w_i)$, $w_i \in W_D$.

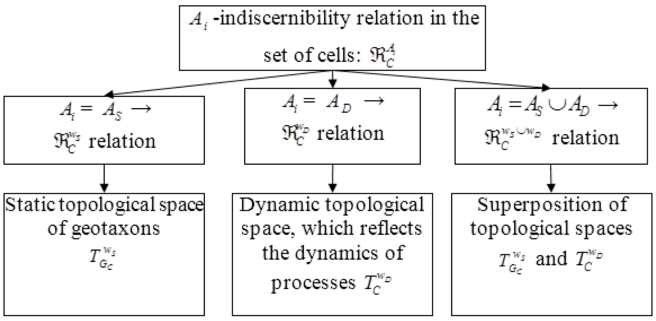

Fig. 2. The indiscernibility relations used to construct of the spatial model

Let $Def\left(\Upsilon_{W_D}\right)$ be a family of composite sets of the soft set Υ_{W_D}. Obviously, that $Def\left(\Upsilon_{W_D}\right) = Def(apr_C^{w_D})$. Thus, the topological space can be represented as the soft set $T_C^{w_D} = \left(C, Def\left(apr_C^{w_D}\right)\right) = \left(C, Def\left(\Upsilon_{W_D}\right)\right)$ [16].

Consider q-indiscernibility relation on the set of cells that generates dynamic equivalence classes, which constitute a dynamic topological space T_C^q representing homogeneous regions in respect of relative hazard, threat, or risk assessments at any given time t. A subset of cells belonging to one class of equivalence forms a zone, each of which does not necessarily have to be connected. At any time t the set of cells can be represented as a dynamic soft set, which splits the set of cells into zones from the set Q: $\Upsilon_Q(t) = \{(w, \Upsilon_Q(q,t)) : w \in 2^Q, \Upsilon_Q(q,t) \in 2^C\}$, where $\Upsilon_Q(q,t)$ is a set of cells, which are within a zone $q \in Q$ at a time t. The decomposition of the subspace C of approximation space X using q-indiscernibility relation is a topological space $T_C^q = (C, Def(apr_C^q))$.

Each i-class of equivalence $(apr_C^q)_i$ can be represented as the value of the soft set $\Upsilon_Q(w_i)$, $q \in Q$. Let $Def(\Upsilon_Q)$ be a family of composite sets of the soft set F_Q. Obviously, that $Def(\Upsilon_Q) = Def(apr_C^q)$. Thus, the topological space can be represented as a soft set $T_C^q = (C, Def(apr_C^q)) = (C, Def(\Upsilon_Q))$. The set Υ_{W_S} is static while the sets Υ_{W_D} and Υ_Q are dynamic. The spatial model Ξ can be represented as a multilayer topological space, which is a superposition of topological spaces in the form of soft sets:

$$T = (C, \{Def(\Upsilon)\}) \big| \Upsilon \in \{\Upsilon_{W_S}, \Upsilon_{W_D}, \Upsilon_Q\},$$

where $Def(\Upsilon)$ is a family of composition sets generated by the soft sets Υ_{W_S}, Υ_{W_D}, and Υ_Q, each of which constitutes a separate layer of the spatial model. Fig. 2 shows three types of A_i-indiscernibility relations in the set ($A_i \subset A$) [16]. Table 1 reflects the properties of the considered topological spaces, which are shown in Fig. 3 [16].

4 Approximate topological space

Since the spatial model of AOI is blurred, in order to build an approximate topology we need to generalize (blur) a strict indiscernibility relation $\Re_C^{w_D}(t)$. Using the approximated soft sets we can represent topological spaces of terrain conditions (geotaxons), dynamic conditions (phase), estimations, etc. The blurring of topological spaces will be considered on the example of the topological space of the dynamic conditions, which is important for obtaining risk assessment.

Let us build a generalization of the relation $\Re_C^{w_D}(t)$ into the similarity relation $\tilde{\Re}_C^{w_D}(t)$, which can be replaced with the fuzzy soft set $\tilde{\Upsilon}_{W_D}$ [17]. As a result, at each time t we obtain a fuzzy approximation space $\widetilde{apr}_C(t) = (C, \tilde{\Re}_C^{w_D}(t)) = (C, \tilde{\Upsilon}_{W_D}(t))$ and a fuzzy topology, which represents a partition of all cells in C into fuzzy sets of cells $\tilde{C}^{w_i}(t)$, $i = 0, \dots n-1$ that enumerate all possible phases of the set W_D [18].

Let L denotes the interval [0,1], 2^C denotes a family of crisp subsets of C, and L^C denotes a family of all fuzzy subsets of C, where each fuzzy set is a mapping

$\tilde{\mathcal{E}}^{w}(t): C \to L$. Thus, we can represent the fuzzy soft set, which divides the set of cells into phases and define the state at time t as

Table 1. Properties of the topological spaces [16]

		Cartographic object or zone			
		Geotaxons	Cells	Processes	Assessments
1	topological space	$T_{G_C}^{A_S}$	T_C	$T_C^{w_D}$	$T_C^{\,q}$
2	The base of formation	A_S - indiscernibility relation $\Re_C^{A_S}$ and soft set Υ_{W_S}	Sampling space on equal objects	w_D - indiscernibility relation $T_C^{w_D}$ and soft set Υ_{W_D}	q- indiscernibility relation $T_C^{\,q}$ and soft set Υ_Q
3	Connection components	Geotaxon	Cell	A set of cells, which belong to the same phase w_D	A set of cells, which corre-sponds to the assessment q
4	Attribute values of connection components	Static	Dynamic	Dynamic	Dynamic
5	The composition of connection components set (variability in space)	Static	Static	Dynamic	Dynamic
6	Elements of the equivalence class	Homogeneous with respect to a set of certain attributes of A_S	Homogeneous with respect to a set of certain attributes of A	Homogeneous with respect to a certain phase w_D	Homogeneous with respect to a certain as-sessment q

$\tilde{\Upsilon}_{W_D}(t) = \left\{ \left(w, \tilde{\Upsilon}_{W_D}(w,t) \right) : w \in 2^{W_D}, \tilde{\Upsilon}_{W_D}(w,t) \in L^C \right\}$, where $\tilde{\Upsilon}_{W_D}(w,t) = \left\{ \left(c, \tilde{\Upsilon}_{W_D}(w,t)(c) \right) : c \in C \right\} = \tilde{\mathcal{E}}^{w}(t) = \left\{ \left(c, \tilde{\mathcal{E}}^{w}(c)(t) \right) : c \in C \right\}$ is the fuzzy set of cells, which belong to the phase $w \in W_D$ at time t, and $\tilde{\Upsilon}_{W_D}(w,t)(c) = \tilde{\mathcal{E}}^{w}(c)(t)$ is a degree of membership of the cell c to fuzzy set of cells $\tilde{\mathcal{E}}^{w}$, which belong to the phase w (w- element of the fuzzy soft set $\tilde{\Upsilon}_{W_D}$) at time t. In this relation, we use the fuzzy w- elements instead of crisp ones, so the soft set becomes the fuzzy soft set.

5 Fuzzy-rough soft topology

The above-considered model of the topological space is based on the fuzzy splitting of the set of cells into phases, the number of which in the general case can be unlimited. However, it is not always possible to find a way to determine the degrees of membership of cells to certain phases. If such degrees are not known, then instead of fuzzy sets, it is convenient to use rough sets defined by a lower approximation (as a subset of cells that uniquely belong to an approximate set), an upper approximation

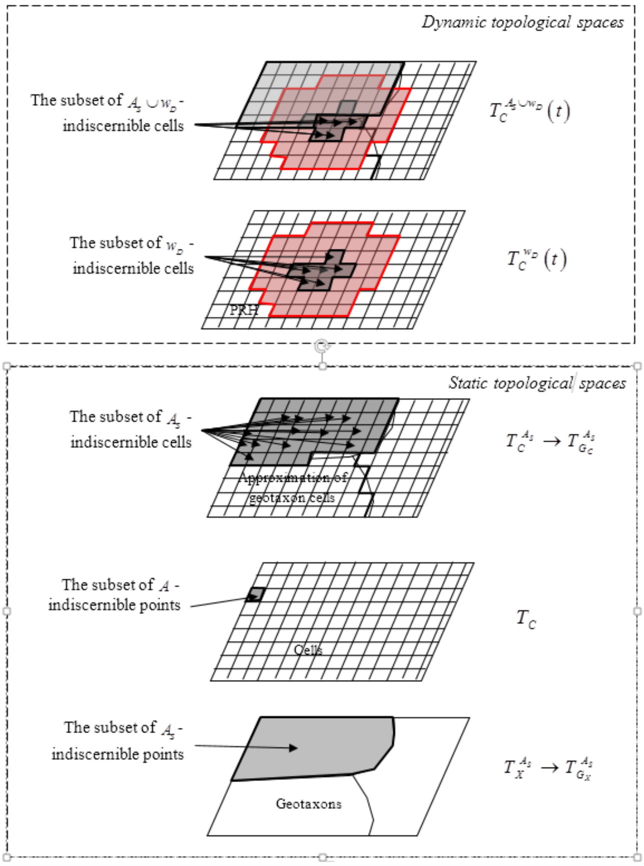

Fig. 3. The topological spaces of the spatial model

(as a subset of cells that may belong to an approximate set), and a boundary region (as a subset of cells, whose degree of membership is unknown with respect to the approximated set).

The approximate indiscernibility relation at a time t generates an approximation space $apr_C(t) = \left(C, \Re_C^{w_D}(t)\right)$ and an approximate topology, that is, the partition of the set of cells C on the approximate subset of cells $C^{w_i}(t)$, $i = 0,...n-1$, which belong to each of the possible phases of the set W_D. To build an approximated soft set of cells, we should blur the crisp soft set by introducing the Pawlak lower and upper rough approximations.

Let $apr_C^{w_D} = \left(C, \Re_C^{w_D}\right)$ be Pawlak space approximation, and $\Upsilon_{W_D} = (\Upsilon, W_D)$ be a soft set within C. Denote the lower and upper rough approximation of the soft set Υ_{W_D} in $(C, \Re_C^{w_D})$ by $\underline{\Upsilon_{W_D}} = (\underline{\Upsilon}, W_D)$ and $\overline{\Upsilon_{W_D}} = (\overline{\Upsilon}, W_D)$ respectively. Clearly, they are the soft sets $\underline{\Upsilon_{W_D}}(w) = \{c \in C | \Re_C^{w_D}(c) \subseteq \Upsilon_{W_D}(w)\}$ and $\overline{\Upsilon_{W_D}}(w) = \{c \in C | \Re_C^{w_D}(c) \cap \Upsilon_{W_D}(w) \neq \varnothing\}$ for all $w \in W_D$. In the case, when $\underline{\Upsilon_{W_D}}(w) = \overline{\Upsilon_{W_D}}(w)$, the w-element of soft sets Υ_{W_D} is the crisp set, otherwise, it is the rough set.

The above definition is a rough approximation of the soft set [9]. The approximate set of cells that belong to a certain phase w at time t is determined by two approximations as $\hat{\Upsilon}_{W_D}(w,t) = \{\underline{\Upsilon_{W_D}}(w,t), \overline{\Upsilon_{W_D}}(w,t)\}$, where $\underline{\Upsilon_{W_D}}(w,t)$ is a lower approximation, which contains all cells that belong to the set $\hat{\Upsilon}_{W_D}(w,t)$ clearly and necessarily (i.e., they belong to the phase w); and $\overline{\Upsilon_{W_D}}(w,t)$ is an upper approximation, which contains all cells that may belong to the set $\hat{\Upsilon}_{W_D}(w,t)$.

The negative region of the rough set $\hat{\Upsilon}_{W_D}(w,t)$ is called a set of cells of the universe C, which do not reliably belong to $C^w(t)$: $NEG(\hat{\Upsilon}_{W_D}(w,t)) = C - \overline{\Upsilon_{W_D}}(w,t)$. A boundary region of the rough set $\hat{\Upsilon}_{W_D}(w,t)$ is called a set of cells of the universe C, which belong to the upper approximation $\overline{\Upsilon_{W_D}}(w,t)$ but does not belong to the lower approximation $\underline{\Upsilon_{W_D}}(w,t)$: $BND(\hat{\Upsilon}_{W_D}(w,t)) = \overline{\Upsilon_{W_D}}(w,t) - \underline{\Upsilon_{W_D}}(w,t)$

During the monitoring of destructive processes, it is often possible to obtain information for the cells about the graduation of their degree of membership with respect to the boundary region of the certain rough set. For this purpose, it is convenient to represent the state of the destructive process as a fuzzy rough soft set of cells, which divides the set of cells into phases at each time t and can be represented as a triple consisting of the upper and lower approximations of the rough set, and the boundary region of the rough set represented as the fuzzy set: $\tilde{\Upsilon}_{W_D}(t) = \langle \underline{\hat{\Upsilon}_{W_D}}(t), \overline{\hat{\Upsilon}_{W_D}}(t), B\tilde{N}D(\hat{\Upsilon}_{W_D}(t)) \rangle$ [19].

Fuzzy-rough soft set splits the set of cells into w-elements, each of which is a fuzzy rough set of cells, which belong to a certain phase w, $w \in W_D$: $\tilde{\Upsilon}_{W_D}(w,t) = \langle \underline{\hat{\Upsilon}_{W_D}}(w,t), \overline{\hat{\Upsilon}_{W_D}}(w,t), B\tilde{N}D(\hat{\Upsilon}_{W_D}(w,t)) \rangle$, where $B\tilde{N}D(\hat{\Upsilon}_{W_D}(w,t))$ is a fuzzy set of cells, which belong to the boundary region of the w-element of the rough set $\hat{\Upsilon}_{W_D}(t)$: $B\tilde{N}D(\hat{\Upsilon}(w,t)) = \{c, B\tilde{N}D(\hat{\Upsilon}(w,c,t)) : c \in BND(\hat{\Upsilon}(w,t))\}$. $B\tilde{N}D(\hat{\Upsilon}(w,c,t))$ is the degree of membership of the cell c, which belongs to the boundary region of the rough set $\hat{\Upsilon}_{W_D}(t)$, to the fuzzy set $B\tilde{N}D(\hat{\Upsilon}_{W_D}(w,t))$ at a time t.

Fig. 4 shows the blurring of the boundaries of w-elements of the soft set Υ_{W_D} using fuzzy rough soft set $\tilde{\Upsilon}_{W_D}$. The top of the figure shows the state of the destructive process in the form of the soft sets Υ_{W_D}, which splits the set of cells into three subsets

(w_0 - elements, w_1 - elements, and w_2 - elements), each of which is a crisp set.

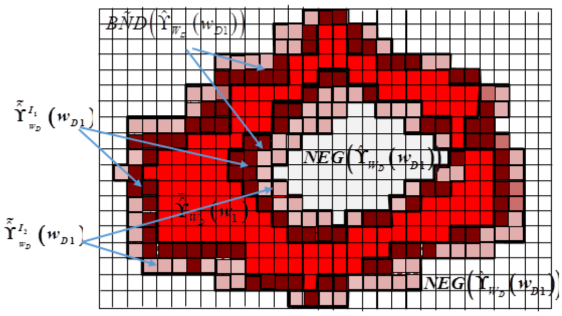

Fig. 4. Blurring the boundaries between the elements of the soft set using the fuzzy rough sets

We obtain a fuzzy rough soft set by generalizing the soft set $\tilde{\Upsilon}_{W_D}$ and blurring the boundaries between its elements. Two lower figures show fuzzy rough sets, which are elements of the fuzzy rough soft set: w_2 - item ($\tilde{\Upsilon}_{W_D}(w_2)$) and w_1 - item ($\tilde{\Upsilon}_{W_D}(w_1)$). The boundary regions of the approximate sets are represented by fuzzy sets. Cells having different degrees of membership to the rough set are represented in different colors. Thus, the state of the process at a time t can be defined as a fuzzy rough soft set of cells $\tilde{\Upsilon}_{W_D}(t)$: $\tilde{\Upsilon}_{W_D}(t)=\left\{\left(w,\tilde{\Upsilon}_{W_D}(w,t)\right):w\in W_D\right\}$, where each fuzzy rough set of cells $\tilde{\Upsilon}(w,t)$ is the w-element of the fuzzy rough soft set $\tilde{\Upsilon}_{W_D}(t)$. Let $C/\tilde{R}_C^{w_D}(t)$ be a factor-set, consisting of fuzzy sets of cells $\tilde{\Upsilon}_{W_D}(w,t)=\tilde{C}^w(t), w\in W_D$, generated by fuzzy rough relation $\tilde{R}_C^{w_D}(t)$. In this case, $\tilde{apr}_C=\left(C,\tilde{R}_C^{w_D}(t)\right)=\left(C,\tilde{\Upsilon}_{W_D}(t)\right)$ is a fuzzy rough approximation space and $Def\left(\tilde{apr}_C\right)=Def\left(\hat{\Upsilon}_{W_D}\right)$ is a family of fuzzy rough sets representing the cells, which belong to a certain phase; $\forall t\in T$ the set $\tilde{\tau}(t)=Def\left(\tilde{\Upsilon}_{W_D}(t)\right)$ is a fuzzy rough topology on C. At any $t\in T$ a couple $\tilde{T}_C^{w_D}(t)=\left(C,\tilde{\tau}(t)\right)$ is fuzzy rough topological space. Each element of $\tilde{\tau}(t)=Def\left(\tilde{\Upsilon}_{W_D}\right)$ is the fuzzy rough open set in C [20].

6 Developing a spatial model of the destructive process

In order to diagnose the situation during destructive processes in real-time systems, in terms of time limit there is no need to allocate a large number of phases of a cell. It is quite enough for a certain time to allocate a subset of cells not yet covered by the destructive process, a subset of the cells covered by the destructive process, and a subset of cells destroyed as a result of the destructive process (which were covered at the previous moments of time). To do this, we use three possible values of the cell phase c in the spreading area of the destructive process F (Fig. 4) [16]:

- "Not covered by *F*" ($w_D(c,t)=w_{D0}$)
- "Covered by *F*" ($w_D(c,t)=w_{D1}$)
- "Phase not defined" ($w_D(c,t)=w_{D2}$).

As a rule, information on being cells in these phases can be obtained during monitoring with different sensors. The cells covered with the destructive process belong to the phase w_{D1} and form a zone limited by the internal and external contour of the process. These contours are blurred and can be represented as boundary regions of an approximate set of cells in this phase. During the monitoring of the dynamics of the process with various sensors, it is often possible to obtain information about the possibility of covering the cells that belong to the blurred contour. Often, we can also determine the gradation of the possibility of covering cells that belong to a blurred contour of the destructive process. The state of the process can be represented as the fuzzy rough set of cells, covered by the destructive process, given by a triple, consisting of the upper and lower approximations of the rough set, as well as the boundary region of the rough set, presented as the fuzzy set: $\tilde{\hat{\Upsilon}}_{W_D}(w_{D_1},t)=\left\langle \underline{\hat{\Upsilon}_{W_D}}(w_{D_1},t), \overline{\hat{\Upsilon}_{W_D}}(w_{D_1},t), B\tilde{N}D\left(\hat{\Upsilon}_{W_D}(w_{D_1},t)\right)\right\rangle$, where $B\tilde{N}D\left(\hat{\Upsilon}_{W_D}(w_{D1},t)\right)=\left\{\left(c,\tilde{\hat{\Upsilon}}_{W_D}(w_{D1},c,t)\right)\right\}$.

For some destructive processes, the concept of the inner contour does not make sense (e.g., for floods). Therefore, it is often advisable to consider only the outer contour of the process, which we will call simply a contour. The location and dynamics of the outer contour are decisive for assessing the time-level threat, representing the time for which the outer contour of the process can reach a certain object (Fig. 5).

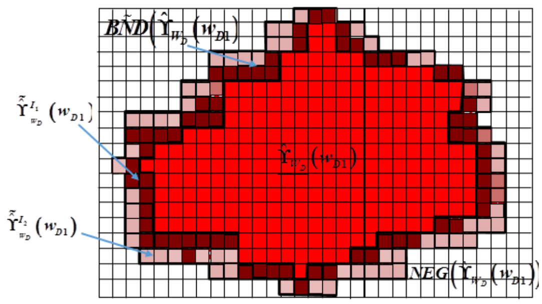

Fig. 5. State of the destructive process defined by the fuzzy rough soft set

In this case, we consider two phases of cell dynamics:

- "Not covered by F" ($w_D(c,t) = w_{D0}$)
- "Covered by F" ($w_D(c,t) = w_{D1}$).

During monitoring it is possible to determine the areas covered and destroyed as a result of the destructive process (lower approximation $\underline{\hat{\Upsilon}_{W_D}}\left(w_{D_1},t\right)$, that is, the set of cells belonging to the phase w_{D1}), and areas not yet covered by the destructive process (negative region $NEG\left(\hat{\Upsilon}_{W_D}\left(w_{D_1},t\right)\right)$, that is, the set of cells belonging to the phase w_{D0} . There is a blurred territory between these areas, which constitute a blurry contour of the destructive process (fuzzy set $B\tilde{N}D\left(\hat{\Upsilon}_{W_D}\left(w_{D_1},t\right)\right)$). Based on the monitoring data it is not difficult to construct a fuzzy rough soft approximation space $a\tilde{p}r_C(t) = \left(C, \tilde{\hat{\Upsilon}}(t)\right)$ and a fuzzy rough soft topology $Def\left(a\tilde{p}r_C\right)(t) = \tilde{\tau}\ (t)$, that is, partitioning the set of cells C at each time t on the fuzzy rough subset of cells $\tilde{\hat{\Upsilon}}_{W_D}(w_{Di},t)$, which belong to each of the possible phases w_{Di} of the set W_D .

7 Experiment Results

The proposed spatial model has been implemented using Visual C and tested on computer based on the Pentium i5-7400 3 GHz processor and 16 GB RAM. The developed spatial model of the spreading destructive processes based on the fuzzy rough soft topology was used in the GIS-based real-time DSS providing the geospatial analysis of emergencies in real time disaster situations [21]. The developed DSS allows evaluating a number of indicators, e.g. danger degrees, threats, and risks, for target objects, as well as providing the geospatial analysis of emergencies in real time disaster situations. To obtain such estimates, it is necessary to build a spreading model of the destructive process and track the movement of its contour in real time by monitoring using UAVs.

To examine the developed model, we use real-time DSS in the forest fire response operations. Fig. 6 shows the representation of the forest fire front based on the fuzzy rough soft topology, which has been obtained during the monitoring. Fig. 7 depicts a fuzzy-rough cut of the forest fire front evaluated by the possibility of burning obtaining during the forest fire monitoring.

The results of the experiment show that the proposed spatial model provides acceptable performance in terms of accuracy and speed for all kind of topology. The fuzzy rough soft topology shows sufficient results on the speed with enough accuracy.

8 Conclusions

The approximate spatial model for the real-time GIS-based DSS based on the fuzzy-rough soft topology is proposed. The model of the destructive process is represented as the model of dynamic change of states of the subset of cells covered by the process within the spatial model. As a result, the spatiotemporal structure of AOI is represented as a topology space, which includes subspaces of cells that belong to the same phase.

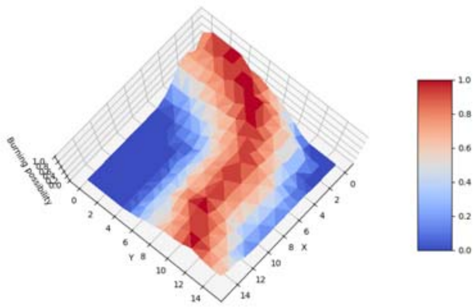

Fig. 6. Representation of the forest fire front based on the fuzzy rough soft topology

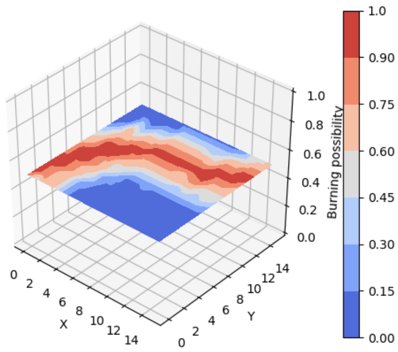

Fig. 7. A fuzzy-rough cut of the forest fire front assessed by the possibility of burning

The soft topological spaces are used to build a spatial model, as well as the fuzzy-rough method is used for its blurring. Since the belonging of each cell to the certain phase is approximately determined due to the uncertainty of remote sensing, the topological space is blurred and the boundaries of the dynamic contour of the destructive process are also blurred. The proposed spatial model representing uncertain information about the disaster reduces the computational complexity and provides flexible and timely decision-making in real time.

9 References

1. Sherstjuk, V., Zharikova, M., Sokol, I.: Approximate spatial model based on fuzzy-rough topology for real-time decision support systems. In Proc. on IEEE First Ukraine

Conference on Electrical and Computer Engineering (UKRCON), pp 1037–1042, Kyiv (2017).
2. Chen, G., Zhao, J., Yuan, L., Ke, Z., Gu, M., Wang, T.: Implementation of a geological dis-aster monitoring and early warning system based on multi-source spatial data: a case study of Deqin Country, Yunnan Province. Hazards Earth Syst. Sci., pp. 15 (2017).
3. Chi, M., Plaza, A., Beneditsson, J.A., Sun, Z., Shen, J., Zhu, Y.: Big data for remote sensing: challenges and opportunities. In Proc. of the IEEE, 104(11), 2207–2219 (2016).
4. Yuan, C., Zhang, Y., Liu, Z.: A Survey on Technologies for Automatic Forest Fire Monitoring, Detection and Fighting Using UAVs and Remote Sensing Techniques. Canadian Journal of Forest Research 45(7), 783–792 (2015)
5. Martínez de Dios, J., Arrue, B., Merino, L., Ollero, A., Gómez-Rodríguez, F.: Computer vision techniques for forest fire perception. Image and Vision Comp. 26(4), 550–562 (2007).
6. Merino, L., Caballero, F., Martinez-de-Dios, J., Maza, I., Ollero, A.: An Unmanned Aircraft System for Automatic Forest Fire Monitoring and Measurement. Journal of Intelligent & Robotic Systems 65, 533–548 (2012).
7. Zharikova, M., Sherstjuk, V.: Development of the Model of Natural Emergencies in Decision Support System. EasternEuropean Journal of Enterprise Technologies 4(1), 62–69 (2015).
8. Zadeh, L.A.: Fuzzy sets. Information and Control 8, 338–353 (1965).
9. Pawlak, Z., Jerzy, W., Slowinski, R., Ziarko, W.: Rough Sets. Comm. of ACM 38(11), 88–95 (1995).
10. Molodtsov, D.A.: Soft set theory – first results. Computer & Mathematics with Applications, pp. 19–31 (1999).
11. Maji, P. K., Roy, A. R., Iswas, R. B.: An application of soft sets in a decision-making problem. Computers and Mathematics with Applications 44, 1077–1083 (2002).
12. Maji, P. K., Iswas, R. B., Roy, A. R.: Fuzzy soft sets. Journal of Fuzzy Mathematics 9, 589–602 (2001).
13. Meng, D., Zhang, X., Qin, K.: Soft rough fuzzy sets and soft fuzzy rough sets. Computers and Mathematics with Applications 62, 4635–4645 (2011).
14. Zharikova, M., Sherstjuk, V.: Threat assessment method for intelligent disaster decision support system. Advances in Int. Systems and Computing 512, 81–99 (2016).
15. Allam, A.A., Bakeir, M.Y., Abo-Tabl, E.A.: Some Methods for Generating Topologies by Relations. Bull. Malays. Math. Sci. Soc. 2(31), 35–45 (2008).
16. Zharikova, M.: Methodological basis of geoinformation technology of decision support in combined natural and man-made systems in destructive processes conditions. Dr Thesis, KNTU, Kherson, p. 503 (2018)
17. Varol, B.P., Aygun, H.: Fuzzy soft topology. Hacettepe Journal of Mathematics and Statistics 41(3), 407–419 (2012).
18. El-Diafy, S.N.: Comparative study of fuzzy topology. Thesis submitted in partial fulfillment of the requirement for the degree of master of mathematics, Gaza, The Islamic University of Gaza, 76 p. (2014).
19. Meng, D., Zhang, X., Qin, K.: Soft rough fuzzy sets and soft fuzzy rough sets. Computers and Mathematics with Applications 62, 4635–4645 (2011).
20. Tang, W., Wu, J., Zheng, D.: On fuzzy rough sets and their topological structures. Mathematical problems in engineering 4, 17 (2014).
21. Sherstjuk, V., Zharikova, M., Sokol, I.: Forest Fire-Fighting Monitoring System Based on UAV team and Remote Sensing. In Proc. of 2018 IEEE 38th International Conference on Electronics and Nanotechnology, pp. 663–668 (2018).