Computer Simulation as a Method of Learning Research in Computational Mathematics

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Abstract. The paper deals with a problem of students' independent work through learning research. By solving research problems students gain experiences in a way closest to the real scientific study. A student does not receive the "ready" knowledge and decisions, but a teacher puts for him the goal, the cognitive problem that this student should solve during independent research activity. We suggest appropriate computer models in Mathcad environment (dynamic support synopsis) to provide such learning research in the "Methods of Computing" course. Students research some problems of global polynomial interpolation, spline interpolation, choose the appropriate function for approximation, verify the efficiency (accuracy and convergence rate) of some methods of numerical calculation of integrals and solving of non-linear equations. The results of approbation of this learning methods in the training process of H. S. Skovoroda Kharkiv National Pedagogical University are shown and discussed. The efficiency of our approach was checked with use of such criteria: completeness of knowledge and depth of knowledge. Statistical analysis of obtained data shows advisability of implementation of learning research in training process.

Keywords: Computer Simulation, Students' Independent Work, Computational Mathematics.

1 Introduction

The existence of the wide software spectrum should be reflected in the technique of the numerical methods learning by future experts in applied fields of science and technology. The main questions of teaching are the following: interpretation of the whole picture of possible approaches to numerical problem solving; realization of the idea and peculiarities of each method; conditions of the method application; advantages and disadvantages in comparison with other methods; determination of problems classes, for which the method can be the most efficient. We should mention that theoretical analysis of possibilities of using some methods becomes very complicated in practice in many cases, so the investigator's intuition is considered to be very important in process of the proper method choice. All these facts can explain the necessity of carrying out the special teaching methods, which will give the future

specialists the opportunity to gain their own experience in the use of different methods and to make proper conclusions.

The realization of these methods is a very difficult task and is determined by the role of computers in the process of teaching. According to the tradition, computer is considered as a calculating device in the process of learning of numerical methods, so practical work in this course is transformed to programming where certain algorithms of methods are translated into one or another language. The availability of software for supporting professional mathematical activity directs us to transfer the practical work to the higher level by the use of computers as a means of scientific research. In this case, we can avoid the necessity of programming, but the process of calculation is hiding from a user and it can reduce the laboratory work to mindless copying the numbers from the computer screen. It is possible to give the laboratory work real educational and research character only by means of the problem situation that can fill in the whole work with purport by putting its final aims and determining the leading questions. We have prepared our guidebook for students – future teachers to put such problem situation and show the steps of problem solving [1].

The fundamentals of numerical methods have been published in scientific and education books of well-known authors [2], [3], [4], [5], [6], [7], [8], [9] etc. Modern applications of numerical methods are undoubtedly connected with using computers. So computer-based environment became a significant element in teaching numerical methods. There are enough scientific works in the field of mathematics education that use a computer-based environment to show the calculation procedure at using numerical methods, for example, the authors [10] use MS Excel "... to perform numerical integration, specifically trapezoidal rule and Simson's rule ..." and the procedure of generation of Lagrange's interpolation polynomial [10]. The other direction of studies in the field of education in numerical methods is to use special teaching software [11]. Graphical visualization is another interesting approach to this difficult educational problem. Thus, the authors [12] take attention for visualization of complicated iterative and recursive process for solving non-linear equations and use GeoGebra software that "... makes possible to deal with these methods by means of their geometrical interpretation and to visualize their behavior and procedure" [12]. Last time, great attention is paid to particularities of using the numerical methods in some non-standard conditions [1], [9]. Therefore, teaching methods, which oriented on using computer-based environment for learning research of these particularities, for deeper mastering in computational mathematics, need in further development.

The goal of this work is to verify the methods of learning research organizing with suggested computer-based models for numerical methods investigation in Mathcad environment (dynamic support synopsis).

2   Curricula and objectives

The purpose of teaching the discipline "Methods of Computing" at Physics and Mathematics department of the H. S. Skovoroda Kharkiv National Pedagogical University is to familiarize students with the general characteristics of the basic

methods of numerical solution of applied tasks based on the construction of mathematical models; to form students skills in mathematical-oriented computer environments and the ability to conduct computational experiments in these environments; to equip students with research skills and analysis skills; to provide students with the skills of competent use of numerical methods in practice.

The main objectives of studying the discipline "Methods of computing" are the such:

- to reveal the place and significance of knowledge on numerical methods in general and vocational education; to show psychological and pedagogical aspects of mastering the subject; to establish relationship of the discipline "Methods of computing" with other educational subjects; to show the practical significance of numerical methods, mathematical modeling, computational experiment, their applicability to solving of humanitarian, technical and scientific problems; to present examples of realization of great opportunities that open the efficient use of computers;
- to provide future teachers with grounded study of the concepts and methods of computing that can be used by them in the process of teaching some topics of the school courses of mathematics and computer science, familiarizing pupils with elements of numerical methods, conducting optional classes in secondary schools;
- to form students with sufficient knowledge, skills and abilities, necessary for the practical conduct of educational work in the secondary schools with the wide use of modern information technologies.

According to the requirements of the educational-professional program, students must know the main stages of solving the problems on a computer, the basic numerical methods for solving mathematical problems and the peculiarities of its computer-based implementation. Students should be able substantiate the choice of a numerical method for solving a mathematical problem, realize the method algorithm using the programming languages or some specialized computer-based environment (such as Excel, Mathcad, etc.), perform the necessary calculations and analyze the obtained results.

According to curricula, students study such chapters of numerical methods: Elements of the Error Theory, Functions Approximation (Polynomial Interpolation, Spline Interpolation, Least Squares Method), Numerical Integration, Solution of One-Variable Equations, Solution of Systems of Linear Equation, First-Order Differential Equation and its Systems. In this paper we focus on approximation and integration problems.

3   Theoretical background

There are two main tasks at the statement of the approximation problem:

- Choosing of approximation function;
- Choosing the criterion of proximity of the function.

Different kinds of approximation may emerge depending on this choice. One case of approximation problem is the polynomial interpolation problem, where it is required that interpolating polynomial should take the same values as the interpolated function at the given points $x_0, x_1, ..., x_n$ of the interval $[A; B]$. A system of linear equations should be solved to obtain the coefficients of polynomial and some useful formulas are well known (Lagrange's and Newton's polynomial interpolation formulas). Students study these formulas and should use it in practice, this part of educational material is reproductive. The interpolation polynomial of degree not above $n = N–1$ always can be construct, if N interpolation nodes are given. But problem is in error estimation of approximation the given function by polynomial. To solve this problem a student should work on high levels of cognitive activity.

We may get the error estimation of the function approximation by interpolation polynomial:

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!}|x - x_0||x - x_1| ... |x - x_n|, \text{ where } M_{n+1} = \max_{x \in [x_0; x_n]}\left|f^{(n+1)}(x)\right|.$$

This formula for polynomial interpolation error estimation can be simplified for equidistant nodes:

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!}h^{n+1}|q(q-1) ... (q-n)|.$$

Thus, $|R_n(x)| = 0(h^{n+1})$, that is interpolation error should decrease as h^{n+1} with decreasing of the interpolation step. But many factors affect at this error:

- the form of the function $f(x)$, in particular, the value of its derivative of the $(n+1)$ order;
- the interpolation nodes arrangement on the interval $[A; B]$ and the size of the interval.

The factor of the function form is very essential, as an example, we can remember the classical Runge's function, which have growth of derivatives with its order so high that the interpolation of this function with polynomial of high degree leads to unacceptable error growth. It is not easy to analyze the function derivative on all interval $[A; B]$ in many practical cases. In addition, the total error of calculation of the function value according to an interpolation formula doesn't only consist in interpolation error (method error) but includes irremovable error, which is connected with inaccuracy of ascertainment of the function values in nodes and calculation error. So we should provide the student with some competence, may be intuition, for an expedient choice between global interpolation, piecewise interpolation and spline interpolation as well as choice of nodes arrangement. Such competence can be formed only by own experience in study of interpolation of different functions in different conditions, that is, practical solving problems or learning research.

A similar situation takes place with the numerical integration, because its error also depends on the form of the function at the interval $[A; B]$. We should provide our students with competences to make an expedient choice between global or piecewise

quadrature formulas, to recommend the order of the quadrature formula, to use Newton-Cotes or Gaussian formulas, to estimate the real accuracy of the integration.

4 Methods of students' learning research management

The laboratory works on the discipline "Methods of Computing" is constructed as a series of learning research of gradually increasing complexity. Students do not obtain ready-made knowledge, ready decisions, but the problem is posed that becomes a basis of students' independent research activity. Gradual complication of the content of the cognitive tasks, which are proposed for experimental study, is an important element in organizing such laboratory practice. The possibility of using the research method in the laboratory practice is conditioned by the advent and improvement of professional-oriented environments that provide the students with comfortable tools for the computational experiments. As a basis for the learning research in numerical methods we have taken the Mathcad environment, which is widely used to solve applied mathematics problems, and, at the same time, has a number of certain attractive qualities that makes it convenient for use in learning: intuitive interface, developed system menu and help, ability to input textual, symbolic and graphic information in any place of the workspace, fairly flexible embedded programming language, etc. [13].

To provide students' learning activity we have designed some models that realize certain algorithms of numerical methods in Mathcad environments. The interface of these models is designed as a set of special electronic pages that play the role of dynamic support synopsis (DSS) that is convenient for entering the input data of the task and structured displaying the result according to algorithm. Thus, the student actually receives a virtual laboratory for a computational experiment.

Management of the learning research was carried out according to the following scheme [13]:

- setting objectives;
- providing observation and accumulation of experimental data;
- doing evaluation and comparative analysis of the obtained data;
- predicting characteristics of the investigated method;
- formulating hypothesizes;
- studying the features of the method being studied;
- analyzing the received data;
- correcting the hypothesis;
- summarizing the results of work and formulating conclusions.

We have developed didactical support for the laboratory works in the form of plans-reports to provide meaningful and targeted student's activities and to ensure the achievement of the predicted learning effect. Plans-reports are built according to a single scheme and consist of two parts – informative and instructional. The informative part contains the subject of the work, its purpose, description of the software, characteristic of entered and displayed numeric and graphic data. The instructional part contains the order of execution of the work, where its key points are marked and fixed.

To motivate the student to a study, he is initially offered a chain of appropriately selected questions. Then the work is carried out according to the proposed plan, which defines the stages of the study, tasks that are solved at each stage, experimental material to be obtained, the form of its submission, etc. As students gain experience, the instructions to them become less detailed. Some experiments the student must plan, put and implement on their own. Individual variants of sets of tasks are suggested to each student to perform the laboratory work. These tasks are selected to show features of every algorithm that are studied and correspond to the purpose of the work [13]. The results of the work are proposed to be done in the form of conclusions, outlines of which with a greater or lesser detailing are fixed in the plan-report. It helps the student to record the results of work, to structure them, to pay his attention to those moments of research that can remain unnoticed.

5   Learning research in polynomial interpolation

We suggest students to investigate the problems of polynomial interpolation during three laboratory works:

- Lagrange's interpolation formula – the students study the influence of interpolation polynomial degree on the accuracy of global interpolation for different functions at the given interval [A; B]. Each student works with the individual set of functions, including functions with great growth of its derivative of high orders, polynomials, even and odd functions etc. An example of such student's investigation are shown in Table 1. Part 2 of this laboratory work is devoted to influence of the interpolation nodes localization on the interpolation accuracy and using Chebyshev's polynomial roots as nodes of interpolation;
- Newton's interpolation formula – the students are introduced with technics of building the polynomial across the table of function values as well as the methods of accuracy control and its limitation in "bad" situations;
- Piecewise interpolation, splines – the students operate with the various kind of interpolation models and get an experience in choice the appropriate method in different cases.

The last laboratory work is integrative. So let analyze it more detail. The aim of the work is to investigate interpolation of the function using piecewise interpolation polynomial and cubic spline. The software (DSS) contains models of spline, global and piecewise polynomial interpolation and gives the students possibilities to choose parameters of this model easy (Fig. 1, 2). Students entered the input data: A, B are the ends of the interpolation interval; $f(x)$ is the interpolated function.

At the beginning of the work students discuss some leading questions to actualize theoretical knowledge and formulate problems for experimental study:

- Are there cases, when increasing of the global interpolation polynomial degree leads to increasing, but not decreasing of the interpolation error?

- What conditions does the global interpolation polynomial become useless for function approximation under?
- What factors limit the interpolation polynomial degree in practice?
- How does the number of interpolation nodes, which have been taken into account, influence on the computational complexity with global, piecewise polynomials and spline interpolation?
- How does the number of interpolation nodes, which have been taken into account, influence on the volume of data, which must be kept for calculation of the value of global or piecewise interpolation polynomials and spline?
- What are the differences between the function approximation by piecewise interpolation with cubic polynomials, global polynomial interpolation and cubic spline interpolation? Consider cases, when there are $3m+1$ (where $m = 1, 2, ...$) nodes on the interpolation interval.
- Can one forecast such situations, when it is necessary to prefer the global interpolation polynomial, piecewise interpolation polynomials or spline?

Table 1. Student's investigation of influence of the global polynomial degree n on interpolation accuracy

n	Error of interpolation in cases:					
	e^x [0.5; 1.5]	$1/x^2$ [0.5; 1.5]	$2x^4 - x^3 + 3x^2 - x + 1$ [−1; 1]	$\dfrac{1}{1 + 25x^2}$ [−1; 1]	$\sin x$ [−2; 2]	$\cos x$ [−2; 2]
1	0.35	1.4	5.1	0.96	0.39	1.5
2	0.024	0.48	0.87	0.64	0.39	0.14
3	$1.6 \cdot 10^{-3}$	0.18	0.40	0.71	$3.5 \cdot 10^{-2}$	0.11
4	$8.7 \cdot 10^{-5}$	0.065	$1.0 \cdot 10^{-15}$	0.44	$2.5 \cdot 10^{-2}$	$7.4 \cdot 10^{-3}$
5	$4.4 \cdot 10^{-6}$	0.025	$3.6 \cdot 10^{-15}$	0.30	$1.4 \cdot 10^{-3}$	$5.3 \cdot 10^{-3}$
6	$2.0 \cdot 10^{-7}$	$9.2 \cdot 10^{-3}$	$1.8 \cdot 10^{-15}$	0.61	$9.7 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$
7	$7.9 \cdot 10^{-9}$	$3.6 \cdot 10^{-3}$	$3.6 \cdot 10^{-15}$	0.25	$3.4 \cdot 10^{-5}$	$1.6 \cdot 10^{-4}$
8	$2.9 \cdot 10^{-10}$	$1.4 \cdot 10^{-3}$	$4.5 \cdot 10^{-15}$	1.0	$2.4 \cdot 10^{-5}$	$4.5 \cdot 10^{-6}$
9	$9.4 \cdot 10^{-12}$	$5.1 \cdot 10^{-4}$	$6.2 \cdot 10^{-15}$	0.28	$5.4 \cdot 10^{-7}$	$3.2 \cdot 10^{-6}$
10	$2.8 \cdot 10^{-13}$	$2.0 \cdot 10^{-4}$	$8.9 \cdot 10^{-15}$	1.5	$3.8 \cdot 10^{-7}$	$6.2 \cdot 10^{-8}$

According to recommendations each student investigate global and piecewise polynomial interpolation and spline interpolation for given individual set of function. Students construct the global interpolation polynomial, piecewise interpolation polynomials of different degree and spline, using a fixed set of interpolation nodes N for the given functions. They find the error of each function approximation.

Then the students use the necessary number of interpolation nodes to provide given accuracy (10^{-4}) of function approximation with spline, global and piecewise interpolation polynomial of the degree $n = 2, 3, 4, 5$. The example of such investigation is shown in the Table 2. Students also investigate dependence of approximation accuracy at a fixed point from the number of considered nodes from a fixed set of nodes for spline interpolation.
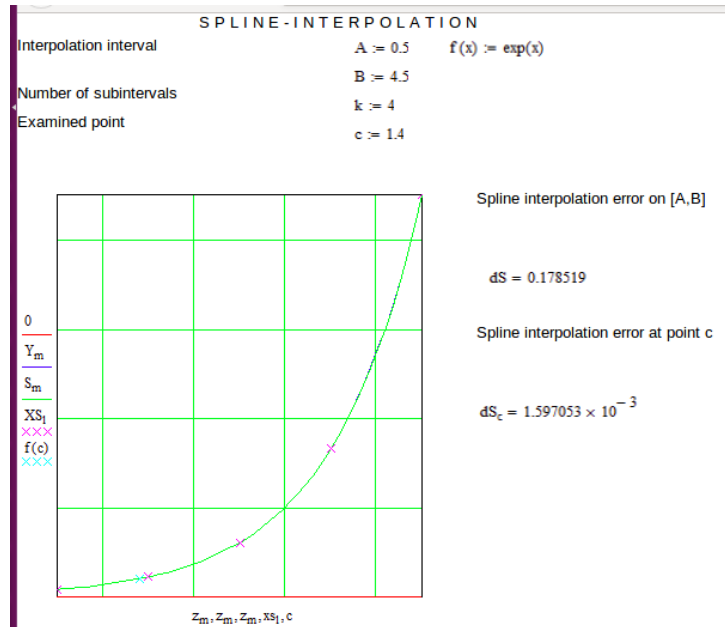
Fig. 1. Interface of the spline interpolation model
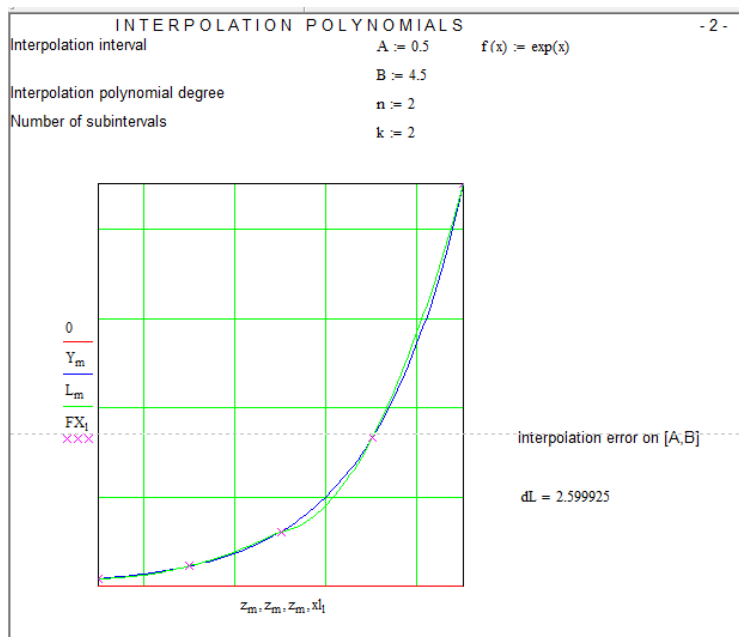
Fig. 2. Interface of the piecewise and global polynomial interpolation model

Table 2. Student's investigation of the number of nodes, which provide the given accuracy of interpolation

Approximation polynomial		Number of subintervals	Number of considered nodes	Interpolation error
type	degree			
$f(x) = e^x$, interpolation interval $[0.5; 4.5]$				
Global interpolation polynomial	9	1	10	$5.5 \cdot 10^{-5}$
Piecewise interpolation polynomial	2	76	153	$9.9 \cdot 10^{-5}$
	3	19	58	$8.4 \cdot 10^{-5}$
	4	8	33	$6.7 \cdot 10^{-5}$
Spline	3	28	29	$9.6 \cdot 10^{-5}$
$f(x) = \dfrac{1}{1 + 25x^2}$, interpolation interval $[-1; 1]$				
Global interpolation polynomial	Necessary accuracy was not reached; interpolation error increases with increasing the interpolation polynomial degree			
Piecewise interpolation polynomial	2	71	143	$1.0 \cdot 10^{-4}$
	3	32	97	$8.7 \cdot 10^{-5}$
	4	17	69	$5.0 \cdot 10^{-5}$
	5	14	71	$6.0 \cdot 10^{-5}$
Spline	3	27	28	$9.3 \cdot 10^{-5}$
$f(x) = \sin x^2$, interpolation interval $[-3; 3]$				
Global interpolation polynomial	Necessary degree of interpolation polynomial exceeds 20, calculation was stopped because the time is beyond			
Piecewise interpolation polynomial	2	140	281	$1.0 \cdot 10^{-4}$
	3	53	160	$9.6 \cdot 10^{-5}$
	4	26	105	$9.4 \cdot 10^{-5}$
	5	17	86	$9.6 \cdot 10^{-5}$
Spline	3	81	82	$9.9 \cdot 10^{-5}$

As a combined result of the investigations, students formulate the conclusions.

6   Learning research in numerical integration

Numerical integration is built on the polynomial interpolation of the function. So, all competences that students obtained, when they studied the polynomial interpolation, are the basic and should be actualize. The aim of the students' learning research is to investigate effectiveness of Gauss quadrature formulas in comparison with Newton-Cotes formulas. At the first, students investigate the application of composite Newton-Cotes quadrature formulas and formulate conclusions about influence of the integration step and the degree of the interpolation polynomial that is used for quadrature formula on the integration error. Also, they use Runge's rule for integration error estimation and investigate the limits of its applicability. Then students make a comparative analysis of composite formulas of not high order of accuracy (trapezoid formula and Gauss formula with one node) and composite or single formulas of high order of accuracy (Newton-

Cotes formula using interpolation polynomial of 8-th order and Gauss formula with 8 nodes). Leading questions on this second stage of the work are the such:

- Will the choice of integration nodes influence on the error of integral calculation? What is a level?
- How can we find out nodes location at integration by Gauss formulas?
- Will the error be sensible to slight error of node position from optimal?
- When is Gauss formula required to be used? Are there any occasions, when it is not possible to use it?
- What occasions need to use composite Gauss formulas?

Appropriate DSS (dynamic support synopsis) are designed on the base of models of numerical integrating with Gauss and Newton-Cotes quadrature formulas. One page of DSS (the last) is shown in Fig. 3.

Gauss Formula and Newton-Cotes Formulas of Different Order of Accuracy

Integrand:

Limits of integration : $a := 0.5$ $f(x) := \exp(x)$

$b := 2.5$

Integral value (calculated by MathCAD): $I_0 = 10.533773$

Quadrature Formula	Number of subintervals, m	Approximated integral value	Error of integration
	$m := 16$		
Trapezoid formula		$I_{N1} = 10.547485$	$\left|I_0 - I_{N1}\right| = 0.013712$
Mean rectangle formula		$I_{G1} = 10.526918$	$\left|I_0 - I_{G1}\right| = 6.854801 \times 10^{-3}$
	$m := 2$		
Newton-Cotes formula, built on interpolation polynomial of 8th degree		$I_{N8} = 10.533773$	$\left|I_0 - I_{N8}\right| = 5.988099 \times 10^{-12}$
Gauss formula that is constructed on 8 nodes		$I_{G8} = 10.533773$	$\left|I_0 - I_{G8}\right| = 8.881784 \times 10^{-15}$

Fig. 3. Interface of the model for numerical integration

Results of computational experiments are collected in tables, forms of which are suggested to the students in the corseware. One of such table with student's results are shown (Table 3).

The final students' conclusions are connected with such numerical integration features:

- choice of the quadrature formula at limited number of nodes of integration;
- difference between Gauss formula and Newton-Cotes formula by construction;
- comparison of accuracy of Gauss and Newton-Cotes quadrature formulas;

- sensitiveness of integration error by Gauss formulas to accuracy of determining the nodes;
- functions, for which high order quadrature formulas are recommended;
- choice of the quadrature formula for numerical integrating of given function at given conditions;
- situations, where the error of applied numerical integration formula turned out to be comparable with calculation error.

Table 3. Student's investigation of the accuracy of various methods of numerical integration

Quadrature formula	N	m	Error of integral calculation		
			$\int_{0.5}^{2.5} e^x dx$ ≈ 10.53377	$\int_{0.5}^{2.5} \frac{dx}{x^2} = 1.6$	$\int_{-1}^{1} \frac{dx}{1+25x^2}$ ≈ 0.549360
I_{N1}	9	8	0.055	0.079	0.0075
I_{G1}	8	8	0.027	0.038	0.0078
I_{N8}	9	1	$5.5 \cdot 10^{-9}$	0.0022	0.25
I_{G8}	8	1	$8.9 \cdot 10^{-15}$	$7.2 \cdot 10^{-6}$	0.041
I_{N1}	17	16	0.014	0.020	$1.3 \cdot 10^{-4}$
I_{G1}	16	16	0.0069	0.010	$4.2 \cdot 10^{-5}$
I_{N8}	17	2	$6.0 \cdot 10^{-12}$	$5.8 \cdot 10^{-5}$	0.001
I_{G8}	16	2	$7.1 \cdot 10^{-15}$	$2.2 \cdot 10^{-8}$	$3.0 \cdot 10^{-5}$

7 Results of the pedagogical experiment and discussion

In order to confirm the idea of using learning research with DSS in the study of numerical methods, a pedagogical experiment was carried out on the basis of H. S. Skovoroda Kharkiv National Pedagogical University in the process of teaching the discipline "Methods of Computing". The sample population was created by students of the III-IV year of the Physics and Mathematics Department. As a result of this experiment, there was fixed in 1999 that educational discipline "Methods of Computing" became more significant, more informative and interesting, it became possible to expand the set of methods that are considered. Students' activity and motivation increased, it was increased completeness of knowledge, depth of knowledge, research competency. Statistical analysis of obtained data showed advisability of implementation of learning research in training process [14].

To obtain new data we design the test of 57 items for yes/no answers for checking the completeness of knowledge in function approximation. The problems of this test items were reproductive, so we estimate the student's completeness of knowledge as

the ratio of the correct answers number to the number of item in the test. All students (11 future teachers of informatics) show the completeness of knowledge above 60%. But some test items where correctly answered by all students, these items cannot be used for pedagogical measurements. We also extract the items with not satisfactory discrimination and obtain the test of 18 items for measurement of learning achievements in the approximation (Table 4). According to the results, its reliability was estimated with Cronbach alpha as $\alpha=0.91$. To check the depth of knowledge, the students were suggested with two problems for full answer: 1) to characterize the influence of global interpolation polynomial order on the interpolation error; 2) to characterize the conditions of using the global polynomial interpolation, piecewise polynomial interpolation, spline interpolation and least square approximation. Depth of knowledge was estimated as a part of essential conditions of expedient application of interpolation formulas that the student shows in the answer. The results are shown in Fig. 4.

Table 4. Test Items Specification

Test Item	Correct answer	Index of difficulty	Correlation with the test score
Newton interpolation formula is more convenient for calculating the values of a function given by a table	Yes	0,91	0,28
Spline gives a piecewise-continuous polynomial of the second degree	No	0,91	0,21
Spline is a cubic polynomial between each pore of neighboring nodes	Yes	0,91	0,21
The replacement of the function by an interpolation polynomial is used for numerical differentiation	Yes	0,82	0,37
The approximating function for the least squares approximation is chosen so that the sum of the deviations of the function from the given points should be less than the specified EPS	No	0,82	0,66
The approximating function for the least squares approximation is chosen so that the sum of the squares of the deviations of the function from the given points is minimal	Yes	0,82	0,52
In interpolation nodes, the value of the interpolation polynomial and the value of the interpolated function differ in absolute value no more than a given EPS	No	0,73	0,75
Newton interpolation formula provides less interpolation error than the Lagrange formula	No	0,73	0,75
The approximating function for the least squares approximation is chosen so that it passes through all given points	No	0,73	0,71
The approximating function for the least squares approximation is chosen so that the largest deviation of the function from the given points is less than the given EPS	No	0,73	0,42

Test Item	Correct answer	Index of difficulty	Correlation with the test score
In the case of approximation by the method of least squares, the number of given experimental values should be equal to the number of parameters of the approximation function	No	0,73	0,58
Interpolation formulas of Lagrange and Newton define different interpolation polynomials	No	0,64	0,67
Spline uses global interpolation	No	0,64	0,59
In the case of approximation by the least squares method, the more values are given, the better the approximating function describes the experimental dependence	Yes	0,64	0,79
In the interpolation nodes, the values of the polynomial must coincide with the values of the function	Yes	0,55	0,92
The formulas of Lagrange and Newton give the same polynomial	Yes	0,36	0,86
The mechanical analog of spline is a flexible rod	Yes	0,36	0,86
Coincidence of the values of the source and the approximating functions at some points is not obligatory in the case of approximation of the function	Yes	0,27	0,73
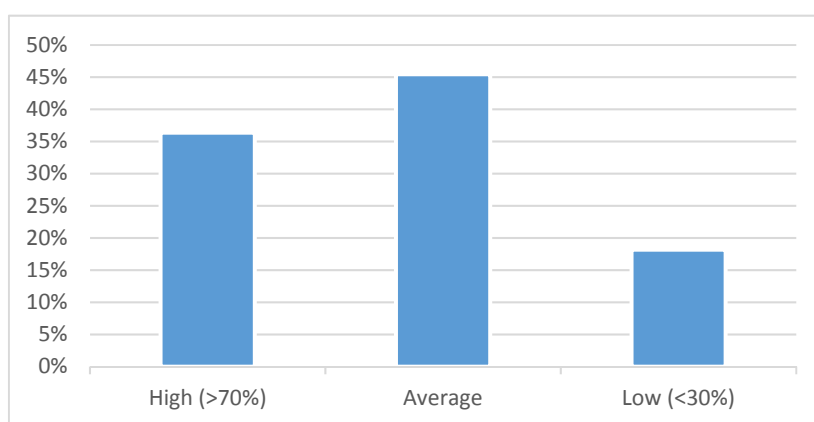
Fig. 4. Percent of students with high, average and low depth of knowledge as a result of testing

Based on many years of experience in the implementation of learning research using a professional computer-based environments, pedagogical conditions of the effectiveness of such method of teaching are formulated: student's formation of the necessary knowledge and skills at the reproductive level; the formation of basic research skills; possession of modern ICT tools for research in a particular subject area; presence of motivation, emotional and volitional attitude to such activity; existence of a multilevel, individualized system of management of students' independent work.

8 Conclusions

Method of learning research is used in special laboratory practice with DSS (dynamic support synopsis – some models of numerical methods) for studying numerical methods of polynomial interpolation and numerical integration.

Pedagogical observations show that suggested methods work towards forming students' motivation and research competency. Students' completeness and depth of knowledge are on enough level according to results of testing and observations during the practical work.

Positive influence of suggested methods on educational results is possible only with developed methodology of investigations, appropriate didactic materials, individualized management of students' independent work and enough level of students' readiness to research activity: satisfactory knowledge and skills at the reproductive level; presence of basic research skills; ability to use modern ICT tools for research in a particular subject area; presence of motivation, emotional and volitional attitude to research activity.

We see prospects for further work in designing the methods of detailed diagnostic of the readiness of the student to learning research and approbation of new methods of individualized management of students' independent work.

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