Parametric Virtual Design-based Multi-Objective Optimization for Sustainable Building Design

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Abstract. With the development of Building Information Modelling aiming for automatic, the automating of sustainability analysis will be a certain requirement in the future. Unlike the current research stream, this paper investigates a novel approach of directly linking parametric architectural models to sustainability optimization through an automatic design-through analysis workflow with the support of parametric virtual design techniques. Data required for optimization was directly extracted from architectural models and then being evaluated. After the optimization, architectural models are automatically updated with optimal parameters through parametric virtual modelling steps. A case study was carried out with optimization of daylight and energy performance for a residential building. The result demonstrates the advantages of directly using architectural models instead of energy models and the possibility of further development.

1. Introduction

With the increasing demand for sustainable building design, a practical and complete system of sustainability analysis has been gradually developed by the research community over the past years as an important step in the Building Information Modeling (BIM) design loop. Useful analysis tools have been developed and put in practice in commonly-used industrial design software such as Autodesk Insight® (AUTODESK. Autodesk Insight, 2019) for Revit (AUTODESK. Autodesk Revit, 2019), EnergyPlus® (EnergyPlus, 2019), and Autodesk Green Building Studio® (GBS) (AUTODESK. Autodesk Green Building Studio, 2019), also known as DOE-2. However, these analysis tools often involve a large amount of manual work in modeling, remodeling and adjustment. Moreover, to run sustainability analysis, two types of information are required, which are building geometry and material properties. Except for some energy analysis which could be done in a well-developed analyzing system with proper software environment (Asl, et al., 2015), common energy analyzing tools allow designers to input these two types of information separately with no constraints which could have potential risks. For example, solar and energy analysis in Autodesk Insight® is based on energy models created from conceptual masses providing basic geometry information and typed-in energy settings providing other remaining information. Where Autodesk® has recommended three common approaches for automatically generating energy models from architectural models (AUTODESK. Autodesk Revit, 2019), but the approaches are not exactly automatic. In the early design stages, where the geometries of architectural models are simple, box-shaped conceptual masses are manually created representing the geometries of target buildings and used for analysis. When the building models become more complex with large amounts of architectural elements, the energy model can only be created with a large amount of manual work to prepare the architectural model. After analysis, the effects of the building parameters on the total sustainability performance of the building will be provided individually. Then, the engineer has to make several attempts to determine an optimal solution and manually update the original model. However, this is where potential risks exist, for example, that conflicting parameters are selected for one architectural element, or the theoretically optimal result (e.g. high glass-to-wall ratio) is unrealistic or does not fit into the specific project. These two shortcomings, before and after analysis, demonstrate the limit of manual sustainability analysis. Sustainability analysis and optimization should ideally be conducted in early design stages. In later design stages, both the working load of preparing analytical model, and the impact scale of parameter change due to sustainability optimization, is significantly increased.

In recent years, many researchers have investigated different automatic approaches for sustainability designs (Wang, et al., 2006; Tuhus-Dubrow & Krarti, 2010; Evins, 2013; Hachem, et al., 2013). However, the current state-of-the-art in this field has significant limitations for practical design tasks. Most of the studies have been performed based on conceptual energy models that cannot directly be used for architectural design in the BIM design process, or based on architectural models with a limited set of parameters, which are too simple to provide critical ideas for the building's initial design. For example, Asl, et al. (2015) optimized the daylight and thermal performances of a manually predesigned building in Revit with only changing window size and window type as parameters. Backer (2017) optimized the Daylight Factor, Solar Heat Gains and Thermal Energy Losses with varying room depth, window size and building orientation for a highly simplified L-shape building. The research outcomes of these papers are therefore only a step towards parametric optimization of sustainability, which is however not detailed enough for informing an initial practical design.

In conclusion, both the official manual optimization method provided by Autodesk® and the automatic methods being explored in recent research are useful but not efficient or automated enough considering the development of higher maturity level BIM in the future. In real-design conditions, it is more reasonable to directly extract the building geometry and construction information from architectural models and use them for analysis. After optimization, the model can be automatically updated, with the optimal parameters acquired from calculation. However, since BIM has grown out of the design tools for interactive object-based parametric design (Eastman, et al., 2008), and recently developed tools enable the combination of the BIM object-based concept with parametric modelling techniques by also embedding some intelligence in the relationship between objects (Boeykens, 2012). This could be used to bridge the limitations in sustainable building optimization. Therefore, in this paper, we attempt to overcome the barrier of current software and implementation by using parametric virtual design technique. Instead of using conceptual masses to evaluate the building, by combining parametric virtual design with optimization techniques, the feasibility and efficiency of linking multi-objective sustainable analysis directly to an architectural model is investigated.

2. Methodology

In this paper, we propose an approach for the sustainable building design loop by extracting information from an architectural model, running automated optimisation of building sustainability in terms of daylight and energy performance, and updating the original model using parametric virtual design technique. The performance of this approach is explored with a predefined case study of a target building. The methodology includes parametric virtual design, energy performance assessment, sensitivity analysis and optimisation. To cover the relevant areas in the initial sustainability design, parameters of both building geometry and construction materials are considered. Prior to optimization, a sensitivity analysis is performed to decrease the range of parameters, which helps to accelerate the convergence. Then, during the optimization, similar to Generative Design (Zarzycki, 2012), possible solutions are investigated using an optimization algorithm within predefined ranges, and a range of optimal solutions is obtained through an iterative process. Then, the results are automatically applied with a parametric virtual design technique to generate an example of an optimal building model.

2.1 Linking Architectural Model and Sustainability Analysis

An obvious disadvantage of using architectural models in optimization tasks is that standard design software currently spends too much time in automatically regenerating models with large amounts of architectural elements. However, two alternative approaches can be explored to avoid this shortcoming. The first approach is by applying parametric virtual design, where a conceptual geometry model (e.g. surfaces created in Dynamo) is constructed following the same algorithm as the architectural model. In this case, these two models could change synchronously with the variation of parameters. The conceptual geometry model can be applied in the optimization process without generating an actual design model. After optimization, the same optimal architectural model can be built by directly applying the optimal parameters. This method works quickly with simple calculations, but appears to be insufficient for the analysis software, which require a large amount of specific data to finish the calculation. The second approach is to use an architectural model during the optimization, but only for the critical part instead of the whole model. This means that only instances described with the parameters relevant for the analysis will be considered, while other excess elements which are not influencing the result or not worth studying will be neglected. An example for selecting the critical parts of the model is demonstrated in the case study. This approach can be adopted for different types of buildings, such as residential and office buildings, where the most important rooms for sustainability design are normally identical, with a large amount of repetition inside the building. In this case, only simplified models appear in the optimization with limited possible variations. In this research, the second approach was explored because it is more suitable for practical applications and can integrate with 3rd-party analysis tools.

2.2 Optimization Algorithm

The optimization algorithm adopted in this research is a Genetic Algorithm, specifically the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) (Deb, et al., 2002). This is a powerful multi-objective optimization tool for environmental design (Kheiri, 2018). The function of Genetic Algorithm is an improved version of Evolutionary Algorithms. As shown in Figure 1, an initial list of samples is randomly created between predefined lower and upper bounds, then a Fitness Function is used to calculate the series of initial objectives corresponding to the variables. The Fitness Function in this research is a script including steps of extracting geometry and material properties, evaluating daylight and energy performance, and exporting results for further data processing. Then, initial variables and objectives are grouped into a list and regarded as the initial generation, which is then input into a while loop for optimization. Inside the loop, new parameter values are created after crossover and mutation. The Fitness Function is applied again for generating the new generation and continued until the iteration number approaches a predefined boundary. A special advantage for NSGA-II is that the elitism samples among each child generation are compared with the last elitism before the creation of the next generation, so that the variables in the next generation will automatically be close to the value of the last elitism (Kheiri, 2018). In this case, the convergence is accelerated as the impact of useless samples among a generation is significantly decreased. The result of optimization is plotted and the best trade-off between the thermal and daylight performance is found. This is followed by a further weighted-sum optimization to optimize the Normalized Average Objective (NAO) (Qu & Suganthan, 2010) calculated from the two performances. After this, the difference between each sample in the Pareto front is compared with the optimal NAO value to select the final best result.

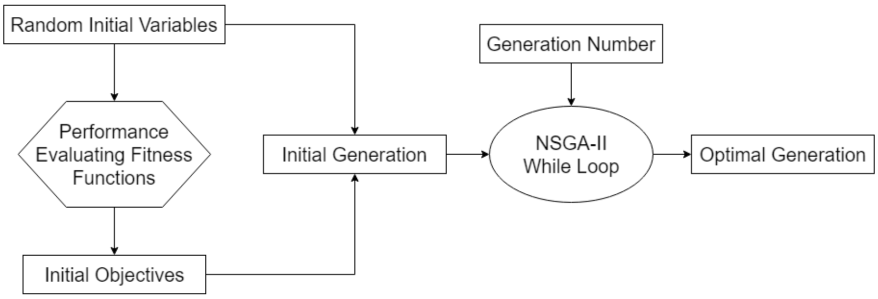

Figure 1: Workflow of NSGA-II

2.3 Daylight Simulation

The daylight simulation in this paper is evaluated by estimating the in-door lux values (lx). For the design purpose, this value needs to be high, to reduce the electricity costs spent by the lighting system. Professional 3rd-party analysing software has been used for this step. In this research, the climate environment data for location in London is used. Building properties include glass-to-wall ratio, façade orientation, glass transmittance and opaque material surface properties such as roughness. In order to be compared with energy performance, the objective Daylight Factor (DF), which is calculated based on the average in-door lux value, is introduced to the optimization system as given in the following equation:

$$DF = \frac{0.01}{n}\sum_{i=1}^{n} lx \qquad 2.3\text{-}1$$

Where:

n is the total amount of individual lx values.

2.4 Energy Analysis

In this research, the Solar Heat Gains (SHG) through glassing areas in summer and Transmission Heat Losses (THL) in winter are calculated. For the design purpose, the value of these factors needs to be lower, in order to reduce the energy consumption. SHG gives the total solar heat absorbed by the walls and transmitted by the glass (Baker, 2017).

$$H_{SHG} = \frac{A_{glass}}{A_{room}} I_t \left(\tau + \frac{\alpha U}{h_0}\right) \ [W/m^2] \qquad 2.4\text{-}1$$

Where:

I_t is the total radiation;

τ is the transmittivity of glass extracted from Revit;

α is the absorptivity of glass defined;

U is the overall heat transfer coefficient for glass extracted from Revit $W/(m^2K)$;

h_0 is the external heat transfer coefficient set as constant in this case study;

The total radiation can be calculated from direct radiation, diffuse radiation and reflected radiation.

$$I_t = I_{DN}\cos[180° - (\gamma + \zeta)] \cdot \cos(\frac{\pi}{2} - |I - d|) + I_d + I_r \qquad 2.4\text{-}2$$

$$I_{DN} = 1080 \times \exp\left[-\frac{0.21}{sin(\frac{\pi}{2} - |I - d|)}\right] \qquad 2.4\text{-}3$$

$$I_d = 0.135 I_{DN} \frac{1 + cos\Sigma}{2} \qquad 2.4\text{-}4$$

$$I_r = (I_{DN} + I_d)\rho_g \frac{1 - cos\Sigma}{2} \qquad 2.4\text{-}5$$

Where:

I is the latitude angle at location;

d is the declination on the selected dates;

Σ is the tilt angle of surface (90° for vertical walls);

ξ is the wall azimuth angle;

ρ_g is the wall reflectivity;

Thermal Losses (TL) is the thermal energy diffused out of the building in winter when the environment temperature is lower than the room temperature, in which the Transmission Heat Losses (THL) are the most direct way. THL gives the heat directly transferred outside the building through the building envelope, including walls and glass:

$$H_{THL} = \sum_{i=1}^{n} \frac{(A_{(wall/glass),i} \cdot U_{wall})(T_{in} - T_{out})(1 + \frac{\alpha}{100})}{A_{floor,i}} \qquad 2.4\text{-}6$$

Where:

U is the heat transfer coefficient extracted from wall properties in Revit W/(m^{2}K);

T_{in} is the temperature in room taken as 20 °C;

T_{out} is the temperature outside the room taken as 0 °C in winter;

α is the proportion of thermal bridges %;

Then, a normalised Thermal Factor (TF) is calculated following the general structure presented below to make a comparison with DF. After finding the pareto front between DF and TF, the final NAO could be calculated as shown:

$$N = a_1(x_1)^{b_1} \pm a_2(x_2)^{b_2} \pm a_3(x_3)^{b_3} \pm \cdots \pm a_n(x_n)^{b_n} \qquad 2.4\text{-}7$$

$$TF = \frac{12.5}{H_{THL}} + \frac{50}{H_{SHG}} \qquad 2.4\text{-}8$$

$$NAO = \frac{1}{TF} + \frac{1}{DF} \qquad 2.4\text{-}9$$

2.5 Sensitivity Analysis

A sensitivity analysis has been carried out before the optimization is carried out in order to find the individual impact of each parameter on the final sustainable performance. During the sensitivity analysis, the target parameter is varied, while other parameters are kept constant. By doing this, the range of each parameter between predefined upper and lower bond can be reduced to a reasonable value based on the evaluated results. Since the overall range of parameters is decreased, the required number of generations for NSGA-II to reach the same optimal result is significantly reduced. As a result, the convergence of optimization is accelerated. Moreover, by comparing the resultant range of different parameters, the most influential one can be found. In conclusion, it is efficient to run a sensitivity analysis before the optimization. However, it needs to be considered that the total time cost of sensitivity analysis is significantly increased with an increasing number of parameters in the optimization task.

2.6 Parametric Virtual Design

After getting the optimal range of parameters from sustainability optimization, values are chosen and input to the script for generating building models with a parametric virtual design tool. The building model is generated based on the coordinates of several control points. An algorithm for positioning these points was developed such that the building profile is changed by adjusting predefined geometrical parameters, for example, rotation of several control points according to the orientation of one façade. Construction properties such as the insulation type and the glass type are controlled by the family type parameters of the architectural elements. Hence, the whole building, including geometric and construction parameters, is generated in a fully automatic way, by importing parametrized family instances onto a parametrically generated layout.

3. Implementation

As shown in Figure 2, following the general methodology briefly described in the previous section, the parametric virtual design and multi-objective optimization is carried out using Autodesk Revit® and its visual-programming plug-in Dynamo (Dynamo, 2019). With the support of open-source packages in Dynamo, the optimization task with the Genetic Algorithm is processed with the built-in package Optimo (Asl, et al., 2015) containing NSGA-II and the daylight performance is analyzed by the package Honeybee included in the Ladybug Tools (Mackey & Roudsari, 2017).

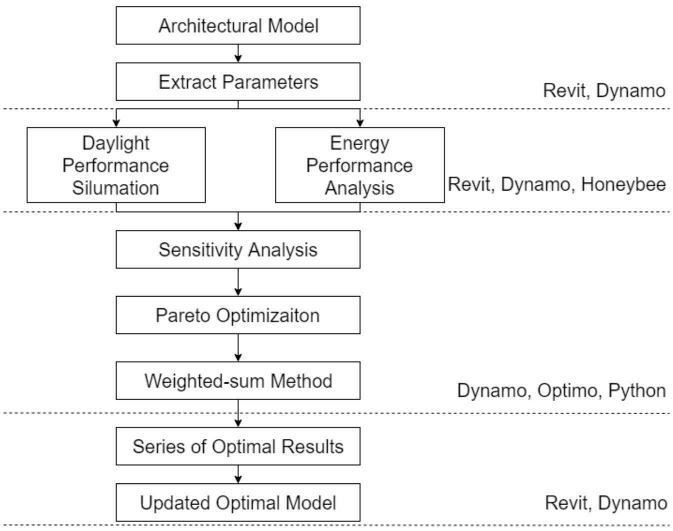

Figure 2: Workflow of the general research method

4. Case Study

A case study was carried out to investigate the feasibility and efficiency of applying sustainable optimization directly with an architectural model. The target building is a standard residential building with 12 floors as shown in Figure 3.

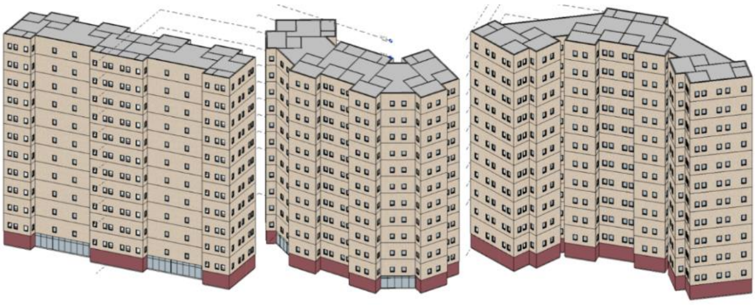

Figure 3: Visualisation of target building in different geometry

As shown in Figure 4, following the general parametric virtual design method described in previous section, the specific architectural model in this case study is generated. The algorithm performs the following steps: i) generation of initial control points and lines for one section; ii) and iii) rotations and symmetry operations for other sections; iv) generation of the reference lines and points for the architectural elements; v) generation of the initial architectural layer; and, vi) extraction of critical sections and generation of whole building.

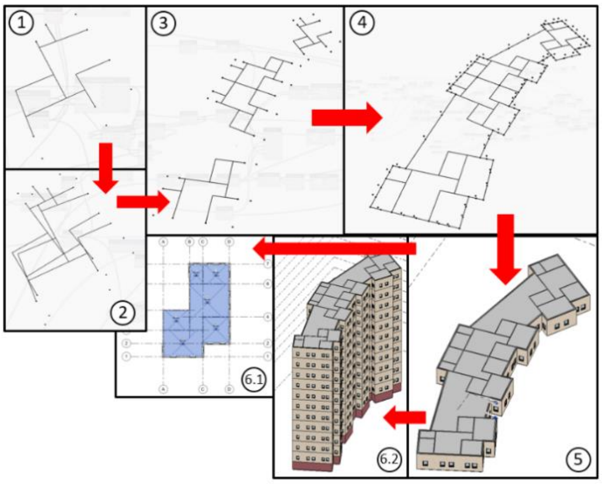

Figure 4: Demonstration of parametric virtual design process

As shown in Figure 5, the profile of each floor is identical and includes three critical parts: west, middle and east. They are three residential areas in this building that are also the focus areas in this case study. Daylight and energy consumption in the remaining areas, such as stair cases and corridors, are not considered because normally no heating/cooling would be provided, and natural lighting is not normally required in these areas.

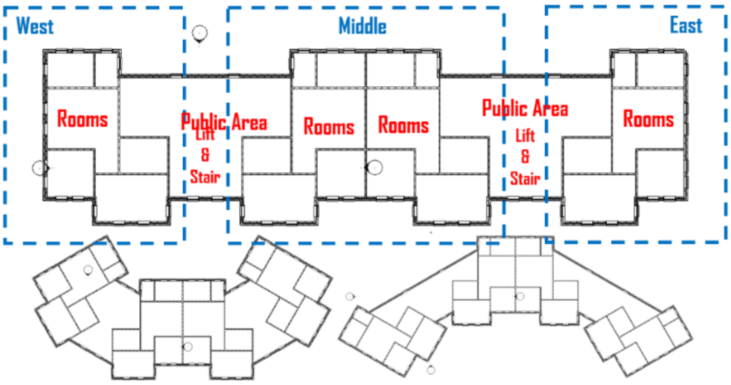

Figure 5: Target building floor plan with shape change

As mentioned in the methodology, the critical parts are then selected as the target model for optimization. The parameters for optimization in the middle critical part are shown in Table 1. The discrete variables are default settings for architectural elements and materials provided by the software. The continuous ones, are independently defined values, which are required for the sustainability performance evaluation but not provided by software. Here, the size of the windows controls the glass-to-wall ratio. Moreover, the angle to project north is used to represent the façade orientation. By doing this, we avoid the rotation of the model during the optimization process.

Table 1: Initial, filtered and final ranges for the middle critical section-Run 1

<table>
<tr><th colspan="3" rowspan="2">Parameters</th><th colspan="2">Initial Ranges</th><th colspan="2">Filtered Ranges</th><th colspan="2">Final Ranges</th></tr>
<tr><th>Lower Bond</th><th>Upper Bond</th><th>Lower Bond</th><th>Upper Bond</th><th>Lower Bond</th><th>Upper Bond</th></tr>
<tr><td rowspan="5">Continuous Variable</td><td colspan="2">Angle to Project North (°)</td><td>-60</td><td>60</td><td>-20</td><td>20</td><td>-9</td><td>8</td></tr>
<tr><td colspan="2">Glass Absorptivity</td><td>0.3</td><td>0.86</td><td>0.45</td><td>0.71</td><td>0.081</td><td>0.089</td></tr>
<tr><td colspan="2">Wall Surface Roughness</td><td>0</td><td>0.2</td><td>0.04</td><td>0.10</td><td>0.045</td><td>0.062</td></tr>
<tr><td colspan="2">Wall Surface Specularity</td><td>0</td><td>0.1</td><td>0.03</td><td>0.09</td><td>0.087</td><td>0.089</td></tr>
<tr><td colspan="2">Wall Surface Reflectivity</td><td>0</td><td>1.0</td><td>0.35</td><td>0.85</td><td>0.687</td><td>0.723</td></tr>
<tr><td rowspan="11">Discrete Variable</td><td colspan="2">Glass Transmittance</td><td>0.3</td><td>0.9</td><td>0.37</td><td>0.9</td><td>0.57</td><td>0.61</td></tr>
<tr><td colspan="2">Wall Heat Transfer Coefficient</td><td>0.0876</td><td>0.1802</td><td>0.0876</td><td>0.1802</td><td colspan="2">0.1056</td></tr>
<tr><td colspan="2">Glass Heat Transfer Coefficient</td><td>1.9873</td><td>6.7018</td><td>0.39</td><td>0.81</td><td>0.36</td><td>0.48</td></tr>
<tr><td rowspan="2">North Window</td><td>Width (mm)</td><td>406</td><td>915</td><td>406</td><td>915</td><td colspan="2">610</td></tr>
<tr><td>Height(mm)</td><td>610</td><td>1220</td><td>610</td><td>1220</td><td colspan="2">915</td></tr>
<tr><td rowspan="2">South Window</td><td>Width (mm)</td><td>406</td><td>915</td><td>406</td><td>915</td><td colspan="2">610</td></tr>
<tr><td>Height(mm)</td><td>610</td><td>1220</td><td>610</td><td>1220</td><td colspan="2">1220</td></tr>
<tr><td rowspan="2">East Window</td><td>Width (mm)</td><td>406</td><td>915</td><td>406</td><td>915</td><td colspan="2">610</td></tr>
<tr><td>Height(mm)</td><td>610</td><td>1220</td><td>610</td><td>1220</td><td colspan="2">915</td></tr>
<tr><td rowspan="2">West Window</td><td>Width (mm)</td><td>406</td><td>915</td><td>406</td><td>915</td><td colspan="2">610</td></tr>
<tr><td>Height(mm)</td><td>610</td><td>1220</td><td>610</td><td>1220</td><td colspan="2">915</td></tr>
</table>

With these initial range of parameters, a sensitivity analysis was performed for each of the critical building parts. A filtered range of parameters has been obtained and applied to the final optimization to get the final range as also shown in Table 1. It can be observed that the optimal results for discrete variables such as glass transmittance and glass heat transfer coefficient are reasonable as the balanced point of higher daylight quality and lower SHG and TL are achieved. However, some of the continuous variables such as wall surface specularity directly converged to the upper bound, which seems unreasonable. This is because the discrete variables are all defined by the type of architectural elements, so they work like passive variables and cannot change independently. In this case, for the glass transmittance reaching the upper bound, the glass heat transfer coefficient reaching the lower bound would be the theoretically best solution. However, the type of glass satisfying these two values does not exist in practice. This is one strong advantage of directly extracting information from architectural models employing real objects, so this unrealistic scenario can be avoided. However, the obtained results for independent continuous variables were relatively valuable, since they are not limited by the practical condition. The error mentioned above could happen if all the parameters are simply used as semantic values to run an analysis without connecting them with real physical objects. The drawback of this result actually proved the advantage of directly extracting information from architectural models.

Finally, the values of objectives during convergence are plotted as DF versus TF and the optimal trade-off between these two objectives has been found. After applying a further weighted-sum optimization, figures of NAO for three critical parts are plotted. One example of the running result for the middle part of the building is shown in Figure 6.

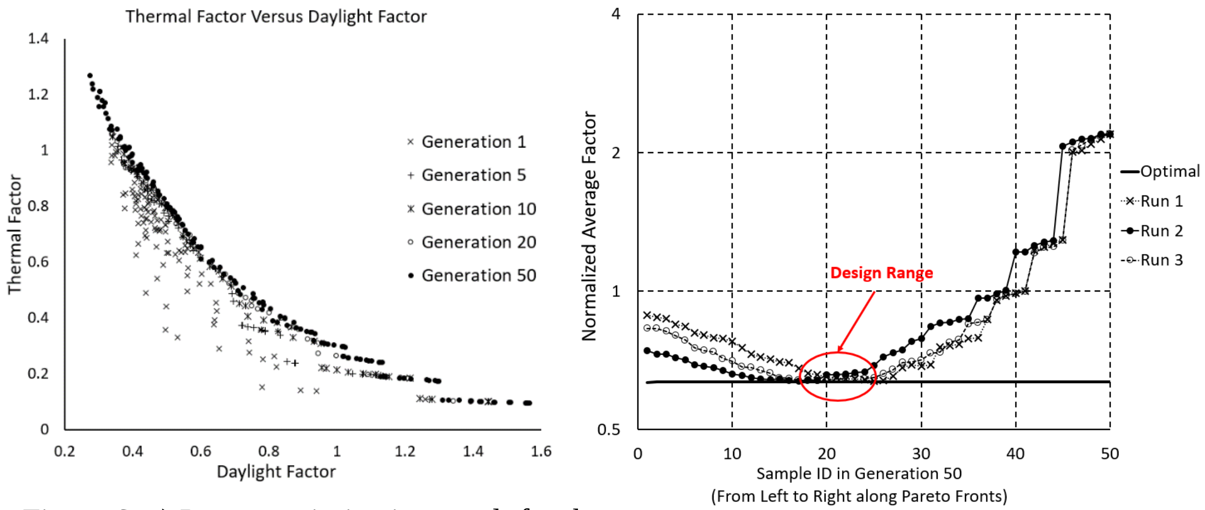

Figure 6: a) Pareto optimization result for the middle critical section-Run 1. b) Normalized optimal results for the critical middle section for 3 runs

After getting the optimal range of parameters, one combination of parameters is selected and input into the custom node for generating an optimal building.

5. Conclusion

In this paper, we proposed a design loop for multi-objective optimization of building sustainability based on the parametric virtual design approach. The methodology of this concept includes setting an automatic link between architectural models and sustainability assessment tools, optimization of sustainable performance based on the genetic algorithm and finally an automatic update of the architectural model with the determined optimal parameters. The case study presented in this paper, demonstrated the potential of the proposed methods to perform a fully automated design-through-analysis-optimization workflow. Furthermore, from the presented case study, the behavior of passive and independent variables revealed the potential risk obtaining unrealistic results when extracting the geometry and semantics from non-related sources, if the input variables are not carefully constrained. Often these constraining steps require professional knowledge and experience in environmental engineering. However, this problem could be solved by developing a method or a tool for direct use of all the information (geometry and semantics) from a single architectural project. By doing this, all the parameters, which are passive variables, are defined by a real physical property of the construction material and the element size. Even though there are limitations in the existing practical implementations, this research area has a large potential for further improvement. Firstly, the enrichment of the environmental database of the standard design tools considering the requirements of different analysis tools, will allow for more user-friendly and efficient analysis. Secondly, the programming environment associated with these tools (open-source and developer-friendly) offers the advantage for exploration of different optimization strategies, since they show different performance for different optimization problems and conditions (Kheiri, 2018). Finally, the strong functionality of parametric virtual design techniques could significantly decrease the workload for the manual update of the design models. However, developing a general algorithm for most common geometrical transformations of building forms that are compatible with the optimization tools, poses a significant challenge for future research. In conclusion, the design approach proposed in this paper has demonstrated that an automated design optimization loop based on a real design model is feasible and also offers a huge space for further developments.

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