

# Multiple Revision in Description Logics

Fillipe Manoel Xavier Resina<sup>\*[0000-0002-8753-3621]</sup>

University of São Paulo, São Paulo - SP, Brazil  
fmresina@ime.usp.br

**Abstract.** The AGM paradigm is the most used framework in Belief Revision. Among its various operations, a revision occurs when a rational agent receives a new information inconsistent with its epistemic state and has to revise it in order to accommodate the new belief in a consistent way. However, this new information may come as a set of beliefs (instead of a single one), a problem known as Multiple Revision. There may be two approaches to this operation: to accommodate the whole new set of beliefs or to incorporate only a subset of it. In the second case, the agent has to be able to determine which part of the set will be added to its state. In this project, we are particularly interested in this last case. In addition to that, the agent may deal with different kinds of belief. Thus, the purpose of this research plan is to define how to operate a multiple revision and, more specifically, how to do this in non-classical logics.

**Keywords:** belief revision · multiple revision · choice revision · description logics · ontology · ontology debugging.

## 1 Introduction

The problem of Knowledge Representation is one of the central areas in Artificial Intelligence. With the advancement of technology, it became clear that we need to enable computers to efficiently represent information regarding a domain in order to be able to proceed with other tasks. In this context, ontologies came out as a valuable option. Among the formalisms that were coined to represent ontologies formally we have Description Logics (DLs)[2]. In addition, DLs form the basis for OWL (*Web Ontology Language*), which is recommended since 2004 by W3C as the standard language to represent ontologies on the Web.

However, according to Gärdenfors[10], it is not very useful to know how to represent knowledge if at the same time we do not know how to change it when we receive new information. The motivation of this idea is that knowledge is not static, what means that we need to be able to deal with its dynamics. That is

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the context of the studies in the area of Belief Revision, which aims to handle the problem of adding or removing new information to a knowledge base in a consistent way. Most of the literature about Belief Revision is based on the AGM paradigm[1].

In the AGM paradigm, given a set of beliefs, there are three possible changes in relation to a new belief: expansion, contraction and revision. Expansion occurs when the base simply absorbs the information without loss. A contraction consists in retracting beliefs from the base until the specified information is not derivable from it anymore. Finally, revision happens when the new belief is added in a consistent way, possibly demanding a repair in order to eliminate inconsistency. In this project, we are particularly interested in this last operation.

In the original framework, the new belief is assumed to be represented by a single formula. Nevertheless, there are situations in which the information by which we are going to revise a set of beliefs comes in block. This generalized operation is called Multiple Belief Revision. It is not the same as applying revision in an iterated way, since in Multiple Revision it is assumed that there is no preference over the input sentences, which means that all of them have equal priority and should be processed at the same time.

However, when you consider the described scenario, there may be two ways to achieve the goal. On the one hand the intention may be to absorb the new sentences completely, which is called *package*. On the other hand, it may be enough to incorporate a subset of the new sentences, which is called *choice*. In this research project we are going to explore this last option in more detail.

From [3], we can visualize a practical scenario for choice revision. Suppose you have a multi-agent system where there are some agents and also a main agent as a coordinator. This last one receives information from the others and has to process it in order to take some decisions. If there is a credibility degree associated to each agent, for example, the coordinator may decide to take into account only the information coming from the most reliable ones, characterizing a kind of multiple version of Selective Revision[4]. On the other hand, it is possible to have some agents with the same (or similar) reliability while you are not sure about the impact of absorbing all the information given by them. In this case, it would be interesting for the coordinator to process the new beliefs in some way before performing a decision. Then it is important to define desirable properties and constructions associated to them that will be able to guide this process.

Combining this and the need to change ontologies, we investigate in this work how to apply Belief Revision in a multiple context, especially for Description Logics, with a particular attention to OWL ontologies.

This proposal is structured as follows. In the next section, we provide a background in Belief Revision and Ontologies. In Section 3, we detail the goals, objectives and methodology of this proposal. Finally, in Section 4 we organize the tasks according to the remaining semesters.

## 2 Background

### 2.1 Belief Revision

In the AGM paradigm, the revision operation is defined from contraction via the *Levi Identity*[10]. Let  $K$  be the belief set of an agent and  $\alpha$  an incoming belief. The referred identity defines the revision operation in the following way:  $K * \alpha = (K - \neg\alpha) + \alpha$ , where  $-$  stands for contraction and  $+$  for expansion.

Besides establishing properties which the operations should obey (known as postulates), constructions were elaborated to give means of executing the desired procedures. Among them, we highlight two: Partial Meet[1] and Kernel[11]. As revision used to be considered just a compound operation, the Partial Meet and Kernel versions of revision were given by just using the corresponding version of contraction in the Levi Identity.

Within the Belief Change area, the operations, properties and constructions were initially defined thinking of the initial beliefs as being a belief set, i.e., a set of beliefs closed under logical consequence. However, since sets conceived this way are usually infinite, there are some disadvantages with this approach, especially for computational purposes. Considering this problem, instead of working with belief sets, some authors began to work also with belief bases, which are knowledge bases not necessarily closed. For practical purposes in change operations it is much more convenient and is of particular interest for us.

### 2.2 Multiple Change and Multiple Revision

In [7] we have a first picture of a change operation that is not necessarily by a single input. The author claims that sometimes we need to withdraw more than one proposition of a belief set at the same time, proposing the name *Multiple Contraction* for this case. Following [7], we also see the discussion of some properties of revision operations that receive as input more than one sentence simultaneously, named *Multiple Revision*. Analogously to Multiple Contraction, when the result of  $K * A$  should contain some elements (but not necessarily all) of  $A$  ( $Cn(K * A) \cap A \neq \emptyset$ ), then we have (*multiple*) *choice revision* (here denoted by  $*_c$ ), while when the result of  $K * A$  should imply everything in  $A$  ( $A \subseteq Cn(K * A)$ ) we have (*multiple*) *package revision* (denoted by  $*_p$ ).

Now, the package revision operation can be defined via a generalized version of the Levi Identity using package contraction:  $K *_p A = (K -_p \neg A) + A$ , where  $\neg A = \{\neg\alpha : \alpha \in A\}$ . By this approach, the properties of a package revision operation become dictated by the ones of the package contraction operation. Nevertheless, considering that some logics lack negation, it would be interesting to study the definition of revision operations in a direct way, i.e., without using contraction as an intermediate step. This is one of the main goals of the work developed in [3], which is focused on package revision.

Still about the generalized Levi Identity, we have in [8] a further exploration of the topics initiated in [7]. Fuhrmann addresses the issue focusing on the *package*

variety and the Partial Meet construction. He also shows how the operations of revision and contraction can be interdefinable.

In multiple operations, one of the questions that emerge is whether it is possible to reduce them to operations by singletons. For the revision case, the following result for finite sets was obtained in [8]: for every finite set  $A$ ,  $K * A = K * \bigwedge A$ . It means that in the finite case it is possible to reduce a multiple revision to a revision by the conjunction of all the elements of the input. However, this identity works only for languages that allow logic conjunction of sentences, which is not the case of Description Logics and some other non-classical logics.

In [3], the authors defined some postulates for a direct operation, most of which were adapted from postulates for singleton revision. By generalizing the techniques from classical belief base revision, the authors defined two kinds of construction: Package Kernel Revision and Package Partial Meet Revision<sup>1</sup>.

In [15], the author explores choice revision (a non-prioritized multiple revision). This means that incoming beliefs do not have total priority with respect to the initial beliefs. Before defining the operation, the author introduces Partial Expansion (denoted by  $\dot{+}$ ), which is a generalization of the traditional expansion operation. Roughly speaking, given a belief set  $K$  and an input set  $A$ , the result of a partial expansion of  $K$  by  $A$  is formed by the union of  $K$  with a subset of  $A$ . Using the definition of negation set of a set of sentences  $A$  ( $\neg A$ ), the Choice Revision operation is defined in [15] as follows:  $K *_c A = K -_c \neg A \dot{+} A$ . Due to the usage of set-negation to perform the contraction part of the choice operation, this approach depends on negation and on disjunction of sentences, which excludes some interesting logics, such as Description Logics.

### 2.3 Selective Revision

In [4], Fermé and Hansson explored a new possibility of revision which is capable of incorporating only a part of the input belief, calling it selective revision. In order to achieve the partial acceptance, we need to apply a *transformation function*  $f$  to every input sentence  $\alpha$  aiming to extract the “most trustworthy” part of it.

With these tools in mind, it is possible to construct a model for selective revision. Let  $K$  be a belief set,  $*$  a partial meet revision for  $K$  and  $f$  a transformation function. The selective revision  $\circ$ , based on  $*$  and  $f$ , is the operation such that for all sentences  $\alpha$ ,  $K \circ \alpha = K * f(\alpha)$ .

However, it is not suitable neither for belief bases (given that it depends on a postulate of closure) nor for Description Logics (once some properties of the transformation function apply negation, disjunction and conjunction of sentences and DLs are not closed under any of these operators).

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<sup>1</sup> In the original article, Package Revision is called Prioritized Change, in the sense that all the sentences from  $A$  should be present in the new belief base. Then, for the constructions, the names originally given were Prioritized Multiple Kernel Revision and Prioritized Multiple Partial Meet Revision.

## 2.4 Ontologies and Description Logics

An ontology provides a description of individuals, their aspects (attributes) and restrictions, how they can be categorized (classes) and how they interact with each other (relations), in relation to a given domain.

Ontologies can be used in a growing range of applications, such as medical diagnosis, biological and biomedical research. In Artificial Intelligence, we can see ontologies in the modeling of Natural Languages and also in Knowledge Representation, being useful for robotic path planning.

As a main language used for describing ontologies, we can cite OWL (Web Ontology Language), part of the standards defined by the World Wide Web Consortium (W3C) to represent ontologies and organize data<sup>2</sup>.

The theoretical basis both for OWL and for the profiles associated to it are formed by Description Logics (DLs), which are structured subsets of First Order Logic and have a well defined formal semantics.

Due to the dynamics of knowledge, it is important to know how to modify an ontology when there is a need for change (a new information, changes dictated by an ontology engineer, etc). As ontologies are usually huge and intricate structures, this problem is of considerable importance, what gives rise to the ontology change issue[6] (also called ontology evolution) and makes clear the relevance of the Belief Change area. The composite kind of ontology change (changes that are applied at the same time) shows the applicability of Multiple Revision in this field.

## 3 Proposal, Objectives and Method

Our main goal with this project is to propose postulates, constructions, theorems and algorithms that make it possible to perform operations of Multiple Belief Revision, especially in Description Logics. We detail below the specific objectives, pointing, when relevant, what has already been done or is in progress.

In order to develop this research plan, we are going to apply a formal mathematical analysis. To accomplish this, our bases will be previous works on Multiple Contraction, Multiple Revision, Merging and on the problems of the Levi identity in non-classical logics, aiming to analyze and extract mathematical formalisms already developed in the area to conduct our steps.

### 3.1 A survey of the field

Before going deeper in the project, we searched for all the works that had already been developed in the area of Multiple Revision. Along the process, it came to us the need to systematize the theories developed, organizing and comparing them.

Then, we have started to produce a survey on the topic, using as main references the works in [7,8,3,15].

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<sup>2</sup> <https://www.w3.org/standards/semanticweb/>  
<https://www.w3.org/TR/owl2-overview/>

### 3.2 Compatibility with Description Logics and Constructions

Firstly, we want to identify and establish all the limitations of the existing theory for Multiple Belief Revision when it comes do DLs, more specifically through the *choice* approach. After that, we have to define all the postulates needed to make feasible the compatibility between Multiple Revision and DLs.

One of the first steps is to analyze and compare the aspects studied in works such as [13] in order to identify all the limitations that DLs bring to classical AGM theory. From the limitations collected in the previous step, it will be possible to make a comparison with the properties and requirement of what was exposed in [8,15] in order to establish precisely in which points the existing theory of Multiple Belief Revision is not applicable to DLs and needs adaptation.

Using Partial Meet and Kernel approaches, it will be possible to create constructions capable of executing (Multiple) Choice Belief Revision in DLs. With these resources in hand, we will be able to propose and demonstrate representation theorems that prove the validity of the connection between the postulates defined and the constructions created.

Regarding this topic, after studying the limitations, it was possible to identify the incompatibilities presented in [7,8,15], what motivated us to use an approach similar to the one found in [3] in order to propose new postulates for Multiple Choice Revision that are appropriate to Description Logics. We defined as well the corresponding Partial Meet and Kernel constructions to this operation. So, we still need to refine the postulates and finally use them to propose and prove representation theorems.

A valuable resource can be obtained analyzing and comparing the properties and constructions used in the development of the theories about Multiple Belief Contraction[9,5], Multiple Belief Revision in Horn Logic[14] and Merging[8], aiming to extract theoretical and mathematical tools possibly useful to develop the theory of Multiple Belief Revision in DLs.

### 3.3 Selective Revision

When we compare the choice approach of Multiple Revision and the operation of Selective Revision it is possible to identify some similarities. Although this last one has been developed to singleton inputs, it shares with *choice* the property of selecting a part of the incoming information to incorporate. It could be a possible way to perform multiple choice revision. Then, we aim to explore the Selective Revision operation in order to study the possibility of extending it to the multiple case and, more specifically, to Description Logics.

### 3.4 Algorithms

Regarding the practical aspects of the theory to be developed, we plan to develop algorithms that correspond to the constructions studied and proposed, in order to pave the way for implementation of the theory developed. In addition, it will

be possible to compare, in this context, the Partial Meet and Kernel approaches in relation to the computational complexity of the operations.

In order to get closer to what has already been developed in the field, we intend to analyze the algorithms for singleton Belief Revision[13] and for Multiple Contraction[12] aiming to analyze the constructions and properties used and, then, create algorithms that correspond to the desired constructions.

In relation to Package Revision, following the approach presented in [3], we have already developed the algorithms (both for Partial Meet and Kernel) and proved their correctness. So, it remains necessary to develop the ones for Choice Revision, after the correct definition of the corresponding constructions.

### 3.5 Comparison with Merging

Due to the non-prioritized nature of Multiple Choice Revision, i.e., the possibility of partial acceptance in relation to the input beliefs, there may be a correlation between this operation and the merging operators.

Merging is also a type of non-prioritized change that happens when you need to harmonize multiple sources of information. Depending on the entrenchment of the beliefs you already have and on the integrity constraints required to the operation, the input knowledge base may be partly or totally discarded, although there is no precedence established to the different sources, i.e., previous and new beliefs have symmetric roles during the process.

Therefore, we intend to study it more deeply to understand what are the differences between the operations and also the similarities, especially if some properties can be shared. For more details about the operation, its properties and constructions, see [3].

## 4 Project Schedule

So far, the student has already accomplished the requisites from the post-graduate program: the mandatory courses and the approval in the qualifying exam. Our research plan is scheduled as follows:

### 2nd Semester of 2019:

- Survey of the field;
- Proposal of the postulates for Multiple Choice Revision;
- Development of Partial Meet and Kernel constructions for Multiple Choice Revision;
- Proposal and proof of representation theorems between the constructions and the postulates;

### 1st Semester of 2020:

- Development and proof of the Partial Meet and Kernel algorithms for Choice Revision;
- Analysis of the complexity of the algorithms developed;

- Implementation and testing of the algorithms developed;
- Identification of the limitations of Selective Revision for Description Logics;
- Analysis of the Selective Revision operation in order to extend it to the multiple case;

#### **2nd Semester of 2020:**

- Proposal of an adaptation of Selective Revision to the multiple case;
- Proposal of an adaptation of Multiple Selective Revision for Description Logics;
- Comparison between Multiple Choice Revision and Merging to identify possible connections;

#### **1st Semester of 2021:**

- Conclusion of the comparison with Merging;
- Final refinements;
- Thesis defense.

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