Human Syllogistic Reasoning: Towards Predicting Individuals' Reasoning Behavior based on Cognitive Principles

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Abstract. During the last decades, a tremendous effort was put into bringing together the techniques developed within the area of computational logic and the findings on human reasoning in cognitive science. In 2012, Khemlani and Johnson-Laird published a meta-study on human syllogistic reasoning, showing that none of the well-established cognitive theories performed well with respect to human data. Recently, a novel cognitive theory using techniques from computational logic seemed to outperform these theories. In this paper we extend this approach by identifying individual reasoning behavior and grouping it by clusters. Each cluster can be characterized by a set of cognitive principles or by additional heuristic strategies. We evaluate our approach with CCOBRA, a benchmark tool in which cognitive models can learn from individual human patterns and adapt their strategies accordingly.

1 Introduction

During the last decades, a tremendous effort was put into bringing together the techniques developed within the area of computational logic and the findings on human reasoning in cognitive science. It seems that the effort had effect, as now terms such as computational cognition, computational cognitive science or computational psychology get more and more attention within the area of AI.

The meta-study on human syllogistic reasoning by Khemlani and Johnson-Laird published in 2012 [1] provides an excellent overview of the most established cognitive theories, and their performance on predicting human responses with respect to six psychological experiments in syllogistic reasoning. The result was sobering: There are more than twelve different cognitive theories, but none of them predicted the human data well. This meta-study gave a good starting point to test own models and to compare them with the other ones. In 2017, the first syllogism challenge took place, where the performance of the submitted models was tested on previously unseen data.[1] In 2019, the challenge was further developed, providing the CCOBRA framework, an acronym

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[1] www.cc.uni-freiburg.de/modelingchallenge/challenge-2017

Natural Language	First-order Logic	Mood
all a are b	$\forall X(a(X) \rightarrow b(X))$	Aab
some a are b	$\exists X(a(X) \wedge b(X))$	Iab
no a are b	$\forall X(a(X) \rightarrow \neg b(X))$	Eab
some a are not b	$\exists X(a(X) \wedge \neg b(X))$	Oab

Table 1: The four moods and their formalization.

Figure	Premise 1	Premise 2
1	$a - b$	$b - c$
2	$b - a$	$c - b$
3	$a - b$	$c - b$
4	$b - a$	$b - c$

Table 2: The four figures.

for Cognitive COmputation for Behavioral Reasoning Analysis, an environment where models are given the individual participants' response patterns and adapt their strategies accordingly. Different than in the meta-study of [1] and in the syllogism challenge in 2017, not the aggregated results of the psychological experiment, but the non-aggregated results the individual participants' response patterns had to be dynamically predicted. In contrast to the challenge in 2017, the challenge in 2019 considered two additional aspects: (1) Not only the majority's response, but the response with respect to an individual participant had to be predicted, and (2) the model had to dynamically adapt its strategy, depending on the previously given (individual) participants' responses. Consider the following pair of syllogistic premises:

All bakers are artists.   *All chemists are bakers.*   (1)

Given these two premises, which conclusion on the relation between *artists* and *chemists* necessarily follows? One conclusion that follows according to FOL is *All chemists are artists* (Aca). Different to classical First-order Logic, but according to the Aristotelian interpretation [2], universally quantified entities are assumed to exist, i.e. *Some chemists are artists* (Ica) and *Some artists are chemists* (Iac) are also valid conclusions. The majority of participants in experimental studies concluded *All artists are chemists* (35%) and *All chemists are artists* (48%) [1]. Consider another pair of syllogistic premises:

No artists are bakers.   *Some bakers are chemists.*   (2)

Some chemists are not artists (Oca) follows according to FOL. Again, according to [1], this conclusion does not comply with the majority's responses: 53% concluded *No artists are chemists* (Eac) and 19% concluded *No valid conclusion* (NVC).

Syllogisms have originally been introduced by Aristotle [3], and each syllogism consists of a pair of syllogistic premises, according to the four classical moods and figures shown in Table 1 and 2, together with one conclusion about a and c expressed in one of the four moods in Table 1. A conclusion can take one of the following forms: Aac Eac Iac Oac Aca Eca Ica Oca There are $4^2 \times 4 = 64$ different pairs of premises that can be uniquely specified by the abbreviations of the moods and figures in Table 1 and 2: The pairs of syllogistic premises in (1) and in (2), are uniquely specified as AA2 and EI1,

Abbreviation	FOL[2]	Majority of participants
AA2: Aba, Acb	Aca, Iac, Ica	Aac (35%), Aca (48%)
IE3: Iab, Ecb	Oac	Oac (20%), Eca (29%), NVC[3](26%)
EI1: Eab, Ibc	Oca	Eac (53%), NVC (19%)
EA2: Eba, Acb	Eac, Eca, Oac, Oca	Eac (28%), Eca (51%)

Table 3: Four pairs of syllogistic premises and the majority's responses from [1].

respectively. Together with the 8 possible conclusions above, $64 \times 8 = 512$ different syllogism exist (out of which only 48 are valid according to FOL).

The responses given by the majority of participants' for AA2 and three other pairs of syllogistic premises are shown in Table 3. Note that, we understand *majority* as in [1], i.e. any conclusion that is drawn by more than 16% of the participants, is too high to be chosen randomly. Therefore, these conclusions are interesting enough to be considered relevant for modeling. Two observations follow immediately from the responses in Table 3: (i) Participants' responses do not always comply with the conclusions that are valid according to FOL, and (ii) Participants do not agree on a given conclusion, i.e. they seem to reason differently among each other. Furthermore, in psychological experiments, participants are usually required to give only one response. Thus, even if Aac and Aca do not need to exclude each other according to FOL, in experiments participants that concluded Aac did not conclude Aca, and vice versa. As the above example and recent investigations on psychological experiments show (cf. [4]), it seems that different humans represent and reason about conditional information differently. This implies that *the* human reasoner does not exist, instead there seem to exist various (human) reasoning clusters. The identification of these differences and the specification of these clusters is central for the development of an adequate cognitive theory. Only recently approaches have been proposed that account for individual differences (or clusters) in syllogistic reasoning (cf. [4,5]).

The central requirements for the cognitive theory that we establish here is that it has to be comparable to other theories and that it should provide qualitative justifications for its predictions. In Section 2 we present the underlying theory together with the relevant cognitive principles. Section 3 introduces the environment for this year's syllogism challenge, the CCOBRA framework, and the implementation of our theory and its learning strategy within this framework. Finally, Section 4 provides an evaluation on the best performing set of clusters and discusses new findings.

[2]First-order logic: Valid conclusions according to FOL when existential import is assumed.

[3]No valid conclusion

2 Clustering by Principles

In the following, we suggest a principle- and cluster-based cognitive model. For this purpose we establish the following two hypotheses according to the previously made observations: (i) Humans make (not necessarily classical logic valid) assumptions, and (ii) *the* human reasoner does not exist, instead there are various reasoners. Accordingly, our goal is two-folded: First, providing one framework for all (possibly not valid according to FOL) assumptions through the characterization of *cognitive principles*, and second, characterizing *reasoning clusters* by means of the cognitive principles. As a starting point, we take the approaches presented in [6,5], which have developed eight principles for quantified statements mostly justified by the cognitive science literature. In the following section we recall the principles and heuristic strategies that are relevant for the purpose of this paper. Thereafter we briefly discuss the reasoning clusters.

2.1 Principles

In [6,5], eight principles and two heuristic strategies have been formalized under the Weak Completion Semantics [7,8], a logic programming approach that understands models under the three-valued Łukasiewicz logic [9]. In this section, we will not show their formal representation but rather give an intuitive understanding by means of examples.

We distinguish between basic and advanced principles. Basic principles hold for all reasoners, i.e. for all participants' responses, for which we believe they have been derived through reasoning. On the other hand, advanced principles might not hold for all reasoners.

There are five basic principles, which are *quantified statements as conditionals* (conditionals), *licenses for inferences* (licenses), *existential import* (import), *unknown generalization* (unknownGen) and *converse premise* (converse).

For premises that are universally quantified, (conditionals), (licenses) and (import) apply. Consider the first premise in AA2 from the introduction, *All bakers are artists*: This natural language statement is to be understood as a conditional statement with a license for inference, which we represent by means of explicitly stating whether something is abnormal and include it within the conditional:

All bakers, that are not abnormal, are artists.	(conditionals & licenses)
By default, no baker is abnormal.	(licenses)

The abnormality predicate within the rule allows for defeasible reasoning and seems to be adequate for modeling rules within human reasoning [10]. Additionally, by the (import) principle, which states that entities which are universally quantified need to exist, we also assume the following statement:

Some *bakers* exist. (import)

The second premise of AA2 can be represented analogously by the above four principles.

For premises that are existentially quantified, additionally, (unknownGen) applies. Consider the second premise in EI1 from the introduction, *Some bakers are chemists*. The idea behind (unknownGen) applies by specifying ab in the conditional statement as follows:

All bakers, that are not abnormal, are chemists.	(conditionals & licenses)
Some *bakers* exist, that are not abnormal.	(licenses & unknownGen)
Some *bakers* exist, for whom it is unknown, whether they are abnormal.	(licenses & unknownGen)

As (unknownGen) is not symmetric, i.e. when we assume *Some bakers are chemists* or *Some chemists are bakers*, the (possibly) resulting derivations from the above assumptions might be different. However, when we additionally introduce the (converse) principle, i.e. *Some bakers are chemists* implies *Some chemists are bakers*, then (unknownGen) applies in both directions. Thus, additionally to the three assumptions above, the (converse) principle applied to the second premise of EI1 leads to the following three (symmetric) statements:

All chemists, that are not abnormal, are bakers.	(converse & conditionals & licenses)
Some *chemists* exist, that are not abnormal.	(converse & licenses & unknownGen)
Some *chemists* exist, for whom it is unknown whether they are abnormal.	(converse & licenses & unknownGen)

There are also advanced principles, not assumed by all reasoners: *Generalization* (generalization), *search for alternative conclusions* (searchAlt) and *contraposition* (contraposition).[4]

Consider again the pair of syllogistic premises, EI1 from the introduction: Recall that *Some chemists are not artists* (Oca) follows according to FOL, by the contrapositive of the first premise *No bakers are artists*, shown by the X in the Venn Diagram in Figure 1. However, it doesn't seem the case that all humans reason with the contrapositive when applicable. Therefore, (contraposition) belongs to the advanced principles. In fact, about half of the participants concluded *No artists are chemists* (53%). First, assume that (import) holds for the first premise in EI1, thus *artists exist*. By (conditionals) and (licenses), we can derive that *these artists are not bakers*. Second, assume that (converse) holds for the second premise in EI1, thus *Some chemists are bakers*. The advanced principle *existential as universal quantification* (generalization) captures the idea that some participants generalize over the existential quantification. Thus for EI1, some participants might understand the second premise and its converse universally as *All bakers are chemists* and *All chemists are bakers* (generalization). Consequently, all artists are not chemists, i.e. *No artists are chemists*, which corresponds to what 53% of the participants concluded.

The (searchAlt) principle is motivated by the assumption that, if participants cannot straightforwardly derive any relation between *artists* and *chemists*,

[4] (contraposition) is assumed together with (converse) for E.

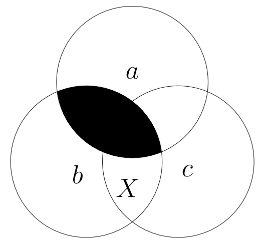

Fig. 1: In FOL, *Some chemists are not artists* follows from: *No artists are bakers. Some bakers are chemists.*

rather than responding NVC, they might search for explanations. Consider yet another pair of syllogistic premises, IA2:

Some bakers are artists. *All chemists are bakers.*

A possible reason why the majority of participants' responses in [1] were *Some artists are chemists* (27%) and *Some chemists are artists* (52%), can intuitively be explained as follows: The first premise implies that *some bakers exist*. This can be explained backwardly, by the information in the second premise, *these bakers are chemists*.

Some conclusions given by a significant amount of participants cannot be explained by the previously introduced (combinations) of principles. Possibly, these participants simply guess or apply some strategy that is not explainable by reasoning. For instance, according to the *atmosphere bias* [11], some humans might be affected by the moods of the premises, in the sense that universal conclusions (Aac, Aca, Eac, Eca) are excluded when one of the premises is existential (Iac, Ica, Oac, Oca) and affirmative conclusions (Aac, Aca, Iac, Ica) are excluded when one of the premises is negative (Eac, Eca, Oac, Oca). In the case of identical moods, the conclusion must have this mood as well. Consider again EI1 from the introduction: Because the second premise is existential, any conclusion with the E and A mood is excluded, and because the first premise is negative, any conclusion with A and I mood is excluded. The only possible conclusions which are left are Oac and Oca, where Oca is sometimes given by participants (see participant B in Section 4). Consider yet another pair of syllogistic premises, EA2:

No bakers are artists. *All chemists are bakers.*

As none of the premises are existential, universal conclusions are not excluded. However, the second premise is negative, thus all affirmative conclusions are excluded. The only conclusions that are left are Eac (28%) and Eca (51%), and correspond to the majority's responses in [1].

2.2 Clusters and Profiles

Recall that *Cognitive principles* (and heuristic strategies) are assumptions made by humans. The relevant ones for this particular case of human syllogistic reasoning have been introduced in Section 2.1. *Reasoning clusters* are sets of cognitive principles, where each cluster, i.e. each set of principles, characterizes a group of reasoners. The population's responses that are to be predicted can be represented by a profile, where *profiles* are sets of clusters, covering the reasoning groups within a population. Khemlani and Johnson-Laird [4] identified the profile that consisted of three clusters of reasoners in human syllogistic reasoning: *intuitive*, *intermediate* and *deliberative*. A clustering approach based on cognitive principles was first introduced in [5] and was specified by five clusters. Three of these clusters were based on a combination of basic and advanced principles and two clusters were additionally based on heuristic strategies. This profile achieved an overall match of 92% with the aggregated data provided in [1]. In the following, we do not specify the profile beforehand but rather choose the profile that performed best during training.

3 Modeling Challenge

One of the major goals of the (cognitive) modeling challenges is to provide a competition among cognitive models that predict human behavior best. In May 2019, the PRECORE challenge took place, where submissions were asked to predict individual syllogistic reasoning[5]. The modeling task is as follows: First, the model is given a syllogistic reasoning problem and the participant, whose response will serve as measure for the predictive accuracy. Second, the model predicts a single-answer response between the nine different options according to a strategy based on the previous responses of that participant (if they exist). Finally, the models' response is evaluated by means of its predictive accuracy with respect to that participant. The predictive accuracy of a model with respect to a modeling task is the probability that its predicted response matches with the participants' response. For instance, consider a model that randomly predicts a response:[6] For the above specified modeling task, i.e. for the prediction of a single-answer response given nine different answer possibilities, the probability that the model will correctly predict the participants' response is $\frac{1}{9} = 11.1\%$. The goal of the challenge is to provide cognitive models that optimize their predictive accuracy.

The CCOBRA Modeling Framework, where CCOBRA is an acronym for <u>C</u>ognitive <u>CO</u>mputation for <u>B</u>ehavioral <u>R</u>easoning <u>A</u>nalysis, is a benchmark tool in which the cognitive model can adapt the optimal prediction strategy through the pre-train phase, by matching the participants' responses.[7] In particular, in the pre-train phase, the model is asked to predict the participants' responses, and then adapt its own strategy according to their responses. So far, the best performance was just above 50%, which has been achieved through machine learning techniques.[8]

[5]`https://www.cc.uni-freiburg.de/modelingchallenge`

[6]This model corresponds to the UniformModel in Figure 4 in the following section.

[7]`https://github.com/CognitiveComputationLab/ccobra`

[8]`https://www.cc.uni-freiburg.de/staff/files/2019-04-dresden`

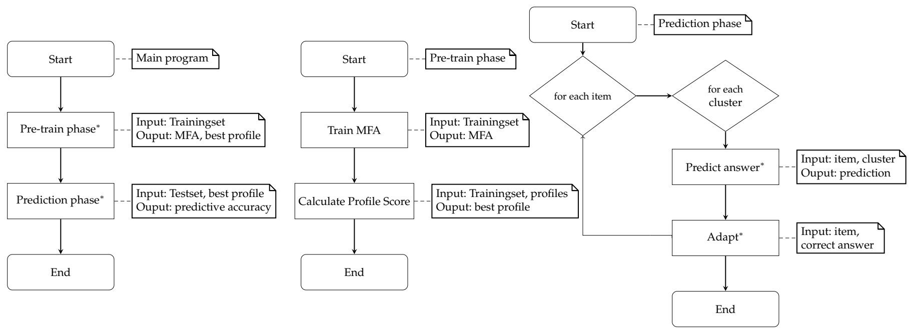

Fig. 2: Flowchart of program (left), pre-train (middle) and prediction (right).

3.1 Clustering by Principles

The idea of our approach is to match the participants' responses with the responses given by a certain set of cognitive principles during the pre-train phase. These sets of principles (clusters) should then help as a guideline for the generation of responses in the prediction phase.

Figure 2 (left) shows an overview of our implementation within the CCOBRA framework, which consists of two main phases: The pre-train phase and the prediction phase, both specified in more details in Figure 2 (middle) and Figure 2 (right), respectively. During the pre-train phase, for each pair of syllogistic premises, the most frequent answer (Train MFA) and the predictive accuracy of each profile (Calculate Profile Scores) is computed. Note, that given n principles, there are 2^n clusters, and thus there are 2^{2^n} profiles to be tested. The responses given by MFA and the profile with the best score, will then be used during the prediction phase. How the answers are predicted and adapted is shown in Figure 3.

In Figure 3 (left) the response(s) for a pair of syllogistic premises (item) with respect to the given cluster is predicted. In case only one response is predicted, this one is chosen, and we are done. As the modeling task is specified such that a single-answer response is required we need to make a choice when a cluster predicts more than one response. This is done by taking the intersection of the predictions of the chosen cluster and the predictions of the cluster with the next highest accuracy (get response from cluster with next best score). If the intersection is non-empty, the process is repeated until at most one predicted response is left. If the intersection is empty, then the predicted responses of a (new) cluster, possessing the next highest predictive accuracy, is checked. If there is such a cluster, the process is repeated again. If not, then the first response from the initially chosen cluster is selected (select first response).

Figure 3 (right) specifies the adapt strategy. First, for each cluster that predicts the participants' response correctly during the adapt phase, its corresponding predictive accuracy (score) is increased dynamically, similar to MFA (most frequent answer). Initially, when no information about the participants' response

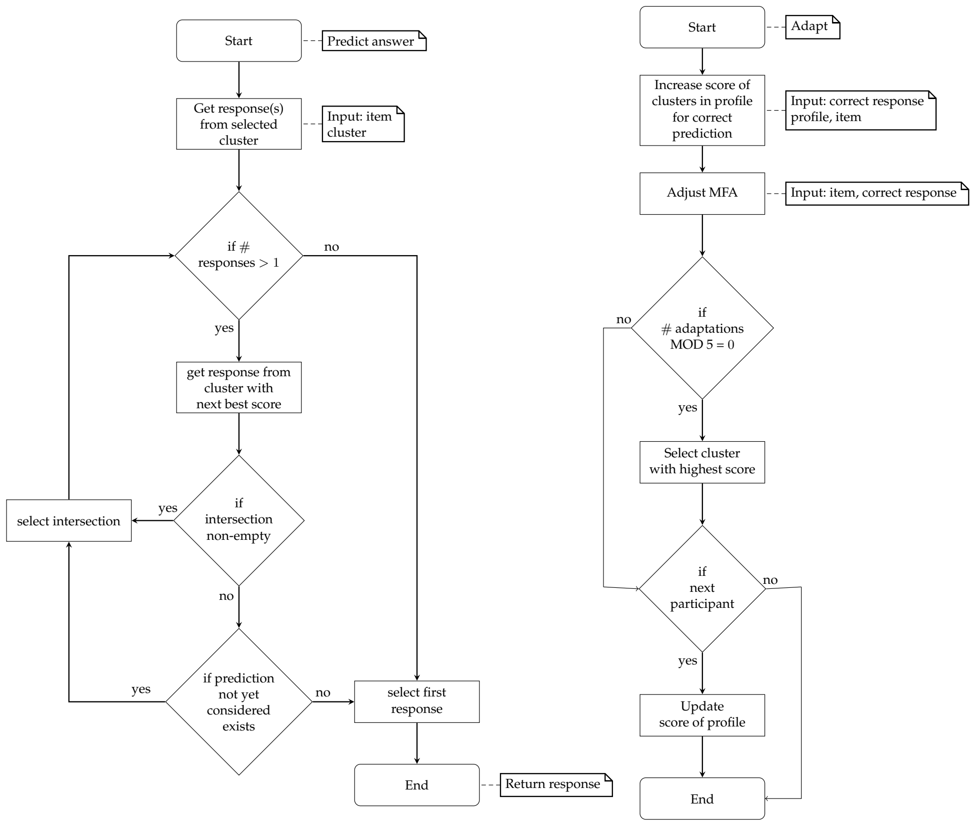

Fig. 3: Flowchart for answer prediction (left) and adaption of predictions (right).

behavior is known, the predicted answer will be the one corresponding to MFA. After each five predictions, the cluster that matched the last responses of the participants best, is chosen for the next five predictions, until the next participants' responses are to be predicted. After all 64 responses for one participant are predicted, the predictive accuracy (Update score of profile) of the profile is updated.

4 Evaluation

For each of the 32 participants, the test set used in the prediction phase had 64 entries.[9] Figure 4 gives an overview of the predictive accuracy (above) and the respective box plot (below) of the benchmark models on the data provided by CCOBRA,[7] compared to *Clustering by Principles*. From left to right the figure shows the results of the following models: uniform (when the responses are

[9] `https://github.com/CognitiveComputationLab/ccobra` (Veser2018.csv)

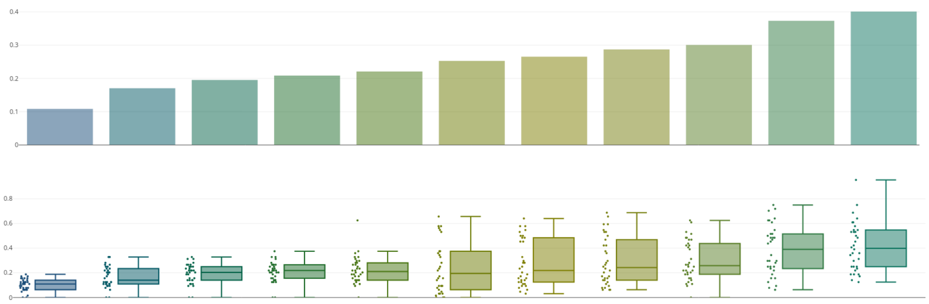

Fig. 4: Pred. accuracy of benchmarks and *Clustering by Principles* (rightmost).

chosen randomly), matching bias [12], probability heuristics model [13], atmosphere bias [11], mental model theory [14], NVC (when the chosen responses are always *no valid conclusion*), (logic-based) PSYCOP model [15], illicit conversion heuristics [16], verbals models theory [17], and *clustering by principles*. In the respective box plots in Figure 4 (below), min and max refer to the worst and best participants' prediction, where the dots refer to the average predictive accuracy of the individual participants.[10]

The profile of the *Clustering by Principles* model that predicted best the participants' responses achieved a predictive accuracy of just above 40%, which is the highest score, compared to the performance of other well known cognitive models.[9] This profile consists of the following six clusters (1) Basic and Abduction, (2) (variations of) Basic and Abduction and Contraposition, (3) (variations of) Basic and Abduction and Generalization, (4) (variations of) Atmosphere, (5) Most frequent answer and (6) NVC. From a qualitative point of view, the first four clusters are the most interesting ones, as they explain the participants' responses. In particular, during the prediction phase, these clusters' predictive accuracy were as follows:

Basic and Abduction predicted well at least 58% of 16% (of the responses).
Basic and Abduction and Contraposition predicted well at least 58% of 19%.
Basic and Abduction and Generalization predicted well at least 41% of 16%.
Atmosphere predicted well at least 27% of 31%.

In total, this profile predicted between 27 and 58% of the responses 82% of the participants correctly. One participant (participant D in Table 4) always answered NVC, for which the NVC cluster achieved a predictive accuracy of 100%.

Consider the four pairs of syllogistic premises from the introduction again. Table 4 provides the answers of four participants of the test set provided in the CCOBRA framework. For AA2, all three participants agree on the conclusion, *All chemists are artists*, which is also valid in Classical Logic. For EI1 however, participants differ in their responses. In the case of IE3, only participant B and C agree on *Some artists are not chemists*, whereas participant A *generalizes* the

[10]Both figures have been computed by CCOBRA.

Participant[11]	AA2	IE3	EI1	EA2	Cluster
A	Aca	Eca	Eac	Eca	⇒ Basic, Abduction and Generalization
B	Aca	Oac	Oca	Eac	⇒ Atmosphere
C	Aca	Oac	NVC	Eca	⇒ Basic, Abduction and Contraposition
D	NVC	NVC	NVC	NVC	⇒ *No valid conclusion*

Table 4: Pairs of premises of the introduction and four participants' responses.

conclusion to *No chemists are artists*. On the other hand, in the case of EA2, only participant A and C agree on the conclusion.

5 Conclusions and Future Work

We presented *clustering by principles*, an approach that predicts individual human behavior for syllogistic reasoning. Beforehand, we specified a set of principles, motivated by the literature. Depending on which set of principles is considered, different responses are generated. We then specified different clusters, each through a set of principles. With the help of the CCOBRA Modeling framework, in which cognitive models can evaluate their performance on real (human) data, and dynamically change their predictions, we suggest an adaptive *clustering by principles* approach. During the pre-train phase, the best performing profile was computed, which was then chosen for the prediction phase. For the training data provided by CCOBRA, the best performing profile consists of six clusters, from which four are based on principles and heuristics. Compared with the benchmark models, this profile reached the highest score with a prediction of just above 40%. Even though the machine learning techniques still achieve a higher score (50%),[12] we believe that *clustering by principles* has the advantage of explaining its predictions with the underlying principles.

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[11] Participant A, B, C and D are participant 1, 3, 7 and 22, respectively in Veser2018.csv.
[12] `https://www.cc.uni-freiburg.de/staff/files/2019-04-dresden`

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