Whiwap: Checking Iterative Belief Changes

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Abstract. In the framework for iterative belief revision by Darwiche and Pearl, epistemic states are equipped with a total preorder on possible worlds. Belief changes in this framework not only affect the set of formulas that are considered to be true but the whole preorder. Different postulates were proposed to confine the changes of total preorders. We present Whiwap, a tool which allows the user to enter a belief change with a propositional formula α, in particular to specify the total preorder of the prior and the posterior state and the models of α. Whiwap checks whether the entered belief change fulfils certain postulates or not. Common belief change postulates for revision and contraction are implemented.

Keywords: belief change, postulate, total preorder, epistemic state

1 Introduction

In the well established framework for iterative belief revision by Darwiche and Pearl [6], epistemic states are equipped with a total preorder on possible worlds. Belief change in this framework not only affects the set of formulas that are considered to be true, but the whole preorder. The motivation for using these more complex structure is the insight by Darwiche and Pearl, that using belief sets is not expressive enough for the task of iterative belief change. Moreover, total preorders are the right semantics for iterative change in the light of very fundamental principles of change, which respect conditional beliefs.

Belief revisions, belief contractions and other kinds of belief changes in the framework of total preorders are still a subject of interest, and so far a lot of different postulates have been proposed to confine the changes on total preorders. And sometimes you want to check if a certain belief change fulfils some postulate or not. Researchers in the field are often searching for counterexamples - or are just curious about the properties of some belief change operator. To avoid doing such checks manually, which is not only tedious but also error-prone, we developed Whiwap (short for *"What sHoud I do With All these Postulates?"*).

Typically, iterative belief change operators are defined syntactically and then a representation theorem connects the syntactical definition with a semantic equivalent, where the semantics is commonly a total preorder semantics. Whiwap works mainly on the semantic side, it allows to enter a total preorder before and after a change with a certain formula and analyses what postulates are fulfilled by this change. Whiwap has implemented checks for common revision postulates as well as contraction postulates, and the set of postulates is extended continuously.

In this paper we present Whiwap as follows: In Section 2 the basics on belief change in the Darwiche and Pearl-framework of total preorders are presented. After that Whiwap is described in more detail in Section 3. Finally we give a discussion and an outlook to future work in Section 4.

2 Preliminaries and Background

Let Σ be a propositional signature and $\mathcal{L}$ be the propositional language over Σ. We denote formulas in $\mathcal{L}$ with lower Greek letters $\alpha, \beta, \gamma, \ldots$, and propositional variables with lower case letters $a, b, c, \ldots \in \Sigma$. The set of propositional interpretations Ω, also called set of worlds, is identified with the set of corresponding complete conjunctions over Σ. Propositional entailment is denoted by $\models$, with $[\![\alpha]\!]$ we denote the set of models of α, and $Cn(\alpha) = \{\beta \mid \alpha \models \beta\}$ is the deductive closure of α. A total preorder $\leq$ over Ω is a relation which fulfils for all $\omega, \omega_1, \omega_2, \omega_3 \in \Omega$:

(reflexivity)	$\omega \leq \omega$
(transitivity)	$\omega_1 \leq \omega_2$ and $\omega_2 \leq \omega_3$ implies $\omega_1 \leq \omega_3$
(totality)	$\omega_1 \leq \omega_2$ or $\omega_2 \leq \omega_1$

For a total preorder $\leq$, we denote with $<$ its strict variant, i.e. $x < y$ iff $x \leq y$ and $y \not\leq x$; and we write $x \simeq y$ iff $x \leq y$ and $y \leq x$. For a set of worlds $\Omega' \subseteq \Omega$ and a total preorder $\leq$ over Ω, we denote with $\min(\Omega', \leq) = \{\omega \mid \omega \in \Omega'$ and there is no $\omega' \in \Omega'$ s.t. $\omega' < \omega\}$ the set of all worlds in the lowest layer of $\leq$ that are elements in Ω'.

2.1 Belief Change in the Darwiche-Pearl Framework

The research in the Darwiche-Pearl framework (DP-framework) of belief change focusses on mainly two kinds of belief changes: incorporating new information into the beliefs of an agent (revision) and removing a belief from an agent's beliefs (contraction). This is inherited from the AGM-Theory (named after Alchourrón, Gärdenfors and Makinson [1]), which deals with the change of theories, instead of epistemic states.

Technically, in the DP-framework beliefs are propositional formulas from $\mathcal{L}$. The beliefs of an agent are represented indirectly by a total preorder $\leq_\Psi$ of possible worlds, where Ψ denotes the epistemic state of an agent. Here, possible worlds are identified with propositional interpretations $\omega, \omega_1, \omega_2, \ldots \in \Omega$. The basic idea is, that the total preorder $\leq_\Psi$ represents the plausibility of the different worlds in Ω, where $\omega_1 \leq_\Psi \omega_2$ means that the world ω_1 is more or equally plausible than the world ω_2. Consequently, the minimal elements of $\leq_\Psi$ are interpreted as the most plausible worlds, denoted by $\|\Psi\| = \min(\Omega, \leq_\Psi)$. The beliefs $Bel(\Psi)$ of an agent with epistemic state Ψ are the set of beliefs that are compatible with the most plausible worlds of $\leq_\Psi$, i.e. define $Bel(\Psi) = \{\alpha \in \mathcal{L} \mid$ for all $\omega \in \|\Psi\|$ we have $\omega \models \alpha\}$. This framework is very foundational, since many more meticulous representation formalisms, like ranking functions [16], induce an total preorder.

A belief change operator $\circ$ transforms an epistemic state Ψ and a formula α into a new epistemic state $\Psi \circ \alpha$. Different kind of changes, like revision and contraction, are characterised in the DP-framework by axiomatic properties. Semantically, the task of the operator is reduced to the question of how the worlds should be shifted in the total preorder $\leq_\Psi$ to obtain $\leq_{\Psi\circ\alpha}$ such that the change obeys the axiomatic description of the changes. In the rest of this paper we restrict ourselves only to the semantic side of (iterative) belief revision and contraction, which is used by WHIWAP.

Darwiche and Pearl showed that AGM revision [1], denoted here by $*$, can be reduced to one axiom in the DP-framework.

Proposition 1 (AGM Revision for Epistemic State [6]). *A belief change operator $*$ is an* AGM revision on epistemic states *if and only if $*$ satisfies:*

$$\text{(AGMes}^*_\leq\text{)} \qquad \|\Psi * \alpha\| = \min([\![\alpha]\!], \leq_\Psi)$$

Proposition 1 characterises AGM revision in the DP-framework as the operation which makes the most plausible worlds of α ultimately plausible.

More recently, a similar characterisation of contraction has been worked out [5, 10]. Contraction of α is the process of making the minimal counter worlds of α maximally plausible, while retaining the plausibility of non-counter worlds.

Proposition 2 (AGM Contraction for Epistemic State [10]). *A belief change operator $\div$ is an* AGM contraction on epistemic states *if and only if $\div$ satisfies:*

$$\text{(AGMes}^{\div}_\leq\text{)} \qquad \|\Psi \div \alpha\| = \|\Psi\| \cup \min([\![\neg\alpha]\!], \leq_\Psi)$$

Observe that neither (AGMes$^*_\leq$) nor (AGMes$^{\div}_\leq$) constrain how worlds in layers above the minimal layer of $\leq_\Psi$ should be ordered. Darwiche and Pearl showed that this is problematic, as it lead to counter-intuitive changes [6]. In the following, we present postulates for different solutions to the problem of how changes of the non-minimal layers should be constrained.

2.2 General Postulates for Iteration

Proposition 1 and Proposition 2 define revision and contraction in the DP-framework. However, iterative belief change also requires to handle maintenance information for the change strategy of Ψ, which is encoded in the total preorder $\leq_\Psi$. Neither the postulate (AGMes$^*_\leq$) nor (AGMes$^{\div}_\leq$) give much insight on how to maintain the strategy informations.

Darwiche and Pearl [6] propose that the order of plausible worlds should be retained under reasonable circumstances. This is a natural extension of the principle of minimal change from the original AGM approach [1].

(DP1) if $\omega_1, \omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi * \alpha} \omega_2$

(DP2) if $\omega_1, \omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi * \alpha} \omega_2$

(DP3) if $\omega_1 \in [\![\alpha]\!]$ and $\omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 <_\Psi \omega_2 \Rightarrow \omega_1 <_{\Psi * \alpha} \omega_2$

(DP4) if $\omega_1 \in [\![\alpha]\!]$ and $\omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi * \alpha} \omega_2$

A revision with α separates the possible worlds in two kinds: the ones that model α and the ones that do not model α. The postulates (DP1) and (DP2) guarantee that worlds of each kind keep their order. Moreover, (DP3) and (DP4) guarantee that models of α stay more plausible then counter-models if they were more plausible before.

For contraction a similar set of postulates has been proposed [5, 10]:

(IC1) if $\omega_1, \omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi \div \alpha} \omega_2$

(IC2) if $\omega_1, \omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi \div \alpha} \omega_2$

(IC3) if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 <_\Psi \omega_2 \Rightarrow \omega_1 <_{\Psi \div \alpha} \omega_2$

(IC4) if $\omega_1 \in [\![\neg\alpha]\!]$ and $\omega_2 \in [\![\alpha]\!]$, then $\omega_1 \leq_\Psi \omega_2 \Rightarrow \omega_1 \leq_{\Psi \div \alpha} \omega_2$

The postulates (IC1) and (IC2) implement the same idea as (DP1) and (DP2). Analogue to (DP3) and (DP4) the postulates (IC3) and (IC4) state that a contraction which obeys this properties should never disimprove the relative plausibility of counter-models.

2.3 Strategies for Iterative Belief Change

Another approach to iterated change is the implementation of concrete change strategies. Many concrete strategies have semantic representations in the DP-framework. The class of operators implementing a specific strategy for revision (respectively contraction) are characterised in the DP-framework by demanding additional postulates in addition to the success postulate (AGMes$^*_\leq$) (respectively (AGMes$^\div_\leq$))

Natural Change One of the most well-known strategy for revision is introduced by Boutiller [4], often called *natural revision*. The approach of natural change is to retain as much as possible of the conditional beliefs. An equivalent of this strategy for revision is translated into the DP-framework by Darwiche and Pearl [6]:

(CBR) if $\omega_1, \omega_2 \notin \|\Psi * \alpha\|$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi * \alpha} \omega_2$

Operators which obey the postulate (CBR) keep as much of the old ordering as possible. Ramachandran, Nayak and Orgun [13] formulate *natural contraction* similar to (CBR) as follows:

(NC1) if $\omega_1 \in \|\Psi\|$ or $\omega_1 \in \min([\![\neg\alpha]\!], \leq_\Psi)$, then $\omega_1 \leq_{\Psi \div \alpha} \omega_2$

(NC2) if $\omega_1, \omega_2 \notin \|\Psi\| \cup \min([\![\neg\alpha]\!], \leq_\Psi)$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi \div \alpha} \omega_2$

Lexicographic Change Another revision strategy introduced by Nayak, Pagnucco and Peppas [12] is to order the worlds by whether they satisfy the information α or not. This policy is given by the following postulate:

(SimpLex) $\quad$ if $\omega_1 \in [\![\alpha]\!]$ and $\omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 <_{\Psi * \alpha} \omega_2$

Transferring the principle of lexicographic change to contraction has been proposed, but is not as straightforward as for revision. Due to complexity of the approach we refer the interested reader to the work of Ramachandra, Nayak and Orgun [13].

Admissible and Restrained Revision The strategy of admissible revision with α enforces improvement of worlds that comply with α. This has been proposed under the name *independence* by Jin and Thielscher [9] and is characterised in the light of (AGMes$^{\div}_{\leq}$) as follows:

(PR) $\quad$ if $\omega_1 \in [\![\alpha]\!]$ and $\omega_2 \in [\![\neg\alpha]\!]$, then $\omega_1 \leq_{\Psi} \omega_2 \Rightarrow \omega_1 <_{\Psi * \alpha} \omega_2$

The strategy of admissibility has been further supplemented by Booth and Meyer [3] with the idea of overwriting the believability of counteracting beliefs by a revision. It can be shown that this is equal to the following principle (DR):

(DR) $\;$ if $\omega_1 \in [\![\neg\alpha]\!], \omega_2 \in [\![\alpha]\!]$ and $\omega_2 \notin \|\Psi * \alpha\|$, then $\omega_1 <_{\Psi} \omega_2 \Rightarrow \omega_1 <_{\Psi * \alpha} \omega_2$

Moderate Contraction The idea of moderate contraction or also called priority contraction [14], is that a removal of α should also decrease the plausibility of at least one of the implications $\beta \to \alpha$ or $\neg\beta \to \alpha$. This is captured semantically in the DP-framework by the following set of postulates by Ramachandra, Nayak, Orgun [13] in the light of (IC1) and (IC2):

(MC3) $\quad$ if $\omega_1 \in [\![\alpha]\!],\ \omega_1 \notin \|\Psi\|, \leq_{\Psi})$ and $\omega_2 \models \neg\alpha$, then $\omega_2 <_{\Psi \div \alpha} \omega_1$

(MC4) $\quad$ if $\omega_1 \in \min(\Omega, \leq_{\Psi})$ or $\omega_1 \in \min([\![\neg\alpha]\!], \leq_{\Psi})$, then $\omega_1 \leq_{\Psi \div \alpha} \omega_2$

In the literature many more classes of operators and their representation in the DP-framework can be found (e.g. [14]). In the following we present how Whiwap can be used to check whether these postulates are satisfied or not.

3 Whiwap

Whiwap allows to check whether a certain revision fulfils a given set of belief change postulates. In this section we will first describe the user interface of the tool. Afterwards we will give a short overview of the implementation.

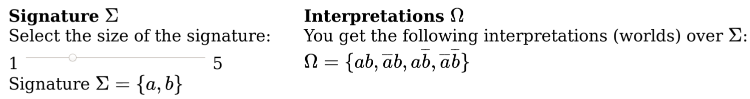
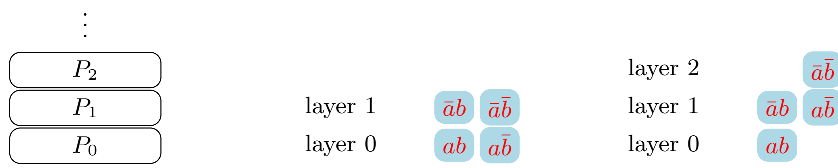


Fig. 1: User interface for setting the signature size. The set of all worlds with this signature is shown on the right side.



(a) Every layer contains a partition. The lower a layer, the more plausible.

(b) Example of a total preorder. The worlds where a is true are considered to be plausible, the worlds where a is not true, not.

(c) Example of a total preorder. The world ab is considered to be plausible, the worlds $\bar{a}b$ and $a\bar{b}$ less plausible and $\bar{a}\bar{b}$ least plausible.

Fig. 2: Representing total preorders with layers.

3.1 User Interface

To analyse a belief change, the tool requires the epistemic states before and after the change as well as a change parameter. One of the main functions of the UI is to allow the user to enter this information.

The user starts by entering the size of the signature with a slider. The tool automatically selects the first characters of the alphabet as the signature Σ and generates the set of all possible worlds Ω over Σ (see Figure 1).

As described in Section 2, an epistemic state can be represented by a total preorder on all worlds in the DP-framework. Every total preorder induces an ordered partition $P_0 \dot{\cup} \ldots \dot{\cup} P_n = \Omega$ of the worlds. Two worlds ω_1, ω_2 are in the same partition, if and only if $\omega_1 \leq \omega_2$ and $\omega_2 \leq \omega_1$. If two worlds $\omega_1 \in P_i$ and $\omega_2 \in P_j$ are in different sets, it holds that $\omega_1 \leq \omega_2$ and $\omega_2 \not\leq \omega_1$ if and only if $i < j$. This partition in turn can be displayed with layers (see Figure 2). The worlds in the set P_0, i.e. the minimal worlds with respect to the total preorder, are placed in the lowest layer, the worlds in P_1 in the second lowest layer and so on. In particular we have $P_0 = \|\Psi\|$.

The user interface uses this concept of layers to realise an intuitive input of total preorders. Layers are indicated by lines, worlds by small boxes above the line of the layer they are placed in. Initially, all worlds are placed in the lowest layer, which corresponds to a total preorder where all worlds are considered equally plausible. From there, worlds can be moved around via 'drag and drop'. If a world is placed in the highest displayed layer, the next layer is added to the visualisation. Empty layers are removed automatically. A screenshot of this part of the user interface is shown in Figure 3.

The parameter α of a belief change is in fact a propositional formula over Σ. As we only consider the semantics of α, the user does not have to type in the formula but only specifies the interpretations that satisfy α. For this, the user interface displays a list of all interpretations. The interpretations can be selected by clicking on them. Selected interpretations are highlighted by different colours (see Figure 3).

After the 'Check belief change'-Button is clicked, the belief change is analysed and the results are displayed in a table (see Figure 4), which shows which postulates are fulfilled. If the mouse hovers over a postulates name, a tooltip with a summary of the postulate is displayed. For the convenience of the user the postulates are grouped.

Example Consider the situation where the initial epistemic state is associated with the total preorder displayed in Figure 2b: The worlds ab and $a\bar{b}$ are equally plausible. The worlds $\bar{a}b$ and $\bar{a}\bar{b}$ are also equally plausible but both less plausible than ab and $a\bar{b}$. We will further assume that the change happens under the proposition $\alpha = b$. The preorder of the resulting epistemic state is the one displayed in Figure 2c: The world ab is considered most plausible. The worlds where one of a or b is false are considered equally plausible and less plausible than ab. The world $\bar{a}\bar{b}$ is even less plausible.

Figure 3 shows how this belief change looks in Whiwap. The total preorders are visualized by their corresponding layers. The proposition b is entered by selecting ab and $\bar{a}b$ which are the interpretations of b. The result table for this change is displayed in Figure 4. The four Darwiche-Pearl postulates are fulfilled by this change, because the worlds fulfilling b (which are ab and $\bar{a}b$) do not change their order among each other, and the worlds not fulfilling $\bar{b}$ (which are $a\bar{b}$ and $\bar{a}\bar{b}$) do not change their order among each other. However, (DR) is not satisfied by this change. For an counter-example consider $\omega_1 = a\bar{b}$ and $\omega_2 = \bar{a}b$. Then we have $\omega_1 <_\Psi \omega_2$ and $\omega_2 \notin \|\Psi \circ \alpha\|$, but $\omega_1 \simeq_{\Psi\circ\alpha} \omega_2$.

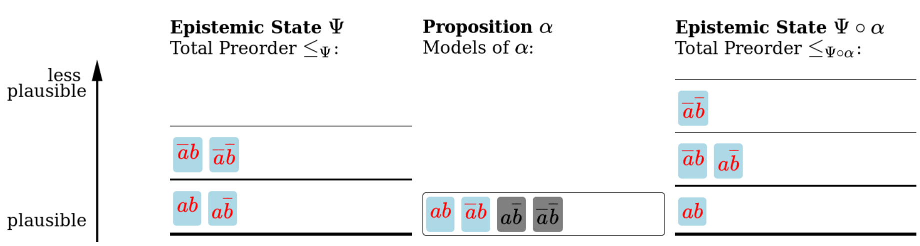


Fig. 3: User interface to enter a belief change. The worlds in the epistemic states can be moved via 'drag and drop'. The worlds which satisfy α can be selected by clicking on them in the proposition UI.

Postulate	$\Psi \to \Psi \circ \alpha$
Darwich-Pearl Iterated Revision	
CR1	true
CR2	true
CR3	true
CR4	true
Restrained Revision	
DR	false
PR	true
RR	false

...

Fig. 4: The result of the analysis is displayed in a table. This figure shows only the first part of the table.

3.2 Some Technical Details

WHIWAP consists of two parts: A frontend to provide the user interface and a backend to check the postulates.

The frontend is a web application build with HTML, CSS and JavaScript. It manages the user interactions and displays information. When the revision postulates are to be checked, the frontend encodes the epistemic states and parameters as JSON and sends them to the backend. As a response it receives a list of postulates, their descriptions, and a table listing which postulates are fulfilled for the revision. The frontend displays this data.

The backend is a small Java application running on a server. It receives the requests from the frontend and replies with a list of postulates, their descriptions, and a table listing which postulates are fulfilled for each revision step. The check for each postulate is implemented in a straightforward way. E.g., consider postulate DP1: Two nested loops iterate over all combinations (ω_1, ω_2) of the worlds that both satisfy α and check if (ω_1, ω_2) is in the preorder before the change if and only if it is in the preorder after the change. The complexity of testing whether a change fulfils a postulate is polynomial[1] in the size of the total preorders. However, a total preorder orders all interpretations, and thus, its size is exponential in relation to the signature. We refer the interested reader to an article by Liberatore on the general complexity of iterative change [11].

[1] for most of the above mentioned postulates

4 Discussion and Future Work

In this paper we introduced Whiwap, a tool that allows to quickly check what postulates are fulfilled by a certain change on epistemic states in the Darwiche-Pearl framework. In Section 3 we described the user interface and gave an overview over the technical realisation. Before we gave a presentation on the technical background in Section 2. The tool Whiwap is available online at:

https://www.fernuni-hagen.de/wbs/alchourron/

To the best of our knowledge there is no tool like Whiwap, which builds on the Darwiche-Pearl framework. However, there are some implementations for belief change in the (classical) setting of belief sets [2, 7, 8].

We believe that total preoders used by the Darwiche-Pearl framework are an intuitive base for belief change and reasoning. While this introduction given here is very technical, there are many applications of the framework. For instance, it might be interesting for psychology, where total preorders could act as mental representation, and thus, help in the task of analysing and formalising belief change of humans. Such a connection has been drawn [15].

For future work, we see many possibilities of advancing Whiwap. So far, Whiwap can only analyse one change at a time. A later version should allow the user to enter several change steps at once. The next step will be to improve the user interface such that counter-examples are presented for postulates that are not satisfied. Furthermore, it is our goal to advance Whiwap in such a way that it will predict the result of a belief change given only two of the ingredients of $\leq_{\Psi}$, α or $\leq_{\Psi\circ\alpha}$ in the light of a selection of belief change postulates.

Acknowledgements

This work was supported by DFG Grant BE 1700/9-1 awarded to Christoph Beierle as part of the priority program "Intentional Forgetting in Organizations" (SPP 1921). Kai Sauerwald is supported by this Grant.

References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. J. Symb. Log. **50**(2), 510–530 (1985)
2. Beierle, C., Kern-Isberner, G.: A verified AsmL implementation of belief revision. In: Börger, E., Butler, M., Bowen, J.P., Boca, P. (eds.) Abstract State Machines, B and Z. pp. 98–111. Springer Berlin Heidelberg, Berlin, Heidelberg (2008)
3. Booth, R., Meyer, T.A.: Admissible and restrained revision. J. Artif. Intell. Res. **26**, 127–151 (2006)
4. Boutilier, C.: Iterated revision and minimal change of conditional beliefs. J. Philosophical Logic **25**(3), 263–305 (1996)

5. Chopra, S., Ghose, A., Meyer, T.A., Wong, K.: Iterated belief change and the recovery axiom. J. Philosophical Logic **37**(5), 501–520 (2008). https://doi.org/10.1007/s10992-008-9086-2
6. Darwiche, A., Pearl, J.: On the logic of iterated belief revision. Artificial Intelligence **89**, 1–29 (1997)
7. Dixon, S., Wobcke, W.: The implementation of a first-order logic AGM belief revision system. In: Fifth International Conference on Tools with Artificial Intelligence, ICTAI '93, Boston, Massachusetts, USA, November 8-11, 1993. pp. 40–47. IEEE Computer Society (1993). https://doi.org/10.1109/TAI.1993.633934
8. Hunter, A., Tsang, E.: Genb: A general solver for AGM revision. In: Michael, L., Kakas, A.C. (eds.) Logics in Artificial Intelligence - 15th European Conference, JELIA 2016, Larnaca, Cyprus, November 9-11, 2016, Proceedings. Lecture Notes in Computer Science, vol. 10021, pp. 564–569 (2016). https://doi.org/10.1007/978-3-319-48758-8_40
9. Jin, Y., Thielscher, M.: Iterated belief revision, revised. Artif. Intell. **171**(1), 1–18 (2007)
10. Konieczny, S., Pino Pérez, R.: On iterated contraction: syntactic characterization, representation theorem and limitations of the Levi identity. In: Scalable Uncertainty Management - 11th International Conference, SUM 2017, Granada, Spain, October 4-6, 2017, Proceedings. Lecture Notes in Artificial Intelligence, vol. 10564. Springer (2017)
11. Liberatore, P.: The complexity of iterated belief revision. In: Afrati, F.N., Kolaitis, P.G. (eds.) Database Theory - ICDT '97, 6th International Conference, Delphi, Greece, January 8-10, 1997, Proceedings. Lecture Notes in Computer Science, vol. 1186, pp. 276–290. Springer (1997)
12. Nayak, A.C., Pagnucco, M., Peppas, P.: Dynamic belief revision operators. Artif. Intell. **146**(2), 193–228 (2003)
13. Ramachandran, R., Nayak, A.C., Orgun, M.A.: Three approaches to iterated belief contraction. J. Philosophical Logic **41**(1), 115–142 (2012)
14. Rott, H.: Shifting Priorities: Simple Representations for Twenty-Seven Iterated Theory Change Operators, pp. 269–296. Springer Netherlands, Dordrecht (2009). https://doi.org/10.1007/978-1-4020-9084-4_14
15. Sauerwald, K.: Student research abstract: Modelling the dynamics of forgetting and remembering by a system of belief changes. In: The 34th ACM/SIGAPP Symposium on Applied Computing (SAC '19), April 8–12, 2019, Limassol, Cyprus. ACM, New York, NY, USA (2019). https://doi.org/10.1145/3297280.3297391
16. Spohn, W.: Ordinal Conditional Functions: A Dynamic Theory of Epistemic States, pp. 105–134. Springer Netherlands, Dordrecht (1988)