On Elastic Image Registration with Varying Material Parameters

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Abstract. In this paper we are concerned with elastic image registration. Usually elastic approaches assume constant material parameters and result in a smooth displacement field. This approach has its shortcomings for images with varying elastic properties, like bones and soft tissue. Here, we present a scheme which not only allows for spatially varying material properties but enables the introduction of discontinuities within the displacement field. First numerical test runs for phantom images as well as for MR images indicate the potential of the discussed scheme.

1 Introduction

Nonrigid image registration is a challenging field in medical imaging. The task is to find a vector field of displacements such that each point in a template image can be mapped onto a corresponding and meaningful point in the reference image. By the notion 'meaningful' often a type of topology preserving constraint is meant. However, there exist several cases where changes in topology are essential. For instance, structures which are connected in one image may be disconnected in the other image, like the brain-skull interface subject to a brain shift. Additionally, structures may move along each other and thereby causing discontinuities, like the liver and its surrounding tissue.

Typically, the wanted displacement is computed subject to a smoothness constraint. For example, in elastic matching, the constraint is realized by a regularization based on the linear elastic potential of the displacement. In general, the constraint is applied globally with one global regularization parameter. Usually, the method provides satisfactory results due to the underlying physical model. Nonetheless it fails in cases described above, since a global regularization does not allow for any local changes in the topology and does not reflect possible different elasticity properties.

In the literature one may find a few attempts dealing with nonrigid image registration in conjunction with spatially varying material parameters, for example the radial basis functions [1], the Bezier tensor product [2], the damped springs [3], the finite elements [4, 5, 6] or the finite differences [7] based approaches, respectively. However, these methods either do not reflect the elastic behavior of the underlying material, or the registration yields a smooth transformation field, allowing for no discontinuities at all.

In this note, we present a new approach which overcomes the above mentioned shortcomings. We briefly summarize its mathematical formulation and report on some preliminary numerical test runs.

2 Theory and Methodology

Let $R, T : \Omega \to G$ denote the reference and the template image. Here, G denotes a set of grey values and, for simplicity, $\Omega \subset \mathbb{R}^2$ the image region. The registration aims at finding a displacement field $\boldsymbol{u} : \Omega \to \mathbb{R}^2$ such that $T_{\boldsymbol{u}} := T(\mathrm{id} - \boldsymbol{u})$ is similar to R. For the mathematical formulation of the registration problem, we make use of a variational model. Here, the functional $\mathcal{J}$ combined from two building blocks

$$\mathcal{J}[\boldsymbol{u}; T, R] := \mathcal{D}[\boldsymbol{u}; T, R] + \alpha \mathcal{S}[\boldsymbol{u}] \tag{1}$$

has to be minimized. The first ingredient $\mathcal{D}$ denotes a similarity measure, whereas $\mathcal{S}$ indicates a regularizer with regularization parameter α (for further details, see [8]). $\mathcal{D}$ can be chosen as any popular measure whenever its Gâteaux derivative exists. However, this note is restricted to the common sum of squared differences, $\mathcal{D}[\boldsymbol{u}; T, R] = \|R - T_{\boldsymbol{u}}\|_{L^2}$, which assumes monomodal images. Furthermore, we have chosen $\mathcal{S}$ as the widely-used linear elastic potential. However, in contrast to the conventional approach (1), where the weighting factor α is a global constant, α as well as the Lamé parameters λ, μ are assumed to be spatially dependent. Thus, a regularizer based on the linear elastic potential now looks like

$$\mathcal{S}[\boldsymbol{u}] = \mathcal{S}[\boldsymbol{u}; \alpha, \lambda, \mu] = \int_{\Omega} \alpha(\boldsymbol{\phi}) \left(\frac{\mu(\boldsymbol{\phi})}{4} \sum_{i,j=1}^{2} (\partial_{x_j} u_k + \partial_{x_k} u_j)^2 + \frac{\lambda(\boldsymbol{\phi})}{2} (\nabla \cdot \boldsymbol{u})^2 \right) dx, \tag{2}$$

where $\boldsymbol{\phi}(x) = x - \boldsymbol{u}(x)$. The computation of a Gâteaux derivative of (1) yields a necessary condition for $\boldsymbol{u}^*$ being a minimizer of (1). The outcome are non-linear partial differential equations known as the Navier-Lamé equations. Care has to be taken, in order to incorporate the spatial change in the parameters appropriately. Finally, a fixed-point type iteration scheme is applied to solve the partial differential equations.

Now, we briefly comment on how to exploit the additional information due to spatially variant material dependent parameters α, λ, μ. First of all, a meaningful segmentation needs to be computed. That is, we are looking for disjoint regions Ω_l, such that $\Omega = \cup_{l=0}^{m} \Omega_l$ and each region Ω_l^R in R corresponds intrinsically to a region $\Omega_l^{T_{\boldsymbol{u}}}$ in image $T_{\boldsymbol{u}}$. For convenience, let $\Omega_0^R, \Omega_0^{T_{\boldsymbol{u}}}$ denote the background of image R and image $T_{\boldsymbol{u}}$, respectively. Such a segmentation may not be easy to get. However, its computation is outside the scope of this note.

Having computed a segmentation, we are now in a position to let discontinuities be introduced, if these appear to be physically meaningful. More precisely, $\alpha = 0$ is set along a non-overlapping interface between two regions Ω_{l_1} and Ω_{l_2}.

If, in addition, the forces induced by $\mathcal{D}$ in $\Omega_{l_1}^{T\boldsymbol{u}}$ and $\Omega_{l_2}^{T\boldsymbol{u}}$ along the interface do not point in the same direction, a discontinuity in the displacement field will arise. Here, non-overlapping regions can be detected by $\Omega_{l_1}^{R} \cap \Omega_{l_2}^{T\boldsymbol{u}} = \emptyset$ and $\Omega_{l_2}^{R} \cap \Omega_{l_1}^{T\boldsymbol{u}} = \emptyset$.

Also, the segmentation may be used to assign to each subdomain different elastic properties and thereby to simulate diverse elastic behavior of different materials, like bones and muscles.

The system of partial differential equations can be discretized using finite differences, yielding a system of equations of size $2N$ (N being the total number of voxels in $\Omega \backslash \Omega_0^{T\boldsymbol{u}}$). This system has to be solved in every step of the fixed-point iteration. The system matrix corresponds to the Navier-Lamé differential operator and includes the additional information given by the segmentation and local parameters. The righthand-side results from the similarity measure and may be seen as force vector.

To evaluate the deformed template image $T_{\boldsymbol{u}}(x)$, an interpolation scheme has to be exploited. The interpolation, again, has to be done with respect to the segmentation. The grey value $T_{\boldsymbol{u}}(x)$ is calculated by a linear interpolation of $\phi(x)$ with respect to T provided that x and $\phi(x)$ belong to corresponding segmentation regions. Otherwise $T_{\boldsymbol{u}}(x)$ is set to some prescribed background value. This situation indicates a change of topology, since there exist a subregion in $T_{\boldsymbol{u}}$ with no correspondence in R.

3 Results

The proposed method has been applied to 2D phantom images as well as to 2D MR-images.

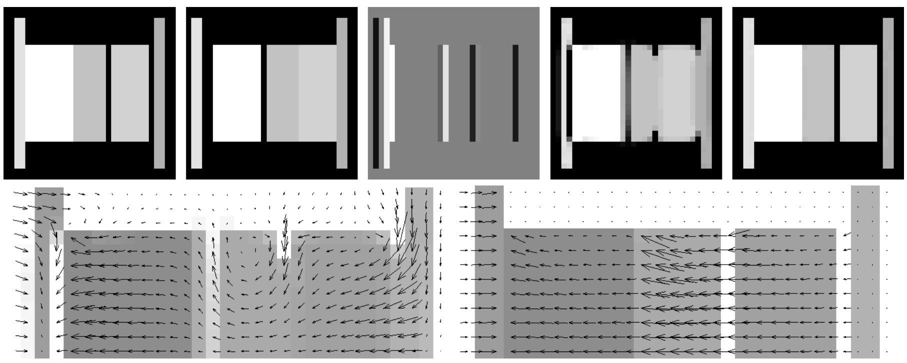
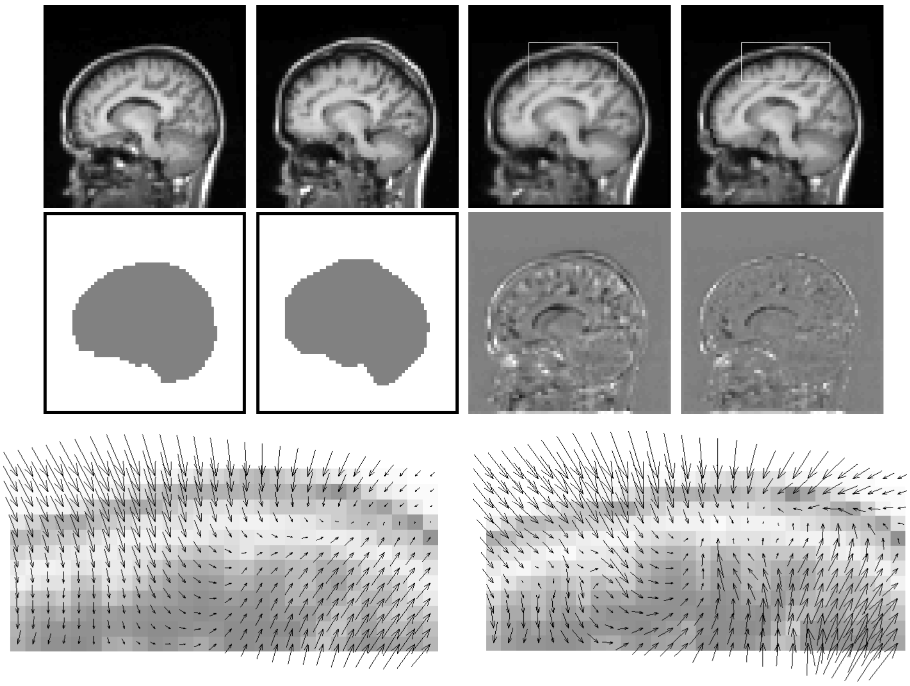


Fig. 1. Registration for a phantom image. In the upper row from left to right are shown the reference, template and subtraction image as well as the deformed template image using constant and varying parameters, respectively. In the lower row, parts of the deformation fields for the constant and the varying case are depicted (images inverted).

Fig. 2. Registration for a 2D MR head image. In the upper left quarter the reference and the template image together with their corresponding segmentation images are shown. The upper right quarter displays the deformed template together with the subtraction image for the case of constant parameters (third column) as well as for the case of varying parameters (last column). Below, parts of the deformation field are depicted (images inverted): left for constant parameters, right for varying parameters.



At first, a phantom image consisting of five rectangular objects is considered (Figure 1). Each object belongs to a single region Ω_l. For the outer objects there is nearly no change in position during transition from the template image to the reference image. The other ones are designed, such that they do change their positions in such a way that gaps between them show up or disappear. As it is apparent from Figure 1, a registration approach with constant parameters fails in such a case, whereas spatially varying parameters allow for a correct transformation of the template image. Note that the corresponding deformation fields nicely display the difference between the two approaches.

Next, slices from two MR data sets showing a human head have been registered. Again, registration is performed using the common elastic matching scheme with no spatially varying parameters and with the proposed method based on a manual segmentation of the brain (cf. Figure 2). Comparing the deformed images as well as the subtraction images and the zoomed displacement fields, an improved registration result is nicely visible. In particular at the brain-skull interface a displacement in opposite directions is expected (cf. the reference and the template image in Figure 2). While the common scheme fails for this task, the proposed method yields an outward displacement of the brain and an inward displacement of the skull.

4 Conclusion and Discussion

We have introduced an elastic matching based registration scheme with local parameters. It is shown that indeed this approach is superior to the conventional schemes, when pathological changes occur in the images. The feasibility of the proposed algorithm has been shown for an academic example as well as for a nontrivial real-life example. The first results are very convincing.

However, it should be noted, that the system matrices are not as nicely structured as for the conventional case. As a consequence, they are no longer solvable by Fourier techniques (see [8]). Currently, we are investigating dedicated multigrid techniques for the solution part.

Also, it would be desirable, if only one of the images needs to be segmented. In particular for time-critical tasks, like brain shift, it is of great interest if a segmentation of the pre-operative generated image is sufficient. If so, a time consuming segmentation of the intra-operative generated image would be redundant. We are currently investigating possible strategies for this nontrivial task.

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