Delta Range Positioning Assisted With Satellites

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Abstract. We know that real global positioning, i.e. in all environments, has been a scientific challenge for several years. The Global Positioning System (GPS) launched the impulse with its democratization in the 2000s: geographical position has gradually become a constituent element of modern digital devices available to general public. Integrating a GPS chip into smartphones was only one step. The rise of the Internet Of Thing announced by the deployment of 5G reinforces this trend. Indeed, more and more connected objects will be present and many of their applications need the geographical position. In this context, satellite signals play a key role in calculating position, especially outdoors using the GPS function. In more challenging environments, or when the need for accuracy becomes crucial, these signals can be used alone or in conjunction with others (or other physical measurements). The recent deployment of several constellations of what are now called the Global Navigation Satellites Systems (GNSS) further reinforces the presence of these signals, which can therefore be used. The previous works [1-2] focused on a positioning method using the measurements of variations of distances between a mobile receiver device and transmitters (pseudolites) placed in the immediate environment. The same principles applied with GNSS satellites give the displacement vectors with a good accuracy. This allows carrying out positioning assisted with satellite whose results are presented here.

Keywords: Least-Square algorithm, GNSS, Carrier Phase measurements; Pseudolites.

1 Delta Range Positioning

1.1 Introduction

We know that positioning with GNSSs has an accuracy of a few meters outdoor (typically 7 to 12 meters User Range Error for GPS [3]). The principle of GNSS positioning is based on signal delay measurements between satellites and receiver. To measure these propagation times, the GPS receiver uses the "code" component of the GNSS signal [4]. This component allows an accuracy of a few meters on the measurements of satellite to receiver distances. To improve this accuracy, the carrier phase of the signal can be used. This one is indeed less sensitive to errors. We can obtain a measurement with an accuracy of a few cm, even a few mm under certain conditions, for instance for geodesic applications [5]. However, its use is problematic. Indeed, the receiver can only measure a fraction φ of the carrier phase whose maximum value is equal to λ, the wavelength of the signal (for example λ = 19 cm for the GPS signal on L1 band). Thus, of the total distance between a satellite and the receiver, only one fragment is known. The remainder is a multiple of the wavelength. The distance can be expressed as in (1):

$$D_{\mathrm{Satellite-Receiver}} = \varphi + N.\lambda \tag{1}$$

Subsequent paragraphs, however, are indented. N is what is called the integer ambiguity. The calculation of precise positioning with satellites can often be reduced to the search for the values of the integer ambiguities. There are many methods that are based on the search for these ambiguities. Perhaps the best known is the phase differential GNSS, whose most famous application is better known as Real Time Kinematic (RTK). This is a Differential GNSS method that requires a base station whose position is known. The pseudoranges (or distances measurements) measured with code and carrier phase are double differenced to form a new set of equation that allows the determination of ambiguities, with determination of a minimum of a cost function [6].

Other older methods exist such as the triple difference. The triple difference introduces a notion of time to solve the ambiguities. The double differences are taken at two different moments and differenced once again [7]. Actually, the movements of the satellites are used to solve the ambiguities. This necessitates waiting several minutes before getting a position [8].

The Delta Range method, which we will return to in the next section, bypasses the problem of complete ambiguity by focusing on variations in distances as the receiver moves. In theory, we no longer need to know the value of the transmitter/receiver distance: its variation is sufficient.

1.2 Delta Range Positioning Method

The proposed method was based on the measurement of the variation of the distance “Pseudolite-Receiver” between two positions. In this section, we present the equation system to be solved, which derives from the basic equations. We present here a simplified version.

Let’s consider that we have eight beacons (there could be more or less, but to ease the demonstration we use eight). These beacons are transmitting towards (or receiving from, depending of the technology) a device which wants to know its position in the area. Fig.1. shows the typical positing situation. Measurements are carried out between two instants t_2 and t_1, we could consider more successive instants [1], but for the sake of clarity, we will limit ourselves to two. The variation of the distances “beacons-device” between t_2 and t_1 are measured. Equation (2) gives the mathematical relationship between the unknowns of the issue and these measurements:

$$\Delta\varphi_{21}^{k} = D_{2}^{k} - D_{1}^{k} + \varepsilon \tag{2}$$

With:

$$D_j^k = \sqrt{\left(x_j - x_{b_k}^j\right)^2 + \left(y_j - y_{b_k}^j\right)^2 + \left(z_j - z_{b_k}^j\right)^2} \tag{3}$$

$k = \{1..8\}$ refers to the kth beacon.
(x_j,y_j,z_j) the coordinates of the position device at the instant t_j
$\left(x_{b_k}^j, y_{b_k}^j, z_{b_k}^j\right)$ the coordinates of the transmitters at the instant t_j
$\Delta\varphi_{21}^k$ the measured difference of distance "beacon k-device" between the instant 2 and the instant 1.This measurement includes any error ε.

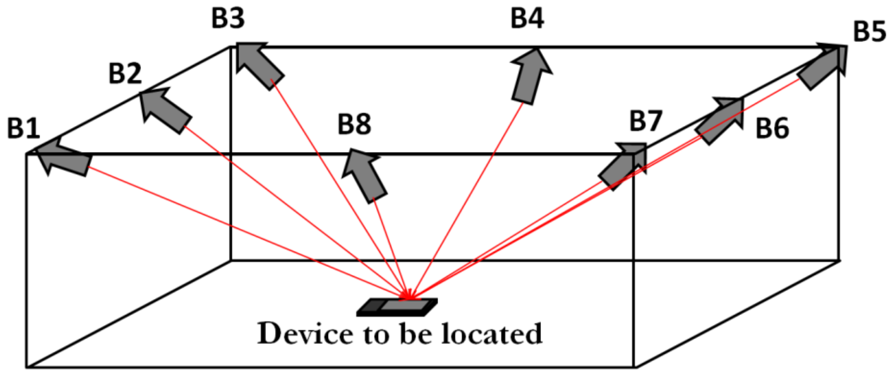

Fig. 1. Principle of positioning with a local constellation of pseudolites.

For 3D positioning, we thus have 6 unknowns (x_1; y_1; x_2; y_2; z_1; z_2). The coordinates of the beacons are fixed and considered known. With 8 beacons, we obtain 8 independent equations which can theoretically solve the issue. As explained in [4], we can add equations by making measurements at a third instant t_3, but this is not necessary here. We will then have asset of 6 unknowns $\{x_1; y_1; z_1; x_2; y_2; z_2\}$ and 8 equations (with the $\Delta\varphi_{21}^k$). This corresponds to an over determined systems of equations.

The next step consists of linearization of the previous equations. We note f_k^{21} as a function of the unknown:

$$f_k^{21}(x_1, y_1, z_1, x_2, y_2, z_2) = D_2^k - D_1^k \tag{4}$$

This function is developed in a first order Taylor series about the approximate "position". The approximate position corresponds to the set: $\hat{X}=\{\hat{x}_1; \hat{y}_1; \hat{z}_1; \hat{x}_2; \hat{y}_2; \hat{z}_2\}$. We note $\Delta X = \{dx_1; dy_1; dz_1; dx_2; dy_2; dz_2\}$, the variations around the approximate position. Equation (2) is the Taylor development of f_k around the approximate position:

$$f_k^{21}(\hat{X} + \Delta X) = f_k^{21}(\hat{X}) + \frac{\partial f_k^{21}}{\partial x_1}(\hat{X})dx_1 + \frac{\partial f_k^{21}}{\partial y_1}(\hat{X})dy_1 + \frac{\partial f_k^{21}}{\partial z_1}(\hat{X})dz_1 + \frac{\partial f_k^{21}}{\partial x_2}(\hat{X})dx_2 + \frac{\partial f_k^{21}}{\partial y_2}(\hat{X})dy_2 + \frac{\partial f_k^{21}}{\partial z_2}(\hat{X})dz_2 + \sigma(\Delta X) \tag{5}$$

With $\sigma(X)$ the second and higher orders terms.

All this is just as in Gauss-Newton GNSS positioning algorithm [4]. However, it is here apply to a difference of Euclidian distances (3) instead of a single distance.

Following equation (6) is obtained:

$$a^2_{x_k}dx_2 + a^2_{y_k}dy_2 + a^2_{z_k}dz_2 - a^1_{x_k}dx_1 - a^1_{y_k}dy_1 - a^1_{z_k}dz_1 = \Delta\varphi^k_{21} - \Delta\hat{\rho}^k_{21} \qquad (6)$$

With:

$$R^k_j = \sqrt{\left(\hat{x}_j - x^j_{b_k}\right)^2 + \left(\hat{y}_j - y^j_{b_k}\right)^2 + \left(\hat{z}_j - z^j_{b_k}\right)^2} \qquad (7)$$

$$a^j_{x_k} = \frac{x_{b_k} - \hat{x}_j}{R^k_j};\ a^j_{y_k} = \frac{y_{b_k} - \hat{y}_j}{R^k_j};\ a^j_{z_k} = \frac{z_{b_k} - \hat{z}_j}{R^k_j} \qquad (8)$$

$$\text{and} \qquad \Delta\hat{\rho}^k_{i1} = R^k_j - R^k_1 \qquad (9)$$

Thus linearized, the system of equations to solve can now be written as a matrix product:

$$H^t H \Delta X^t = H^t d\Delta\varphi \qquad (10)$$

With:

$$H=\begin{bmatrix} a^1_{x_1} & a^1_{y_1} & a^1_{z_1} & -a^2_{x_1} & -a^2_{y_1} & -a^2_{z_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a^1_{x_k} & a^1_{y_k} & a^1_{z_k} & -a^2_{x_k} & -a^2_{y_k} & -a^2_{z_k} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a^1_{x_8} & a^1_{y_8} & a^1_{z_8} & -a^2_{x_8} & -a^2_{y_8} & -a^2_{z_8} \end{bmatrix} \qquad (11)$$

and:

$$d\Delta\varphi = \begin{bmatrix} \Delta\varphi^1_{21} - \Delta\hat{\rho}^1_{21} \\ \vdots \\ \Delta\varphi^k_{21} - \Delta\hat{\rho}^k_{21} \\ \vdots \\ \Delta\varphi^8_{21} - \Delta\hat{\rho}^8_{21} \end{bmatrix} \qquad (12)$$

We use the Least-Square algorithm because the system is overdetermined. The inversion of (10) gives (13):

$$\Delta X^t = (H^t H)^{-1} H^t d\Delta\varphi\, \hat{X} \qquad (13)$$

ΔX^t gives the variation between the approximate position (which is a hypothetic position) and the real position (or more precisely a position that is coherent with the measurement $d\varphi$). The next step consists of updating $\hat{X}$:

$$\hat{X} \leftarrow \hat{X} + \Delta X^t \qquad (14)$$

Then, we go back to (6) to apply (13) with the new $\hat{X}$. The system is thus iteratively solved until the error $|\Delta X^t|$ is equal to zero (in practice and $\varepsilon << 0$). The purpose of the algorithm is finally to find the $\widehat{X}$ which minimizes the set of functions f_k^{21}.

Note that we consider that the variation of the clock bias between the transmitters and the receiver is known. This is not true in practice [1]. We prefer to focus on geometrical issues; the very specific issue of the clock bias will be addressed in future works.

At this stage, we would like to apply the principles of the previously presented Delta Range positioning with GNSS satellites as transmitters.

2 Delta Range Positioning with Satellites

2.1 Displacement Vector with satellites

It is not possible to apply the Delta Range method directly with satellites. In a nutshell, it can be said that the very large distances involved with satellites are responsible for this. This can be understood by looking at equations (8) and (11). If the distance R_j^k is very large with respect to the differences between coordinates (x_2,y_2,z_2) and (x_1,y_1,z_1), the matrix H will not be invertible. Indeed, in this case, even if the satellite is moving, i.e. (x_{bk}, y_{bk}, z_{bk}) change between instant 1 and 2, we have: $a_{x_k}^1 \approx a_{x_k}^2$, $a_{y_k}^1 \approx a_{y_k}^2$ and $a_{z_k}^1 \approx a_{z_k}^2$ for each transmitter k. Then H is not invertible.

However, the algorithm can be adapted. We can demonstrate that the large distances can be compensated by considering approximation taking into account the satellite motion. We can thus obtain not exactly the positions, but the displacement vector $\vec{D}$ between the position of the receiver at t_1 and the position at t_2.

$$\vec{D} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \qquad (15)$$

We will detail the method to obtain this vector in future publications. We are presenting now some simulation results of the determination of the vector $\vec{D}$ for different scenarios.

2.2 Simulations Results with satellites

The simulation results presented here correspond to a displacement of 1 to a few meters, starting from a point on the Earth's surface with the following coordinates:

P_1: Lat: 48.6198530° N, Lon: 2.430451° E, Alt: 105 m

The displacement remains in two dimensions on the Earth's surface. We use 7 GPS satellites with real ephemeris. Their distribution around position P_1 corresponds to a

PDOP of about 1.5 [4]. The results are tested for 1, 5 and 10 seconds of travel, for lengths of 1, 2, 5 and 10 meters.

We try to simulate more or less realistic reception conditions outdoors. A Gaussian error centered with standard deviation of 0.001 m, 0.01 m and 0.1 m is randomly added to the assumed carrier phase measurements. This is typically the kind of error that is obtained on a phase measurement for different reception conditions [9].

Tables I, II and III present the results obtained in terms of error on the displacement vector.

Table I: Simulation Data for an error $\sigma = 0.001$m

Duration	**Distance P_1P_2**			
	1m	***2m***	***5m***	***10m***
1 sec	1.1 mm	1.1 mm	1.1 mm	0.9 mm
5 sec	1.7 mm	1.9 mm	2.2 mm	2.4 mm
10 sec	3 mm	3.3 mm	3.8 mm	5 mm

Table II: Simulation Data for an error $\sigma = 0.01$m

Duration	**Distance P_1P_2**			
	1m	***2m***	***5m***	***10m***
1 sec	7 mm	6.9 mm	6.8 mm	6.7 mm
5 sec	3.8 mm	3.5 mm	2.9 mm	2.8 mm
10 sec	0.3 mm	0.3 mm	1.5 mm	5.4 mm

Table III: Simulation Data for an error $\sigma = 0.1$m

Duration	**Distance P_1P_2**			
	1m	***2m***	***5m***	***10m***
1 sec	60 mm	50 mm	47 mm	46 mm
5 sec	99 mm	73 mm	69 mm	67 mm
10 sec	116 mm	94 mm	97 mm	98 mm

In Table I, we observe a phenomenon of deterioration in performance (albeit small) linked both to the total distance travelled and also to the duration of the movement. This is not surprising, it is simply the illustration that the higher order terms become a little less negligible for larger distance covered and for longer duration of the motion (which results in a larger satellite displacement). In practical terms, this means that it would be better to favor short movements over short durations.

Table II does not confirm all these trends. The presence of a higher measurement noise complicates the analysis a little. For the one-second travel time, the logic is respected: overall deterioration in performance compared to Table I, but general stability of the error is observed. For 5 seconds duration, performance improves with distance. It can be deduced that the effect of second-order terms is superseded by the effect of improvement with respect to noise related to a larger distance. This is a phenomenon already observed with the Deltarange algorithm: the more the distance travelled increases, the less sensitive the algorithm is to noise. This is caused by a phenomenon comparable to that of dilution of precision [1]. For the duration of 10 seconds overall, the performance is better than with less noise, except for a distance of 10 meters covered, for which it is comparable.

Table III shows performances ranging from 5 to 12 cm of error depending on the case. It can be reminded that a 10 cm error on a phase measurement means that we are under difficult reception conditions. We still manage to obtain a displacement vector with an error of a few cm. We are quite within the orders of magnitude of reasonable error for a position calculation.

2.3 Experimentation Results with Satellites assisted Delta Range method

Now we would like to use these results to carry out assistance of Delta Range method. The main advantage of the knowledge of vector $\vec{D}$ is the reduction of the number of unknowns. Indeed, the Delta Range algorithm can consider two successive instants t_1 and t_2, as we presented here, or more (t_i). Then we consider a set of displacement vectors in this form:

$$\vec{D} = \begin{pmatrix} x_i - x_1 & \cdots & x_3 - x_1 & x_2 - x_1 \\ y_i - y_1 & \cdots & y_3 - y_1 & y_2 - y_1 \\ z_i - z_1 & \cdots & z_3 - z_1 & z_2 - z_1 \end{pmatrix} \tag{16}$$

If we note $D_x^i = x_i - x_1$, $D_y^i = y_i - y_1$ and $D_z^i = z_i - z_1$, the knowledge of $\vec{D}$ leads to the reduction of the number of unknowns to the coordinates of the initial position P_1 (x_1,y_1,z_1) only. Indeed, if we consider our example, the set of unknowns {x_1; y_1; z_1; x_2; y_2; z_2} can be written: $\{x_1;\ y_1;\ z_1;\ x_1 + D_x^2;\ y_1 + D_y^2;\ z_1 + D_z^2\}$.

We are illustrating this by using this knowledge as assistance of previous experiments presented in [1], reporting the errors previously obtained with simulations on vector $\vec{D}$ determination. This consists of deploying four transmitters (pseudolites) emitting a GPS signal and measuring the phase variation in an urban canyon-type environment measuring 20 m by 30 m. Table IV indicates the results obtained in terms of determination of the starting point P_1.

Table IV: Error on P_1 determination (x tr = number of transmitters, b = best, w = worst)

	Error applied to vector D					
Numbers of Points	**4 tr**			**3 tr**	**2 tr (b)**	**2 tr (w)**
	1.1 mm	***6.9 mm***	***50 mm***	***1.1mm***	***1.1mm***	***1.1mm***
2	47 cm	40 cm	54 cm	57 cm	28cm	4.3 m
3	36 cm	39 cm	78 cm	31 cm	24 cm	2.1 m

3 Conclusion and Future Works

There are a lot of factors that must be taken into account to explain the results on Table IV. We notice that globally, increasing the error on vector $\vec{D}$ determination increases the error on P_1. This is coherent. We can decrease the number of transmitters, but the quality of the measurements will have a bigger influence than with several transmitters. The last two columns of Table IV show the worst and the best results obtained. These results largely depend of the quality of the measured differences of distance for the two considered transmitters.

Our future works consist of carry out experiment of vector $\vec{D}$ determination and integrating the assistance to local transmitters positioning method.

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