A Well-founded Semantics for Hybrid MKNF Knowledge Bases*

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Abstract. In [10], hybrid MKNF knowledge bases have been proposed for combining open and closed world reasoning within the logics of minimal knowledge and negation as failure ([8]). For this powerful framework, we define a three-valued semantics and provide an alternating fixpoint construction for nondisjunctive hybrid MKNF knowledge bases. We thus provide a well-founded semantics which is a sound approximation of the cautious MKNF model semantics, and which also features improved computational properties. We also show that whenever the DL knowledge base part is empty, then the alternating fixpoint coincides with the classical well-founded model.

1 Introduction

One of the major open research questions in Description Logic (DL) research is how to combine the open-world semantics of DLs with the closed-world semantics featured by (nonmonotonic) logic programming (LP). Much of this research effort is being driven by the needs of the Semantic Web initiative. Indeed, the addition of rules, in LP style, on top of the DL-based ontology layer has been recognized as an important task for the success of the Semantic Web, and initiatives are being taken to define such a rule layer (cf. the Rule Interchange Format working group of the W3C). Combining LP rules and DLs indeed is a non-trivial task since these two formalisms are based on different assumptions: the former is nonmonotonic, relying on the closed world assumption, while the latter is based on first-order logic under the open world assumption.

Accordingly, several proposals have been made for dealing with knowledge bases (KB) which contain DL and LP statements (see e.g. [2–4, 7, 10, 12]). But apart from [4], they rely on the stable models semantics (SMS) of logic programs [6]. It is our stance that, especially for use in the Semantic Web, the well-founded semantics (WFS) [14], though being closely related to SMS (see e.g. [5]), is often the better choice. Indeed, in applications dealing with large amounts of information, the polynomial worst-case complexity of WFS is preferable to the NP-hard SMS. Furthermore, the WFS is defined for all programs and allows to

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answer queries by consulting only the relevant part of a program whereas SMS is neither relevant nor always defined.

While the approach in [4] is based on a loose coupling between DL and LP, others are tightly integrated. The most advanced of these approaches currently appears to be that of hybrid MKNF knowledge bases [10] which is based on the logic of Minimal Knowledge and Negation as failure (MKNF) [8]. Its advantage lies in a seamless integration of DL and LP which is nevertheless decidable due to the restriction of reasoning in the program part to known constants by means of DL-safe rules.

In this paper, we define a well-founded semantics for hybrid MKNF knowledge bases, for now restricting to nondisjunctive MKNF rules, which compares to that of [10] as the WFS does to the SMS of LP:

- our well-founded semantics is a sound approximation of the semantics of [10]
- the computational complexity is strictly lower
- the semantics retains the property of [10] of being faithful, but now wrt. the WFS, i.e. when the DL part is empty, it coincides with the WFS of LPs.

We start by recalling basic notions and then introduce models in a 3-valued setting. The paper continues with the definition of the proposed semantics and some of its properties. We end with conclusion and future work. Lack of space prevents us from presenting all proofs, which can be found in the extended report at http://centria.di.fct.unl.pt/∼mknorr/wfmknf-extd.pdf.

2 Preliminaries

MKNF notions. We start by recalling the syntax of MKNF formulas from [10]. A *first-order atom* $P(t_1, \ldots, t_n)$ is an MKNF formula where P is a predicate and the t_i are first-order terms[3]. If φ is an MKNF formula then $\neg\varphi$, $\exists x : \varphi$, $\mathbf{K}\,\varphi$ and $\mathbf{not}\,\varphi$ are MKNF formulas and likewise $\varphi_1 \wedge \varphi_2$ and $\varphi_1 \subset \varphi_2$ for MKNF formulas φ_1, φ_2. We use the following symbols to represent boolean combinations of the previously introduced syntax, i.e. $\varphi_1 \vee \varphi_2$ for $\neg(\neg\varphi_1 \wedge \neg\varphi_2)$, $\varphi_1 \equiv \varphi_2$ for $(\varphi_1 \subset \varphi_2) \wedge (\varphi_2 \subset \varphi_1)$, and $\forall x : \varphi$ for $\neg\exists x : \neg\varphi$. Substituting the free variables x_i in φ by terms t_i is denoted $\varphi[t_1/x_1, \ldots, t_n/x_n]$. Given a (first-order) formula φ, $\mathbf{K}\,\varphi$ is called a *modal* $\mathbf{K}$*-atom* and $\mathbf{not}\,\varphi$ a *modal* $\mathbf{not}$*-atom*. An MKNF formula φ without any free variables is a *sentence* and *ground* if it does not contain variables at all. It is *positive* if it does not contain the operator **not**.

It is assumed that apart from the constants occurring in the formulas the signature contains a countably infinite supply of constants not occurring in the formulas. The Herbrand Universe of such a signature is also denoted $\triangle$. The signature contains the equality predicate $\approx$ which is interpreted as congruence relation on $\triangle$. An *MKNF structure* is a triple (I, M, N) where I is an Herbrand first-order interpretation over $\triangle$ and M and N are nonempty sets of Herbrand first-order interpretations over $\triangle$. For the 2-valued satisfiability of MKNF sentences we refer only to [10], since we will define 3-valued satisfiability in a way that, when restricted to 2-valued trivially coincides with the one of [10].

[3] We consider function-free first-order logic, so terms are either constants or variables.

Hybrid MKNF Knowledge Bases. Quoting from [10], the approach of hybrid MKNF knowledge bases is applicable to any first-order fragment $\mathcal{DL}$ satisfying these conditions: (i) each knowledge base $\mathcal{O} \in \mathcal{DL}$ can be translated into a formula $\pi(\mathcal{O})$ of function-free first-order logic with equality, (ii) it supports *A-Boxes*-assertions of the form $P(a_1, \ldots, a_n)$ for P a predicate and a_i constants of $\mathcal{DL}$ and (iii) satisfiability checking and instance checking (i.e. checking entailments of the form $\mathcal{O} \models P(a_1, \ldots, a_n)$ are decidable[4].

We recall MKNF rules and hybrid MKNF knowledge bases from [10]. For the rationales behind these and the following notions we also refer to [9].

Definition 2.1. *Let $\mathcal{O}$ be a DL knowledge base. A first-order function-free atom $P(t_1, \ldots, t_n)$ over Σ such that p is $\approx$ or it occurs in $\mathcal{O}$ is called a* DL-atom*; all other atoms are called* non-DL-atoms*. An MKNF rule r has the following form where H_i, A_i, and B_i are first-order function free atoms:*

$$\mathbf{K}\, H_1 \vee \ldots \vee \mathbf{K}\, H_l \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n, \mathbf{not}\, B_1, \ldots, \mathbf{not}\, B_m \tag{1}$$

The sets $\{\mathbf{K}\, H_i\}$, $\{\mathbf{K}\, A_i\}$, and $\{\mathbf{not}\, B_i\}$ are called the rule head*, the* positive body*, and the* negative body*, respectively. A rule is* nondisjunctive *if $l = 1$; r is* positive *if $m = 0$; r is a* fact *if $n = m = 0$. A* program *is a finite set of MKNF rules. A* hybrid MKNF knowledge base *$\mathcal{K}$ is a pair $(\mathcal{O}, \mathcal{P})$ and $\mathcal{K}$ is* nondisjunctive *if all rules in $\mathcal{P}$ are nondisjunctive.*

The semantics of an MKNF knowledge base is obtained by translating it into an MKNF formula ([10]).

Definition 2.2. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF knowledge base. We extend π to r, $\mathcal{P}$, and $\mathcal{K}$ as follows, where x is the vector of the free variables of r.*

$$\pi(r) = \forall x : (\mathbf{K}\, H_1 \vee \ldots \vee \mathbf{K}\, H_l \subset \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n, \mathbf{not}\, B_1, \ldots, \mathbf{not}\, B_m)$$

$$\pi(\mathcal{P}) = \bigwedge_{r \in \mathcal{P}} \pi(r) \qquad \pi(\mathcal{K}) = \mathbf{K}\, \pi(\mathcal{O}) \wedge \pi(\mathcal{P})$$

An MKNF rule r is *DL-safe* if every variable in r occurs in at least one non-DL-atom $\mathbf{K}\, B$ occurring in the body of r. A hybrid MKNF knowledge base $\mathcal{K}$ is *DL-safe* if all its rules are DL-safe. Given a hybrid MKNF knowledge base $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, the *ground instantiation of* $\mathcal{K}$ is the KB $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ where $\mathcal{P}_G$ is obtained by replacing in each rule of $\mathcal{P}$ all variables with constants from $\mathcal{K}$ in all possible ways. Then it was shown in [9], for a DL-safe hybrid KB $\mathcal{K}$ and a ground MKNF formula ψ, that $\mathcal{K} \models \psi$ if and only if $\mathcal{K}_G \models \psi$.

3 Well-founded MKNF Semantics

3.1 Three-valued Models

Satisfiability as defined in [10] allows modal atoms only to be either true or false in a given MKNF structure. We extend the framework by allowing a third

[4] For more details on DL notation we refer to [1].

truth value **u**, denoting undefined, to be assigned to modal atoms while first-order atoms remain two-valued due to being interpreted solely in one first-order interpretation. We therefore introduce *consistent* MKNF structures which, for all MKNF formulas φ over some given signature, do not allow φ to be true for all $J \in M$ and false for some $J \in N$ at the same time. Subsequently, we evaluate MKNF sentences in consistent MKNF structures with respect to the set $\{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ of truth values with the order $\mathbf{f} < \mathbf{u} < \mathbf{t}$:

- $(I, M, N)(p(t_1, \ldots, t_n)) = \begin{cases} \mathbf{t} & \text{iff } p(t_1, \ldots, t_n) \in I \\ \mathbf{f} & \text{iff } p(t_1, \ldots, t_n) \notin I \end{cases}$
- $(I, M, N)(\neg\varphi) = \begin{cases} \mathbf{t} & \text{iff } (I, M, N)(\varphi) = \mathbf{f} \\ \mathbf{u} & \text{iff } (I, M, N)(\varphi) = \mathbf{u} \\ \mathbf{f} & \text{iff } (I, M, N)(\varphi) = \mathbf{t} \end{cases}$
- $(I, M, N)(\varphi_1 \wedge \varphi_2) = \min\{(I, M, N)(\varphi_1), (I, M, N)(\varphi_2)\}$
- $(I, M, N)(\varphi_1 \supset \varphi_2) = \mathbf{t}$ iff $(I, M, N)(\varphi_2) \geq (I, M, N)(\varphi_1)$ and $\mathbf{f}$ otherwise
- $(I, M, N)(\exists x : \varphi) = \max\{(I, M, N)(\varphi[\alpha/x]) \mid \alpha \in \triangle\}$
- $(I, M, N)(\mathbf{K}\,\varphi) = \begin{cases} \mathbf{t} & \text{iff } (J, M, N)(\varphi) = \mathbf{t} \text{ for all } J \in M \\ \mathbf{f} & \text{iff } (J, M, N)(\varphi) = \mathbf{f} \text{ for some } J \in N \\ \mathbf{u} & \text{otherwise} \end{cases}$
- $(I, M, N)(\mathbf{not}\,\varphi) = \begin{cases} \mathbf{t} & \text{iff } (J, M, N)(\varphi) = \mathbf{f} \text{ for some } J \in N \\ \mathbf{f} & \text{iff } (J, M, N)(\varphi) = \mathbf{t} \text{ for all } J \in M \\ \mathbf{u} & \text{otherwise} \end{cases}$

The operator max chooses the greatest element with respect to the truth ordering given above and likewise min chooses the least one. We can see that the truth of modal atoms is evaluated just as in the two-valued case (see [10]), we only have to separate additionally false from undefined modal atoms which is done by means of the other set of interpretations in the structure. Note that implications and objective MKNF formulas can never be undefined.

Definition 3.1. *An* interpretation pair (M, N) *consists of two MKNF interpretations* M, N *and* models *a closed MKNF formula* φ, *written* $(M, N) \models \varphi$, *if and only if* $(I, M, N)(\varphi) = \mathbf{t}$ *for each* $I \in M$. *We call* φ consistent *if there exists an interpretation pair modeling it.*

It is straightforward to see (cf. [10]) that (M, M) corresponds to the (two-valued) *MKNF interpretation* M, i.e. a nonempty set of Herbrand first-order interpretations over $\triangle$, since there are no undefined modal atoms in it. In this case, recalling from [10], M is additionally an *MKNF model* if (1) $(I, M, M)(\varphi) = \mathbf{t}$ for all $I \in M$ and (2) for each MKNF interpretation M' such that $M' \supset M$ we have $(I', M', M)(\varphi) = \mathbf{f}$ for some $I' \in M'$.

Example 3.1. Let us consider the following hybrid MKNF knowledge base

$$NaturalDeath \sqsubseteq Pay \qquad Suicide \sqsubseteq \neg Pay$$

$$\mathbf{K}\,Pay(x) \leftarrow \mathbf{K}\,murdered(x), \mathbf{K}\,benefits(y, x), \mathbf{not}\,responsible(y, x)$$
$$\mathbf{K}\,Suicide(x) \leftarrow \mathbf{not}\,NaturalDeath(x), \mathbf{not}\,murdered(x)$$
$$\mathbf{K}\,murdered(x) \leftarrow \mathbf{not}\,NaturalDeath(x), \mathbf{not}\,Suicide(x)$$

based on which a life insurance company decides whether to pay or not the insurance. Additionally, we know that Mr. Jones who owned a life insurance was found death in his living room, the revolver still in his hand. Thus we add $\neg NaturalDeath(jones)$ and the last two rules offer us a choice between commitment of suicide or murder. While immediately obtaining two MKNF models in such a scenario, the three-valued framework allows to assign **u** to both so that we delay this decision until the evidence is evaluated. Until then, by the first rule, also no payment is possible.

3.2 Alternating Fixpoint for Hybrid MKNF

As discussed in [9], since an MKNF model M is in general infinite, instead of representing M directly, a first-order formula φ is computed such that M is exactly the set of first-order models of φ. This is possible for modally closed MKNF formulae and the ideas from [11] are applied to provide a partition (P, N) of modal atoms which uniquely defines φ. We extend this idea by allowing partitions to be partial in the sense that modal atoms may occur neither in P nor in N, i.e. are neither true nor false but supposed to be undefined. To obtain the unique desired partial partition we apply a technique known from logic programming: stable models ([6]) for normal logic programs correspond one-to-one to MKNF models of programs of MKNF rules (see [8]). The well-founded model ([14]) for normal logic programs can be computed by an alternating fixpoint of the operator used to define stable models ([13]).

Here we proceed similarly: we define an operator providing a stable condition for nondisjunctive hybrid MKNF knowledge bases and use it to obtain an alternating fixpoint, the well-founded semantics. We thus start by adapting some notions from [10] formalizing partitions and related concepts.

Definition 3.2. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF knowledge base. The* set of $\mathbf{K}$-atoms of $\mathcal{K}$, *written $\mathsf{KA}(\mathcal{K})$, is the smallest set that contains (i) all $\mathbf{K}$-atoms of $\mathcal{P}_G$, and (ii) a modal atom $\mathbf{K}\,\xi$ for each modal atom $\mathbf{not}\,\xi$ occurring in $\mathcal{P}_G$.*

For a subset P of $\mathsf{KA}(\mathcal{K})$, the objective knowledge *of P is the formula $\mathsf{ob}_{\mathcal{K},P} = \mathcal{O} \cup \bigcup_{\mathbf{K}\,\xi \in P} \xi$. A* (partial) partition *(P, N) of $\mathsf{KA}(\mathcal{K})$ is* consistent *if $\mathsf{ob}_{\mathcal{K},P} \not\models \xi$ for each $\mathbf{K}\,\xi \in N$.*

For a set of modal atoms S, S_{DL} is the subset of DL-atoms of S and $\widehat{S} = \{\xi \mid \mathbf{K}\,\xi \in S\}$.

An MKNF interpretation M induces *the partition (P, N) of $\mathsf{KA}(\mathcal{K})$ if $\mathbf{K}\,\xi \in P$ implies $(M, M) \models \mathbf{K}\,\xi$ and $\mathbf{K}\,\xi \in N$ implies $(M, M) \models \mathbf{not}\,\xi$.*

We now adapt the operators from [10] which allow to draw conclusions from positive hybrid MKNF knowledge bases similarly to the immediate consequence operator for definite logic programs, only that the operators below also are "aware" of possible consequences including the DL knowledge base $\mathcal{O}$.

Definition 3.3. *For $\mathcal{K}$ a positive nondisjunctive DL-safe hybrid MKNF knowledge base, $R_{\mathcal{K}}$, $D_{\mathcal{K}}$, and $T_{\mathcal{K}}$ are defined on the subsets of $\mathsf{KA}(\mathcal{K})$ as follows:*

$R_{\mathcal{K}}(S) = S \cup \{\mathbf{K}\, H \mid \mathcal{K}$ *contains a rule of the form (1) such that* $\mathbf{K}\, A_i \in S$ *for each* $1 \leq i \leq n\}$
$D_{\mathcal{K}}(S) = \{\mathbf{K}\,\xi \mid \mathbf{K}\,\xi \in \mathsf{KA}(\mathcal{K})$ *and* $\mathcal{O} \cup \widehat{S}_{DL} \models \xi\} \cup \{\mathbf{K}\, Q(b_1, \ldots, b_n) \mid \mathbf{K}\, Q(a_1, \ldots, a_n) \in S \setminus S_{DL}, \mathbf{K}\, Q(b_1, \ldots, b_n) \in \mathsf{KA}(\mathcal{K})$*, and* $\mathcal{O} \cup \widehat{S}_{DL} \models a_i \approx b_i$ *for* $1 \leq i \leq n\}$
$T_{\mathcal{K}}(S) = R_{\mathcal{K}}(S) \cup D_{\mathcal{K}}(S)$

The difference to the operators in [10] is that given e.g. only $a \approx b$ and $\mathbf{K}\, Q(a)$ we do not derive $\mathbf{K}\, Q(b)$ explicitly but only as a consequence of $\mathsf{ob}_{\mathcal{K},P}$.

As in [9], it can be shown that $T_{\mathcal{K}}$ is monotonic and yields a least fixpoint $T_{\mathcal{K}} \uparrow \omega$ in the usual manner. We can therefore, in the style of stable models, define a transformation which turns a nondisjunctive hybrid MKNF knowledge base into a positive one allowing to apply the previous operators.

Definition 3.4. *Let* $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ *be a ground nondisjunctive DL-safe hybrid MKNF knowledge base and* $S \subseteq \mathsf{KA}(\mathcal{K})$*. The* MKNF transform $\mathcal{K}_G/S = (\mathcal{O}, \mathcal{P}_G/S)$ *is obtained by* $\mathcal{P}_G/S$ *containing all rules* $\mathbf{K}\, H \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n$ *for which there exists a rule* $\mathbf{K}\, H \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n, \mathbf{not}\, B_1, \ldots, \mathbf{not}\, B_m$ *in* $\mathcal{P}_G$ *with* $\mathbf{K}\, B_j \notin S$ *for all* $1 \leq j \leq m$*.*

On top of that, an operator yielding the fixpoint of $T_{\mathcal{K}}$ is defined.

Definition 3.5. *Let* $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ *be a nondisjunctive DL-safe hybrid MKNF knowledge base and* $S \subseteq \mathsf{KA}(\mathcal{K})$*. We define:*

$$\Gamma_{\mathcal{K}}(S) = T_{\mathcal{K}_G/S} \uparrow \omega$$

This operator is antitonic (cf. extended technical report), so applying $\Gamma_{\mathcal{K}}(S)$ twice is a monotonic operation yielding a least fixpoint by the Knaster-Tarski theorem (and dually a greatest one) and we can iterate as follows: $\Gamma^2_{\mathcal{K}} \uparrow 0 = \emptyset$, $\Gamma^2_{\mathcal{K}} \uparrow (n+1) = \Gamma^2_{\mathcal{K}}(\Gamma^2_{\mathcal{K}} \uparrow n)$, and $\Gamma^2_{\mathcal{K}} \uparrow \omega = \bigcup \Gamma^2_{\mathcal{K}} \uparrow i$, and dually $\Gamma^2_{\mathcal{K}} \downarrow 0 = \mathsf{KA}(\mathcal{K})$, $\Gamma^2_{\mathcal{K}} \downarrow (n+1) = \Gamma^2_{\mathcal{K}}(\Gamma^2_{\mathcal{K}} \downarrow n)$, and $\Gamma^2_{\mathcal{K}} \downarrow \omega = \bigcap \Gamma^2_{\mathcal{K}} \downarrow i$. The least and the greatest fixpoint then define the well-founded partition.

Definition 3.6. *Let* $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ *be a nondisjunctive DL-safe hybrid MKNF knowledge base and let* $\mathbf{P}_{\mathcal{K}}, \mathbf{N}_{\mathcal{K}} \subseteq \mathsf{KA}(\mathcal{K})$ *with* $\mathbf{P}_{\mathcal{K}} = \Gamma^2_{\mathcal{K}} \uparrow \omega$ *and* $\mathbf{N}_{\mathcal{K}} = \Gamma^2_{\mathcal{K}} \downarrow \omega$*. Then* $(P_W, N_W) = (\mathbf{P}_{\mathcal{K}} \cup \{\mathbf{K}\, \pi(\mathcal{O})\}, \mathsf{KA}(\mathcal{K}) \setminus \mathbf{N}_{\mathcal{K}})$ *is the* well-founded partition *of* $\mathcal{K}$*.*

Example 3.2. Continuing our example, the investigation of the police reveals that the known criminal Max is responsible for the murder, though not being detectable, so we cannot conclude $Suicide(jones)$ while $\mathbf{K}\, responsible(max, jones)$ and $\mathbf{K}\, murdered(jones)$ hold. Unfortunately, the person benefitting from the insurance is the nephew Thomas who many years ago left the country, i.e. $\mathbf{K}\, benefits(thomas, jones)$. Computing the well-founded partition yields thus $\mathbf{K}\, Pay(jones)$, so the company contacts the nephew outside the country. However, they also hire a private detective who finds out that Thomas is max, having altered his personality, i.e. we can add $thomas \approx max$ to the hybrid KB. Due to $D_\mathcal{K}$ and grounding we now obtain a well-founded partition which contains $\mathbf{K}\, responsible(thomas, jones)$ and $\mathbf{K}\, benefits(max, jones)$ being true and the insurance is not paid any longer.

One of the results shown in the extended paper is that the well-founded partition is consistent. Besides that, similarly to stable models, we can compute one fixpoint defining the well-founded partition directly from the other.

Proposition 3.1. *Let $\mathcal{K}$ be a nondisjunctive DL-safe hybrid MKNF knowledge base. Then $\mathbf{P}_\mathcal{K} = \Gamma_\mathcal{K}(\mathbf{N}_\mathcal{K})$ and $\mathbf{N}_\mathcal{K} = \Gamma_\mathcal{K}(\mathbf{P}_\mathcal{K})$.*

Knowing this, we can use $\Gamma_\mathcal{K}$ as an alternative characterization of MKNF models if the considered KB is consistent. It should be noted that in case of an inconsistent hybrid MKNF KB due to the operator $\mathcal{D}_\mathcal{K}$ we obtain a well-founded partition where all modal **K**-atoms are true. I.e., even though we always obtain a well-founded partition for any $\mathcal{K}$ the result may not be a desired one.

It was also shown that the information derived in the well-founded partition is contained in any MKNF model.

Theorem 3.1. *Let $\mathcal{K}$ be a nondisjunctive DL-safe hybrid MKNF knowledge base, M an MKNF model of $\mathcal{K}$ with (P, N) induced by M, and (P_W, N_W) the well-founded partition of $\mathcal{K}$. Then $P_W \subseteq (P \cup \{\mathbf{K}\, \pi(\mathcal{O})\})$ and $N_W \subseteq N$.*

Furthermore, the well-founded partition yields a model in the three-valued framework we defined in the previous subsection.

Theorem 3.2. *Let $\mathcal{K}$ be a consistent nondisjunctive DL-safe hybrid MKNF KB and $(\mathbf{P}_\mathcal{K} \cup \{\mathbf{K}\, \pi(\mathcal{O})\}, \mathsf{KA}(\mathcal{K}) \setminus \mathbf{N}_\mathcal{K})$ be the well-founded partition of $\mathcal{K}$. Then $(I_P, I_N) \models \pi(\mathcal{K})$ where $I_P = \{I \mid I \models \mathsf{ob}_{\mathcal{K},\mathbf{P}_\mathcal{K}}\}$ and $I_N = \{I \mid I \models \mathsf{ob}_{\mathcal{K},\mathbf{N}_\mathcal{K}}\}$.*

One of the open questions mentioned in [10] was that MKNF models are not compatible with the well-founded model for logic programs. Our approach, regarding knowledge bases just consisting of rules, does coincide with the well-founded model for the corresponding (normal) logic program.

Finally, though not providing here a detailed study of complexity issues we can recall from [9], assuming that entailment of first-order formulas encountered while computing $T_\mathcal{K}$ is decidable in $\mathcal{C}$, that the data complexity of computing $T_\mathcal{K}$ is in $\mathsf{P}^\mathcal{C}$ (for positive nondisjunctive programs). Since we just apply the same operator n-times we remain in the same complexity class while the data complexity for reasoning with MKNF models in nondisjunctive programs is shown to be $\mathcal{E}^{\mathsf{P}^\mathcal{C}}$ where $\mathcal{E} = \mathsf{NP}$ if $\mathcal{C} \subseteq \mathsf{NP}$, and $\mathcal{E} = \mathcal{C}$ otherwise. Thus computing the well-founded partition ends up in a strictly smaller complexity class than deriving the MKNF models.

4 Conclusions and Future Work

We have continued the work on hybrid MKNF knowledge bases providing an alternating fixpoint restricted to nondisjunctive rules. We basically achieve better complexity results by having only one model which is semantically weaker than any MKNF model defined in [10] but bottom-up computable. The well-founded semantics is not only a sound approximation of any MKNF model but a partition of modal atoms which can seemlessly be integrated in the reasoning algorithms presented for MKNF models in [10] thus reducing the difficulty of guessing the 'right' model. Future work shall include the extension to disjunctive rules, a study on top-down querying procedures, and further investigations on the well-founded model in the three-valued framework.

References

1. F. Baader, D. Calvanese, D. L. McGuinness, D. Nardi, and P. F. Patel-Schneider, editors. *The Description Logic Handbook: Theory, Implementation, and Applications*. Cambridge University Press, 2003.
2. J. de Bruijn, T. Eiter, A. Polleres, and H. Tompits. Embedding non-ground logic programs into autoepistemic logic for knowledge-base combination. In *Proceedings of the Twentieth International Joint Conference on Artificial Intelligence (IJCAI-07)*, Hyderabad, India, January 6–12 2007. AAAI Press.
3. T. Eiter, T. Lukasiewicz, R. Schindlauer, and H. Tompits. Combining answer set programming with description logics for the semantic web. In D. Dubois, C. Welty, and M.-A. Williams, editors, *KR'04*, pages 141–151. AAAI Press, 2004.
4. T. Eiter, T. Lukasiewicz, R. Schindlauer, and H. Tompits. Well-founded semantics for description logic programs in the semantic web. In G. Antoniou and H. Boley, editors, *RuleML'04*, pages 81–97. Springer, LNCS, 2004.
5. M. Fitting. The family of stable models. *Journal of Logic Programming*, 17(2/3&4):197–225, 1993.
6. M. Gelfond and V. Lifschitz. The stable model semantics for logic programming. In R. A. Kowalski and K. A. Bowen, editors, *ICLP*. MIT Press, 1988.
7. S. Heymans, D. V. Nieuwenborgh, and D. Vermeir. Guarded open answer set programming. In C. Baral, G. Greco, N. Leone, and G. Terracina, editors, *LPNMR'05*, pages 92–104. Springer, LNAI, 2005.
8. V. Lifschitz. Nonmonotonic databases and epistemic queries. In *IJCAI'91*, pages 381–386, 1991.
9. B. Motik and R. Rosati. Closing semantic web ontologies. Technical report, University of Manchester, UK, 2006.
10. B. Motik and R. Rosati. A faithful integration of description logics with logic programming. In *Proceedings of the Twentieth International Joint Conference on Artificial Intelligence (IJCAI-07)*, pages 477–482, Hyderabad, India, January 6–12 2007. AAAI Press.
11. R. Rosati. Reasoning about minimal belief and negation as failure. *J. of Artificial Intelligence Research*, 11:277–300, 1999.
12. R. Rosati. Dl+Log: A tight integration of description logics and disjunctive datalog. In P. Doherty, J. Mylopoulos, and C. Welty, editors, *KR'06*. AAAI Press, 2006.
13. A. van Gelder. The alternating fixpoint of logic programs with negation. In *Principles of Database Systems*, pages 1–10. ACM Press, 1989.
14. A. van Gelder, K. A. Ross, and J. S. Schlipf. The well-founded semantics for general logic programs. *Journal of the ACM*, 38(3):620–650, 1991.