Modal Logic Applied to Query Answering and the Case for Variable Modalities

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Abstract. We present a query answering technique based on notions and results from modal Correspondence Theory. It allows us to answer a wide family of conjunctive queries by polynomial reduction to knowledge base satisfiability problem. An advantage of this technique lies in its uniformity: it does not depend on a Description Logic (DL), so that extending a DL does not invalidate the algorithm. Thus, the problem of answering queries in this family is decidable in any decidable DL. The construction also leads to an idea of extending the modal language with so called variable modalities, whose syntax and semantics is introduced in the paper. On the one hand, this yields a broader family of queries that can be answered with the same technique. On the other hand, modal logic with variable modalities is interesting per se, and we formulate some natural (open) questions about this logic.

1 Introduction

Developing languages and algorithms for reasoning with ontologies is a crucial aspect of the Semantic Web activity. Among reasoning tasks, querying is a fundamental mechanism for extracting information from a KB. Two most important reasoning services involving queries are query answering and query containment (also called subsumption); they are mutually reducible (and we focus on query answering here). While the complexity of DLs is now well understood [1, 11], the decidability and complexity issues for query answering in expressive DLs have only recently got partial or complete solutions (see [5, 8] and references therein). For the expressive DLs that underpin the state-of-the-art web ontology languages OWL DL and OWL 1.1, even the decidability of query answering is not established yet.

Usually, query answering techniques are developed for a specific DL, and the more expressive is a DL, the more complex becomes the query answering algorithm. In this paper, we address the problem from a different perspective: we develop a *uniform* technique for answering a certain *family* of queries, which means that the algorithm is independent of a DL in which a KB is formulated. Therefore, extending the expressivity of a DL does not hurt the algorithm (in contrast to other approaches where, e.g., introducing transitive roles can invalidate the algorithm, cf. [5, 8]). The basic and most prominent uniform algorithm for query answering is so called *rolling-up* technique (see, e.g., [5]) applicable to the family of *tree-like* queries whose root is the only distinguished variable. Given such a query $q(x)$, the algorithm transforms it into an *equivalent* concept C, i.e., in any model, the extensions of C and $q(x)$ coincide. Hence, to answer the query $q(x)$ is the same as to retrieve all instances of the concept C (and the latter task is reducible to KB unsatisfiability). The starting point for us is an observation that, in general, the equivalence of a concept to a query is sufficient, but not necessary for them to have the same answers. It turns out that the proper relation between C and $q(x)$ that guarantees them to have the same answers is closely related (or, as we conjecture, even equivalent) to the relation of *local correspondence* known from modal Correspondence Theory [2, 6, 9].

To illustrate how this works, consider a cyclic query $q(x) \leftarrow xRx$; it is not equivalent to any $\mathcal{ALC}$ concept, as follows from the tree model property, so rolling-up is not applicable. Now recall that reflexivity xRx is expressible by (i.e., *locally corresponds* on Kripke frames to) a modal formula $p \to \Diamond p$, where p is a propositional *variable* (its interpretation on a Kripke frame is universally quantified). Then we introduce a *fresh* concept name P (i.e., whose interpretation is not constrained by a KB) and translate this modal formula into a DL concept $\neg P \sqcup \exists R.P$, which, as we prove, always has the same answers as our query $q(x)$.

In general, given a query $q(x)$ from a family specified below, we invoke an algorithm from Correspondence Theory [6] to build a modal formula φ that locally corresponds to $q(x)$; then we translate φ into a concept (usually, in the same DL as the query) by introducing a fresh concept name for each variable in φ; the resulting concept, as we show, has exactly the same answers as the original query $q(x)$ over *any* KB in any DL. The details of this technique are described in Sect. 2. A natural question arises: what if we additionally allow to use fresh role names? In terms of modal logic, this means that a formula φ will contain what we call *variable modalities*. We introduce syntax and semantics for such a modal language in Sect. 3 and generalise the query answering technique to this setting, which results in a wider family of queries that can be answered within this approach. Finally, in Sect. 4 we conclude with formulating some open problems concerning our query answering technique, as well as definability and first-order correspondence for the modal logic with variable modalities.

2 Queries answered by concepts

Since our results are applicable to *any* DL (extending $\mathcal{ALC}$), we do not need to describe expressive DLs, and we only briefly recall the definition of $\mathcal{ALC}$ and $\mathcal{ALCI}$ and fix some notation. The vocabulary consists of finite sets of *concept names* CN, *role names* RN, and *individual names* (or *constants*) IN. Concepts of $\mathcal{ALC}$ are defined by the following syntax:

$$C ::= \ \bot \mid A \mid \neg C \mid C \sqcap D \mid \forall R.C, \qquad \text{where } A \in \mathsf{CN},\ R \in \mathsf{RN}.$$

Other connectives are taken as customary abbreviations, e.g., $C \to D$ stands for $\neg(C \sqcap \neg D)$. In $\mathcal{ALCI}$, inverse roles R^- can additionally be used in place of R.

A *terminology* (or a TBox) $\mathcal{T}$ is a finite set of *axioms* of the form $C \sqsubseteq D$, where C, D are arbitrary concepts. An ABox $\mathcal{A}$ is a finite set of *assertions* of the form $a\colon C$ and aRb, where $a, b \in \mathsf{IN}$, C is a concept and R a role. Finally, a *knowledge base* $\mathcal{KB} = \langle \mathcal{T}, \mathcal{A} \rangle$ consists of a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$.

Definition 1. (Semantics) An *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of non-empty *domain* $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ that maps:

- each constant $a \in \mathsf{IN}$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$,
- each concept name $C \in \mathsf{CN}$ to a subset $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$,
- each role name $R \in \mathsf{RN}$ to a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$;

and is extended to concepts, roles, axioms and assertions as follows:

$\bot^{\mathcal{I}} = \varnothing$	$(R^-)^{\mathcal{I}} = \{\langle e, d\rangle \mid \langle d, e\rangle \in R^{\mathcal{I}}\}$	$\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$(\forall R.C)^{\mathcal{I}} = \{\, e \in \Delta^{\mathcal{I}} \mid d \in C^{\mathcal{I}}$	$\mathcal{I} \models a\colon C$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$
$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$	for all d such that $\langle e, d\rangle \in R^{\mathcal{I}}\}$	$\mathcal{I} \models aRb$ iff $\langle a^{\mathcal{I}}, b^{\mathcal{I}}\rangle \in R^{\mathcal{I}}$

Here $\mathcal{I} \models \Phi$ stands for '$\mathcal{I}$ satisfies Φ'. An interpretation is called a *model* of a KB if it satisfies all its TBox axioms and ABox assertions. A knowledge base $\mathcal{KB}$ *entails* Φ (notation: $\mathcal{KB} \models \Phi$) if $\mathcal{I} \models \Phi$, for all models $\mathcal{I}$ of $\mathcal{KB}$.

Definition 2. (Queries) A *conjunctive query* is an expression of the form

$$q(\vec{x}) \;\leftarrow\; \exists \vec{y}\,\big(t_1(\vec{x}, \vec{y}) \wedge \ldots \wedge t_n(\vec{x}, \vec{y})\big),$$

where $\vec{x}, \vec{y}$ are tuples of (*distinguished*, resp., *non-distinguished*) variables, and each *atom* $t_i(\vec{x}, \vec{y})$ is of the form $u\colon C$ (*concept atom*) or uRv (*role atom*), where C is a concept, R a role, and u, v are either variables from $\vec{x}, \vec{y}$ or constants.[1] A query without concept atoms is called *relational*. Queries with 0 and 1 distinguished variables are called *boolean* and *unary* resp. Given an interpretation $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$, a query q of arity m is interpreted as follows:

$$q^{\mathcal{I}} := \{\ \vec{e} \in \Delta^m \ \mid\ \mathcal{I} \models \exists \vec{y}\ \big(t_1(\vec{e}, \vec{y}) \wedge \ldots \wedge t_n(\vec{e}, \vec{y})\big)\ \}.$$

The *answer set* of a query $q(\vec{x})$ w.r.t. a knowledge base $\mathcal{KB}$ is defined as the set of tuples of constants $\vec{a}$ that satisfy the query q in all models of $\mathcal{KB}$:

$$\mathsf{ans}_{\mathcal{KB}}(q) := \{\ \vec{a} \in \mathsf{IN} \ \mid\ \mathcal{KB} \models q(\vec{a})\ \}.$$

The following is the main notion of our paper (in fact, it can be formulated for an arbitrary first-order formula q in the appropriate language, cf. Example 2).

Definition 3. A unary query $q(x)$ is *answered by* a concept C (written as $q(x) \approx C$) if, for any $\mathcal{KB}$ and any $a \in \mathsf{IN}$, we have: $\mathcal{KB} \models q(a) \Leftrightarrow \mathcal{KB} \models a\colon C$; in other words, if the queries $q(x)$ and $x\colon C$ always have the same answer set.[2]

A boolean query q is *answered by* a concept C (notation: $q \approx C$) if, for any $\mathcal{KB}$, the equivalence holds: $\mathcal{KB} \models q \Leftrightarrow \mathcal{KB} \models a\colon C$, where a is a fresh constant.

[1] In what follows, w.l.o.g. we consider queries without constants, since constants can be eliminated at the price of introducing nominals: xRa is equivalent to $xRz \wedge z\colon \{a\}$.

[2] Strictly speaking, we should define: $q(x) \approx C$ *over* a DL $\mathcal{L}$ iff $q(x)$ and $x\colon C$ have the same answers for any $\mathcal{KB}$ formulated in $\mathcal{L}$. However, we shall not complicate the matters, since in all our results whenever $q \approx C$ holds, it hold in fact for any DL $\mathcal{L}$.

Our task is to determine what kind of queries can be answered by concepts and when these concepts can be found efficiently (and preferably in the same language as the query). For this aim, we will use results from a branch of modal logic called the Correspondence Theory. Briefly, this theory is devoted to questions whether a modally definable class of frames is also first-order definable, and if so, whether the corresponding first-order formula can be found efficiently (and similarly in the other direction). The background information on that theory can be found in [2, Chap. 3]; here we will recall its basic notions.

Formulas of the (multi-)modal language with n modalities $\Box_i$ and a countable set of propositional variables p_i are defined by the following syntax:

$$\varphi \ ::= \ \bot \mid p_i \mid \varphi \to \psi \mid \Box_i \varphi.$$

As K. Schild observed in [10], this language is a notational variant of $\mathcal{ALC}$ with n role names R_i.[3] Exploiting this fact, whenever φ is a modal formula, we denote by C_φ the concept obtained from φ by replacing $\Box_i$ with $\forall R_i$ and p_i with *fresh* concept names P_i (it is convenient to reserve a countable set of fresh concept names, i.e., that will not occur in any KB or query).

Definition 4 (Kripke semantics). A *frame* $F = \langle \Delta, r_1, \ldots, r_n \rangle$ consists of a non-empty set Δ and n binary relations r_i on Δ. A model $M = \langle F, \nu \rangle$ (based on F) consists of a frame F and a *valuation* of variables $p_i^\nu \subseteq \Delta$. The notion "a formula φ is *true* at a point $e \in \Delta$ in a model M" (notation: $M, e \models \varphi$) is defined inductively: $M, e \not\models \bot$; $M, e \models p_i$ iff $e \in p_i^\nu$; $M, e \models \varphi \to \psi$ iff $M, e \not\models \varphi$ or $M, e \models \psi$; $M, e \models \Box_i \varphi$ iff $M, d \models \varphi$ for all $d \in \Delta$ such that $\langle e, d \rangle \in r_i$. A formula φ is *valid* (at a point e) in a frame F (notation: $F, e \Vdash \varphi$ or $F \Vdash \varphi$ resp.) if it is true (at this point) in all models based on F.

Definition 5 (Correspondence). Let φ be a modal formula, $\alpha(x)$ and β first-order formulae in the vocabulary $\{R_1, \ldots, R_n, =\}$ with one and no free variables, resp. We say that $\alpha(x)$ *locally corresponds* to φ (notation: $\alpha(x) \leftrightsquigarrow \varphi$) if $F \models \alpha(e) \Leftrightarrow F, e \Vdash \varphi$, for any frame F and any its point e. Similarly, β *globally corresponds* to φ (notation: $\beta \leftrightsquigarrow \varphi$) if $F \models \beta \Leftrightarrow F \Vdash \varphi$, for any frame F.

For example, a formula $p \to \Diamond p$ corresponds (both locally and globally) to reflexivity, whereas $\Box(\Box p \to p) \to \Box p$ is valid in a frame F iff F is transitive and has no infinite ascending chains, which is a (monadic) second-order property.

We are ready to establish a relationship between the correspondence relation ($q \leftrightsquigarrow \varphi$) and query answering ($q \approx C_\varphi$). We conjecture that they are equivalent; however, we have succeeded to prove only one implication and partially the converse one. For proofs, see a paper [13] and a recent technical report [12].

Theorem 6 (Reduction). *Let $q(x)$ be a unary relational*[4] *query, φ a modal formula. If $q(x)$ locally corresponds to φ, then the query $q(x)$ is answered by the $\mathcal{ALC}$-concept C_φ. In symbols: $q(x) \leftrightsquigarrow \varphi \Longrightarrow q(x) \approx C_\varphi$.*

Similarly for boolean queries and global correspondence: if $q \leftrightsquigarrow \varphi$ then $q \approx C_\varphi$.

[3] Note that the words 'correspondence theory' in the title of his paper refer to this observation only and have nothing to do with modal Correspondence Theory.

[4] Queries that additionally involve concept atoms $y : C$ are covered by Theorem 10.

Lemma 7 (Partial converse). (1) *Suppose that a unary relational query $q(x)$ is answered by a concept C_φ, i.e., $q(x) \approx C_\varphi$, for some modal formula φ. Then:*

a) $F \models q(e) \Rightarrow F, e \Vdash \varphi$, for any frame F and any its point e;

b) $F \models q(e) \Leftarrow F, e \Vdash \varphi$, for any finitely branching[5] *frame F and any point e.*

(2) *The same holds for boolean queries and global validity ($F \Vdash \varphi$), but only for 'finite' instead of 'finitely branching' frames in (b).*

It is undecidable to determine whether a given FO formula corresponds to some modal formula [3]. There were few attempts to find FO fragments for which the problem is decidable.[6] We apply (and extend) those results and identify several families of queries for which the answering concept can be built efficiently. The family $\mathcal{K}$ below stems from so called *Kracht's fragment* [6]; the family $\mathcal{Z}$ contains queries beyond that fragment. Queries from both $\mathcal{K}$ and $\mathcal{Z}$ are answered by $\mathcal{ALC}$-concepts. By "forgetting" the direction of edges[7] in queries, we obtain a family $\mathcal{E}$ of queries that are answered by $\mathcal{ALCI}$-concepts. The formal description of these families of relational queries (and their non-relational analogues), the corresponding algorithm and the proofs can be found in [13].

Corollary 8. *There exist a (polynomial) algorithm that takes a unary relational conjunctive query $q(x)$ from the following families $\mathcal{K}$, $\mathcal{Z}$, and $\mathcal{E}$ and returns a concept in $\mathcal{ALC}$ (for $q \in \mathcal{K} \cup \mathcal{Z}$) or $\mathcal{ALCI}$ (for $q \in \mathcal{E}$) that answers this query. Furthermore: $\mathcal{K} \not\subseteq \mathcal{Z}$, $\mathcal{Z} \not\subseteq \mathcal{K}$, and $(\mathcal{K} \cup \mathcal{Z}) \subset \mathcal{E}$. So, for any DL $\mathcal{L}$, the problem of answering queries within these families has the same complexity as $\mathcal{L}$ itself.*

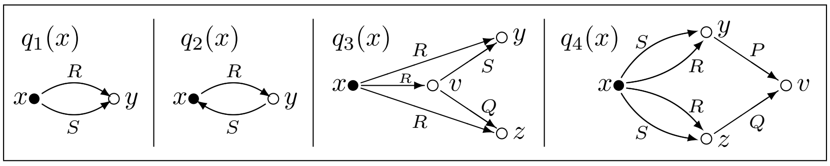
Family $\mathcal{K}$: *take an oriented tree with the root x, and add any number of oriented chains linking x with other nodes (in any direction).*

Family $\mathcal{Z}$: *take an "anti-tree" (i.e., an oriented tree with edges directed from leaves to the root); merge all its leaves and denote the resulting node by x.*

Family $\mathcal{E}$: *the graph of a query is connected and has no cycles that consist of non-distinguished variables only (i.e., any cycle contains the node x).*

Example 1. The queries $q_1(x) \leftarrow xRy \wedge xSy$ and $q_2(x) \leftarrow xRy \wedge ySx$ are answered by the concepts $\forall R.Y \to \exists S.Y$ and $X \to \exists R.\exists S.X$ resp. The following two queries witness that the families $\mathcal{K}$ and $\mathcal{Z}$ are incomparable w.r.t. inclusion:

Query in $\mathcal{K} \setminus \mathcal{Z}$:	$q_3(x) \leftarrow xRv \wedge vSy \wedge vQz \wedge xRy \wedge xRz$
Concept:	$\forall R.(Y \sqcap Z) \to \exists R.(\exists S.Y \sqcap \exists Q.Z)$
Query in $\mathcal{Z} \setminus \mathcal{K}$:	$q_4(x) \leftarrow xRy \wedge xSy \wedge yPv \wedge xRz \wedge xSz \wedge zQv$
Concept:	$\forall R.(Y \sqcap Z) \to \exists S.\big((Y \sqcap \exists P.V) \sqcup (Z \sqcap \exists Q.\neg V)\big)$



[5] I.e., any point of the frame has a finite number of successors.

[6] And much more results in the "modal to first-order" direction; see [4] for an overview.

[7] It is convenient to represent a relational query as an oriented graph, whose nodes are distinguished (•) and non-distinguished (∘) variables and edges are role atoms.

Example 2. Boolean (not necesserily conjunctive) queries can be answered similarly; namely, we can check for *modally definable* properties of roles [2, Ch. 3]. For instance, whether a role R is reflexive, transitive, dense, eucleadean, confluent etc. in a KB can be checked, by Theorem 6, using the DL-translations of the modal formulae $p \to \Diamond p$, $\Diamond p \to \Diamond\Diamond p$, $\Diamond\Diamond p \to \Diamond p$, $\Diamond p \to \Box\Diamond p$, $\Diamond\Box p \to \Box\Diamond p$ resp.

3 Modal logic with variable modalities

The results obtained above inspire to introduce a natural extension of modal logic, which is interesting per se and in addition yields to a wider family of queries that can be answered with the same technique. Recall that in order to answer, e.g., a query $q(x) \leftarrow xRx$, we introduced a fresh concept name P and then proved that $q(x)$ has the same answers as the concept $\neg P \sqcup \exists R.P$. It is not hard to see that, without fresh concept names, only tree-like queries can be answered (and this will coincide with the rolling-up technique). So it were fresh concept names that enabled us to answer cyclic queries. Now, can we gain even more if we allow to use fresh role names? As we show below, the answer is Yes (but additional queries are usually not *conjunctive*, but rather first-order formulae of other kinds).

Recall that in the definition of the *validity* of a modal formula in a frame, we quantify over interpretations of propositional variables (i.e., unary predicates), but interpretation of modalities (i.e., binary predicates) is fixed. In other words, the standard modal logic is a logic of *constant* modalities and propositional *variables*. Therefore it is natural to consider a notion of validity, in which the rôle of unary and binary predicates is symmetric. So, we extend modal logic with *variable* modalities and propositional *constants*.

The vocabulary of the *mixed modal logic* consists of m propositional constants $A_1, \ldots, A_m$, n constant modalities $\Box_1, \ldots, \Box_n$, and countable sets of propositional variables p_i and variable modalities $\boxdot_i$. The syntax for formulae is:

$$\varphi \ ::= \ \bot \ \mid \ p_i \ \mid \ A_i \ \mid \ \varphi \to \psi \ \mid \ \Box_i \varphi \ \mid \ \boxdot_i \varphi$$

Definition 9 (Semantics). A *frame* $F = \langle \Delta, \vec{\alpha}, \vec{r} \rangle$ consists of a non-empty set Δ, a list $\vec{\alpha}$ of m unary predicates $\alpha_i \subseteq \Delta$, and a list $\vec{r}$ of n binary relations $r_i \subseteq \Delta \times \Delta$. A *model* $M = \langle F, \vec{\pi}, \vec{s} \rangle$ consists of a frame F and countable sequences of unary predicates $\pi_i \subseteq \Delta$ and binary relations $s_i \subseteq \Delta \times \Delta$.

The notion "a formula φ is *true* at a point $e \in \Delta$ in a model M" is defined inductively: boolean cases are standard; $M, e \models p_i$ iff $e \in \pi_i$; $M, e \models A_i$ iff $e \in \alpha_i$; $M, e \models \Box_i \varphi$ iff $M, d \models \varphi$ for all $d \in \Delta$ such that $\langle e, d \rangle \in r_i$; and similarly for $\boxdot_i$ and s_i. The notion of *validity* of a formula (at a point) in a frame is standard, but note that here saying "for all models M" involves quantification over unary *binary* predicates s_i (hence it is no longer a *monadic* second-order notion).

The notion of correspondence is defined as in Def. 5, but for first-order formulae in the vocabulary $\{A_1, \ldots, A_m, R_1, \ldots, R_n, =\}$. The mixed modal language is much more expressive. For instance, $F \Vdash \Box p \to \boxdot p$ iff $r = \Delta \times \Delta$; and

$F \models p \to \boxdot p$ iff $|\Delta| = 1$; these properties are not expressible in the standard modal language (see. [12] for more examples). It turns out that almost all the results from the previous section can be generalised to the mixed modal logic, with even more elegant formulations, as we do not need to rule out concept atoms $x{:}C$ from queries now. In what follows, whenever φ is a mixed modal formula, by C_φ we denote a concept obtained from φ by replacing $\Box_i$ with $\forall R_i$, $\boxdot_i$ with $\forall S_i$, and p_i with P_i, where concept names P_i and role names S_i are fresh (symbols A_i are left unchanged). The following results are proved in [12].

Theorem 10 (Reduction). *Let $q(x)$ be a unary query, φ a mixed modal formula. If $q(x)$ locally corresponds to φ, then the query $q(x)$ is answered by the $\mathcal{ALC}$-concept C_φ. In symbols: $q(x) \leftrightsquigarrow \varphi \Longrightarrow q(x) \approx C_\varphi$.*

Similarly for boolean queries and global correspondence: if $q \leftrightsquigarrow \varphi$ then $q \approx C_\varphi$.

Lemma 11 (Partial converse). (1) *Suppose that a unary query $q(x)$ is answered by by a concept C_φ, i.e., $q(x) \approx C_\varphi$, for some mixed modal formula φ. Then $F \models q(e)$ implies $F, e \Vdash \varphi$, for any frame F and any its point e.*

(2) *The same holds for boolean queries and the global validity ($F \Vdash \varphi$).*

Example 3 (Mary likes all cats). Suppose that a $\mathcal{KB}$ contains an individual Mary, a concept Cat and a role Likes, and we want to express a boolean query whether "Mary likes all cats". A straightforward way to do this is to write a concept subsumption: $\mathsf{Cat} \sqsubseteq \exists \mathsf{Likes}^{-}.\{\mathsf{Mary}\}$, but it contains an inverse role and a nominal, even if the language of $\mathcal{KB}$ does not, thus increasing the complexity of reasoning [11]. This query can also be formulated using role negation: $\mathsf{Mary}{:}\forall\neg\mathsf{Likes}.\neg\mathsf{Cat}$, again with an increase of the complexity [7]. The solution we propose enables one to express this query in $\mathcal{ALC}$. To this end, note that a mixed modal formula $\Box p \to \boxdot(A \to p)$ locally corresponds to a first-order formula $q(x) := \forall y\,(A(y) \to xRy)$ (see [12] for a proof). Now let A stand for Cat and R for Likes, then our query "Mary likes all cats" can be represented as $q(\mathsf{Mary})$. Finally, we apply Theorem 10 and conclude that $q(\mathsf{Mary})$ holds w.r.t. $\mathcal{KB}$ iff Mary is an instance (w.r.t. $\mathcal{KB}$) of the following concept (where the concept name $\mathsf{SomeConc}$ and the role name $\mathsf{SomeRel}$ are fresh):

$$\forall \mathsf{Likes}.\mathsf{SomeConc} \;\to\; \forall \mathsf{SomeRel}.(\mathsf{Cat} \to \mathsf{SomeConc}).$$

4 Conclusions and open quesions

One of the achievements of this paper is the established relationship between query answering in DL and Correspondence Theory (Theorems 6 and 10). It allowed us to build a uniform query answering algorithm for some families of conjunctive queries. Furthermore, a modal logic with variable modalities was introduced; although it is quite a natural extension of the standard modal logic, it has not been considered in the literature, to the best of our knowledge.

There is a number of natural problems left open, answers to which would complete the whole picture. Here we mention some of them.

Q1 Do the converses of Theorems 6 and 10 hold?
Q2 Which conjunctive queries locally/globally correspond to modal formulae (with or without $\boxdot$'s)? At least, are these families of queries decidable?
Q3 Which conjunctive queries can be answered by $\mathcal{ALC}$-concepts (with or without fresh role names)? By Theorems 6 and 10, queries from **Q2** form a subset of queries from **Q3**, and by Corollary 8, they contain the families $\mathcal{K}$ and $\mathcal{Z}$.
Q4 The same questions **Q2** and **Q3** for the logics $\mathcal{ALCI}$, $\mathcal{ALCQ}$, and $\mathcal{ALCQI}$.
Q5 The expressive power of the mixed modal logic: What classes of frames are definable (i.e., an analogue of Goldblatt-Thomason theorem [2, Th. 3.19])? Which of them are first-order definable?

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