Long Waves Simulation in Coastal Systems Using Parallel Computational Technologies

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Abstract

The construction and investigation of parallel algorithms for the numerical realization of 3D models of transport and deposition of suspended matter and 2D models of bottom sediment transport in coastal marine systems have already developed. 3D model has been presented for numerical modeling of gravitational waves near the shoreline in this work. It consists of Navier-Stokes equation system, which includes three momentum equations and mass conservation law equation in region with dynamically varying boundaries. It has been shown that presented 3D model gives more realistic description of physical wave process near the coastal line. The practical significance of constructed 3D model, numerical algorithm and the complex of programs consist in the possibility of its application for the study of hydrophysical processes in coastal water systems, as well as in assessing of the hydrodynamic effect on shore protection constructions and coastal structures in the presence of gravitational surface waves.

1 Introduction

The study of the hydrodynamic processes of the coastal waters connected with the investigation of the influence of wave processes generated in the open sea or in the coastal zone of the reservoir. The movement of waves can lead to negative results affecting the operation of the coastal zone: to the transformation of the bottom surface resulting from the rise of bottom sediments, to abrasion, the process of destruction by the waves and surf of the banks of various water systems [Suk05]. The result of the interaction of waves with the bottom surface and the coastal slope is refraction, diffraction and changes in wave structure. The most significant factors are fluctuations in water surface level, wind phenomena, currents, transport of bottom materials and deformation of the coastal slope.

Characteristic feature of the coastal waters is the significant influence of the bottom surface on the wave processes, which makes it difficult to study tidal phenomena in the coastal regions of the seas and river mouths [Gus02, Suk05']. The influence of wave processes on the coastal zone can be ambivalent [Gus04]: wave processes can have a significant effect on the accumulation and abrasion of the coastal zone of the reservoir and directly on coastal structures.

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To simulate the hydrodynamic processes, the problem of practical application of computationally effective methods is actualized, which makes it possible to obtain a fairly accurate approximate numerical solution [Ale13]. There is a need of constructing a set of interrelated models of three-dimensional wave processes intended for modeling wave processes.

2 Statement of the Problem of Wave Hydrodynamics

The initial equations of hydrodynamics of shallow water bodies are [Suk05, Gus02, Suk05']:

- the equation of motion (Navier-Stokes):

u

$$u'_{t} + uu'_{x} + vu'_{y} + wu'_{z} = -\frac{1}{\rho}p'_{x} + (\mu u'_{x})'_{x} + (\mu u'_{y})'_{y} + (\nu u'_{z})'_{z},$$

$$v'_{t} + uv'_{x} + vv'_{y} + wv'_{z} = -\frac{1}{\rho}p'_{y} + (\mu v'_{x})'_{x} + (\mu v'_{y})'_{y} + (\nu v'_{z})'_{z},$$

$$(1)$$

$$v'_{t} + uw'_{x} + vw'_{y} + ww'_{z} = -\frac{1}{\rho}p'_{z} + (\mu w'_{x})'_{x} + (\mu w'_{y})'_{y} + (\nu w'_{z})'_{z} + g,$$

- the equation of continuity in the case of variable density:

$$\rho_{t}^{'} + (\rho u)_{x}^{'} + (\rho v)_{y}^{'} + (\rho w)_{z}^{'} = 0, \qquad (2)$$

Where $V = \{u, v, w\}$ are the components of the velocity vector, p is the pressure, ρ is the density, μ , ν are the horizontal and vertical components of the coefficient of turbulent exchange, g is the acceleration of gravity.

The system of equations (1)-(2) is considered under the following boundary conditions:

- at the entrance

$$u(x, y, z, t) = u(t), \ v(x, y, z, t) = v(t), p'_n(x, y, z, t) = 0, \ V'_n(x, y, z, t) = 0,$$
(3)

- the lateral border (shore and bottom)

$$\rho\mu(u')_n(x, y, z, t) = -\tau_x(t), \ \rho\mu(v')_n(x, y, z, t) = -\tau_y(t), \ V_n(x, y, z, t) = 0, \ p'_n(x, y, z, t) = 0,$$

- the upper limit
$$\begin{array}{l} \rho\mu(u')_n(x,y,z,t) = -\tau_x(t),\\ \rho\mu(v')_n(x,y,z,t) = -\tau_y(t),\\ w(x,y,t) = -\omega - p'_t/\rho g, \ p'_n(x,y,t) = 0 \end{array}$$

where ω is the evaporation rate of the liquid, τ_x, τ_y are the components of the tangential stress [Gus04, Ale13]. The components of the tangential stress for the free surface: $\tau_x = \rho_a C_p(|\vec{w}|) w_x |\vec{w}|, \tau_y = \rho_a C_p(|\vec{w}|) w_y |\vec{w}|,$ where \vec{w} is the vector of the wind speed relative to the water, ρ_a is the density of the atmosphere, $C_p(x)$ is the dimensionless coefficient.



Figure 1: The geometry of the computational domain

Fig. 1 shows the geometry of the computational domain. The components of the tangential stress for the bottom, taking into account the notation, can be written as follows: $\tau_x = \rho_v C_p(|V|) u |V|$, $\tau_y = \rho_v C_p(|V|) v |V|$ where ρ_v is the density of bottom sediments.

The approximation considered below makes it possible to build on the basis of the measured velocity pulsations the coefficient of vertical turbulent exchange, inhomogeneous in depth [Ale13]:

$$\nu = C_s^2 \Delta^2 \frac{1}{2} \sqrt{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2} \tag{4}$$

where $\overline{U}, \overline{V}$ are the time-averaged pulsations of the horizontal velocity components, Δ is the characteristic scale of the grid, and C_s is the dimensionless empirical constant whose value is usually determined on the basis of calculating the decay process of homogeneous isotropic turbulence.

3 The discrete model of hydrodynamics of shallow water reservoirs

The computational domain inscribed in a parallelepiped. For the numerical realization of the discrete mathematical model of the hydrodynamic problem posed, a uniform grid is introduced:

$$\bar{w}_h = \left\{ t^n = n\tau, \, x_i = ih_x, \, y_j = jh_y, \, z_k = kh_z; \ n = \overline{0..N_t}, \, i = \overline{0..N_x}, \, j = \overline{0..N_y}, \, k = \overline{0..N_z}; \\ N_t \tau = T, \, N_x h_x = l_x, \, N_y h_y = l_y, \, N_z h_z = l_z \right\},$$

where τ is the time step, h_x , h_y h_z are steps in space, N_t is the number of time layers, T is the upper bound on the time coordinate, N_x , N_y N_z is the number of nodes by spatial coordinates, l_x , l_y l_z are boundaries along the parallelepiped in the direction of the axes Ox, Oy and Oz accordingly.

To solve the hydrodynamic problem, we used the method of correction to pressure. The variant of this method in the case of a variable density will take the form [Suk11, Bel75]:

$$\frac{\tilde{u} - u}{\tau} + u\bar{u'}_{x} + v\bar{u'}_{y} + w\bar{u'}_{z} = (\mu\bar{u'}_{x})'_{x} + (\mu\bar{u'}_{y})'_{y} + (\nu\bar{u'}_{z})'_{z},$$

$$\frac{\tilde{v} - v}{\tau} + u\bar{v'}_{x} + v\bar{v'}_{y} + w\bar{v'}_{z} = (\mu\bar{v'}_{x})'_{x} + (\mu\bar{v'}_{y})'_{y} + (\nu\bar{v'}_{z})'_{z},$$

$$\frac{\tilde{w} - w}{\tau} + u\bar{w'}_{x} + v\bar{w'}_{y} + w\bar{w'}_{z} = (\mu\bar{w'}_{x})'_{x} + (\mu\bar{w'}_{y})'_{y} + (\nu\bar{w'}_{z})'_{z} + g,$$

$$\hat{p}''_{xx} + \hat{p}''_{yy} + \hat{p}''_{zz} = \frac{\hat{\rho} - \rho}{\tau^{2}} + \frac{(\hat{\rho}\tilde{u})'_{x}}{\tau} + \frac{(\hat{\rho}\tilde{v})'_{y}}{\tau} + \frac{(\hat{\rho}\tilde{w})'_{z}}{\tau},$$

$$\frac{\hat{u} - \tilde{u}}{\tau} = -\frac{1}{\hat{\rho}}\hat{p'}_{x}, \quad \frac{\hat{v} - \tilde{v}}{\tau} = -\frac{1}{\hat{\rho}}\hat{p'}_{y}, \quad \frac{\hat{w} - \tilde{w}}{\tau} = -\frac{1}{\hat{\rho}}\hat{p'}_{z},$$
(5)

where $V = \{u, v, w\}$ are the components of the velocity vector, $\{\hat{u}, \hat{v}, \hat{w}\}, \{\tilde{u}, \tilde{v}, \tilde{w}\}$ are the components of the velocity vector fields on the new and intermediate time layers, respectively, $\bar{u} = (\tilde{u} + u)/2$, $\hat{\rho}$ and ρ is the distribution of the density of the aqueous medium on the new and previous time layers, respectively.

In the construction of discrete mathematical models of hydrodynamics, the fullness of the control cells was taken into account, which makes it possible to increase the real accuracy of the solution in the case of a complex geometry of the investigated region by improving the approximation of the boundary.

Through $o_{i,j,k}$ marked fullness of the cell (i, j, k) [Suk14]. The degree of fullness of the cell is determined by the pressure of the liquid column inside this cell. If the average pressure at the nodes that belong to the vertices of the cell in question is greater than the pressure of the liquid column inside the cell, then the cell is considered to be full $(o_{i,j,k} = 1)$. In the general case, the fullness of the cells can be calculated by the following formula [Suk11]:

$$o_{i,j,k} = \frac{p_{i,j,k} + p_{i-1,j,k} + p_{i,j-1,k} + p_{i-1,j-1,k}}{4\rho g h_z},\tag{6}$$

where p is the pressure.

We introduce the coefficients q_0 , q_1 , q_2 , q_3 , q_4 , q_5 , q_6 , describing the fullness of regions located in the vicinity of the cell (control areas). The value characterizes the fullness of the region

 $\begin{array}{l} D_0: \ \{x \in (x_{i-1}, x_{i+1}), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_1 - D_1: \ \{x \in (x_i, x_{i+1}), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_2 - D_2: \ \{x \in (x_{i-1}, x_i), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_3 - D_3: \ \{x \in (x_{i-1}, x_{i+1}), \ y \in (y_j, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_5 - D_5: \ \{x \in (x_{i-1}, x_{i+1}), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_5 - D_5: \ \{x \in (x_{i-1}, x_{i+1}), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k+1})\}, \ q_5 - D_5: \ \{x \in (x_{i-1}, x_{i+1}), \ y \in (y_{j-1}, y_{j+1}), \ z \in (z_{k-1}, z_{k})\}.\end{array}$

The filled parts of the regions D_m will be called Ω_m , where $m = \overline{0..6}$. In accordance with this, the coefficients q_m can be calculated from the formulas:

$$\begin{split} (q_m)_{i,j,k} &= \frac{S_{\Omega_m}}{S_{D_m}}, (q_0)_{i,j,k} = \frac{1}{2} \left((q_1)_{i,j,k} + (q_2)_{i,j,k} \right), \\ (q_1)_{i,j,k} &= \frac{o_{i+1,j,k} + o_{i+1,j+1,k} + o_{i+1,j,k+1} + o_{i+1,j+1,k+1}}{4}, \\ (q_2)_{i,j,k} &= \frac{o_{i,j,k} + o_{i,j+1,k} + o_{i,j,k+1} + o_{i,j+1,k+1}}{4}, \\ (q_3)_{i,j,k} &= \frac{o_{i+1,j+1,k} + o_{i,j+1,k} + o_{i+1,j+1,k+1} + o_{i,j+1,k+1}}{4}, \\ (q_4)_{i,j,k} &= \frac{o_{i,j,k} + o_{i+1,j,k} + o_{i,j,k+1} + o_{i+1,j,k+1}}{4}, \\ (q_5)_{i,j,k} &= \frac{o_{i,j,k+1} + o_{i+1,j,k+1} + o_{i+1,j+1,k+1} + o_{i,j+1,k+1}}{4}, \\ (q_6)_{i,j,k} &= \frac{o_{i,j,k} + o_{i+1,j,k} + o_{i+1,j+1,k} + o_{i,j+1,k}}{4}. \end{split}$$

In the case of boundary conditions of the third kind $c'_n(x, t) = \alpha_n c + \beta_n$, the discrete analogues of the convective uc'_x and diffusion $(\mu c'_x)'_x$ transfer operators, obtained with the help of the integro-interpolation method, taking into account the partial "fullness" of the cells, can be written in the following form:

$$\begin{split} uc'_x &\simeq (q_1)_i \, u_{i+1/2} \frac{c_{i+1} - c_i}{2h_x} + (q_2)_i \, u_{i-1/2} \, \frac{c_i - c_{i-1}}{2h_x}, \\ (\mu c'_x)'_x &\simeq (q_1)_i \, \mu_{i+1/2} \frac{c_{i+1} - c_i}{h_x^2} - (q_2)_i \, \mu_{i-1/2} \frac{c_i - c_{i-1}}{h_x^2} - |(q_1)_i - (q_2)_i| \, \mu_i \frac{\alpha_x c_i + \beta_x}{h_x}. \end{split}$$

Similarly, approximations for the remaining coordinate directions will be recorded. The error in approximating the mathematical model is equal to $O\left(\tau + \|h\|^2\right)$, where $\|h\| = \sqrt{h_x^2 + h_y^2 + h_z^2}$. The conservation of the flow at the discrete level of the developed hydrodynamic model is proved, as well as the absence of non-conservative dissipative terms obtained as a result of discretization of the system of equations. A sufficient condition for the stability and monotony of the developed model is determined on the basis of the maximum principle [Suk11], with constraints on the step with respect to the spatial coordinates:

$$h_x < |2\mu/u|, h_y < |2\mu/v|, h_z < |2\nu/w| \text{ or } Re \le 2N,$$

where $Re = |V| \cdot l/\mu$ are Reynolds numbers, l is the characteristic size of the region $N = \max\{N_x, N_y, N_z\}$.

Discrete analogs of the system of equations (5) are solved by an adaptive modified alternating-triangular method of variational type.

4 Method for solving grid equations

The resulting grid equations can be written in the matrix form [Suk12]:

$$Ax = f, (7)$$

where A – a linear, positive definite operator (A > 0). To find the solution of problem (7) we will use an implicit iterative process

$$B\frac{x^{m+1} - x^m}{\tau_{m+1}} + Ax^m = f.$$
(8)

In equation (8), m is the iteration number, $\tau > 0$ is an iteration parameter, and B is an invertible operator, called the preconditioner or stabilizer. The inversion of the operator B in (8) must be substantially simpler than the direct inversion of the original operator A in (7). In constructing B, we proceed from the additive representation of the operator A_0 is the symmetric part of the operator A:

$$A_0 = R_1 + R_2, \quad R_1 = R_2^*, \tag{9}$$

where $A = A_0 + A_1$, $A_0 = A_0^*$, $A_1 = -A_1^*$.

The preconditioner operator will be written in the following form:

$$B = (D + \omega R_1) D^{-1} (D + \omega R_2), \ D = D^* > 0, \ \omega > 0,$$
(10)

where D is an operator.

Relations (9)-(10) define modified alternate-triangular method (MATM) for the solution of the problem if operators R_1, R_2 are defined and methods for determining parameters τ_{m+1}, ω and the operator D are indicated.

The algorithm of the adaptive modified alternating-triangular method of minimum corrections for calculating the grid equations with a nonselfadjoint operator has the form:

$$B(\omega_m)w^m = r^m, \ r^m = Ax^m - f, \tilde{\omega}_m = \sqrt{\frac{(Dw^m, w^m)}{(D^{-1}R_2w^m, R_2w^m)}},$$
(11)

$$s_m^2 = 1 - \frac{(A_0 w^m, w^m)^2}{(B^{-1} A_0 w^m, A_0 w^m) (B w^m, w^m)} , \ k_m = \frac{(B^{-1} A_1 w^m, A_1 w^m)}{(B^{-1} A_0 w^m, A_0 w^m)},$$
$$1 - \sqrt{\frac{s_m^2 k_m}{m}} \qquad (A w^m, w^m)$$

$$\theta_m = \frac{1 - \sqrt{(1 + k_m)}}{1 + k_m (1 - s_m^2)}, \tau_{m+1} = \theta_m \frac{(A_0 w^m, w^m)}{(B^{-1} A_0 w^m, A_0 w^m)},$$
$$x^{m+1} = x^m - \tau_{m+1} w^m, \omega_{m+1} = \tilde{\omega}_m,$$

where r^m is the discrepancy vector, w^m is the correction vector, the diagonal part of the operator D is used as the operator.

5 The parallel version of the algorithm for solving grid equations

Consider the parallel algorithm for calculating the correction vector [Suk12]:

$$(D + \omega_m R_1)D^{-1}(D + \omega_m R_2)w^m = r^m,$$

where R_1 is the lower-triangular matrix, and R_2 is the upper-triangular matrix. To this end, we solve successively the systems:

$$(D + \omega_m R_1)y^m = r^m, \quad (D + \omega_m R_2)w^m = Dy^m.$$



Figure 2: The scheme for calculating the vector y^m

First, the vector y^m is calculated, and the calculation starts in the lower left corner. Then the calculation of the correction vector w^m begins from the upper right corner. Fig. 2 shows the calculation of the vector y^m .

The results of calculating the acceleration and efficiency, depending on the number of processors for the parallel variant of the adaptive alternating-triangular method, are given in the table 1.

The table 1 shows that the algorithm of the alternating-triangular iterative method and its parallel realization on the basis of decomposition in two spatial directions can be effectively applied to solve hydrodynamic problems for a sufficiently large number of calculators ($p \leq 128$).

Number of pro-	Time, sec.	Acceleration	Efficiency
cessors			
1	7,490639	1	1
2	4,151767	1,804	0,902
4	2,549591	2,938	0,734
8	1,450203	5,165	0,646
16	0,882420	8,489	0,531
32	0,458085	16,351	0,511
64	0,265781	28,192	0,44
128	0,171535	43,668	0,341

Table 1: The dependence of acceleration and	l efficiency on the r	number of processors.
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6 Results of numerical experiments

The constructed complex of programs allows you to set the shape and intensity of the source of oscillations, as well as the geometry of the surface object. Fig. 3 (a) shows the results of numerical experiments on modeling the propagation of wave hydrodynamic processes in the flow of an aquatic environment around a surface body, taking into account the geometries of the bottom of an object located in a liquid and the bottom of a reservoir. As an example of the practical use of a problem-oriented program complex, the problem of calculating the hydrodynamic effect of support waves on structures is solved. Surface dimensions: 5 m wide, 10 m long, immersion depth 20 cm. The structure is installed at the bottom of the reservoir with the help of 6 supports. The selected section of the simulation has dimensions of 50x50 m and a depth of 1 m. The source of disturbances is set at some distance from the surface object. The boundaries of the hydrodynamic force action on the surface structures. At the initial moment of time, the liquid is at rest. It is required to determine the subsequent movement of the aquatic environment when there is a surface object on the surface and hydrodynamic force loads on the structure supports. To solve this problem, a grid of 100x100 sizes was used, the time step is 0.01 seconds.



Figure 3: The level function of water flow around the surface of the body having a support



Figure 4: Power loads on supports of the surface structure: 1 - on the front pair from the side of the incident wave, 2 - on the central pair, 3 - on the far pair of supports

Fig. 3 shows that during the propagation of a plane wave, which encounters an obstacle in the form of a

surface body, there is a reflection of wave oscillations from a stationary object, which in turn leads to a change in the wave profile. The source of oscillations is distributed along the left border and has a sinusoidal shape. The results of numerical experiments on modeling the propagation of hydrodynamic wave processes and provide an opportunity to assess the impact of waves on structures that have a support at the bottom of the reservoir. Fig. 4 shows the force hydrodynamic effect on the supports of the surface structure installed at the bottom of the reservoir. The ordinate axis shows the power loads in tons, the abscissa axis indicates the time counted from the onset of oscillations in seconds.

On the basis of full-scale data, a three-dimensional model of wave hydrodynamic processes has been developed that describes the motion of an aquatic environment taking into account the wave's output to the shore. A modern software package adapted for simulation of hydrodynamic wave processes is developed, the field of application of which is the construction of the velocity and pressure field of the aquatic environment, and the evaluation of the hydrodynamic impact on the shore in the presence of surface waves. Based on the developed complex of programs, numerical simulation of hydrodynamic wave processes in the coastal zone of a shallow water body was carried out.

The practical significance of numerical algorithms and the complex of programs that realize them consists in the possibility of their application in the study of hydrophysical processes in coastal water systems, as well as in the construction of the velocity and pressure field of the aquatic environment, and the evaluation of the hydrodynamic impact on the shore in the presence of surface waves. The constructed program complex allows you to specify the shape and intensity of the oscillation source, as well as the geometry of the bottom of the reservoir. Fig. 6 shows the results of numerical experiments on the simulation of the propagation of wave hydrodynamic processes when the wave leaves the shore, taking into account the geometries of the bottom of the object located in the liquid and bottom of the reservoir.



Figure 5: The field of the velocity vector of the aquatic environment (XOZ plane cut)



Figure 6: Level and bottom elevation function

As an example of the practical use of a problem-oriented program complex, the problem of calculating the velocity and pressure fields is solved. The selected modeling site measures 50x50 m and a depth of 2 m, the peak point rises above sea level by 2 m. The disturbance source is given at some distance from the shore line. At the initial time, the liquid is at rest. To solve this problem, a grid of 100x100x40 dimensions was used, the time step is 0.01 seconds.

Fig. 6 shows the field of the vector of the velocity of the aquatic environment when the wave rolls to the shore, while the function of elevating the level dynamically changes, zones of flooding and shallowing are formed. Fig. 7 shows that the land area was flooded with an incident wave. Accounting for flooding and dehumidification of coastal areas was carried out by recalculating the occupancy of the calculated cells. The proposed approach makes it possible to solve problems in domains with a complex and dynamically rearranged geometry of the boundary.

It should be noted that the developed software package has a distinctive feature, when modeling the propagation of surface waves, the wave output to the shore is taken into account.

Fig. 7 shows the results of modeling wave propagation towards the shore obtained from two different models with the same input parameters. The calculations are based on a two-dimensional model constructed on the basis of a system of shallow water equations (a) and a mathematical model involving three equations of motion (b). The calculated interval in both cases was 5 sec.



Figure 7: Level and bottom elevation function



Figure 8: Level and bottom elevation function

Fig. 8 shows the functions of level elevation obtained on the basis of a two-dimensional system of shallow water equations and a mathematical model that includes three equations of motion for a section with the plane y=25. The figure shows that the results vary considerably. The wave profiles calculated on the basis of a two-dimensional model of shallow water overtake the wave profiles obtained on the basis of a three-dimensional model. The maximum distance between the two profiles was 32.9 cm.

7 Conclusion

The work is devoted to the development of a model of three-dimensional wave processes, designed to simulate wave processes taking into account wave propagation towards the shore. A full-scale experiment was conducted to measure various parameters of wave propagation in shallow water. On the basis of experimental data, the values of the spectrum of the function of elevating the water level are obtained. The description of the developed program complex is given. The complex of programs constructed allows one to specify the shape and intensity of the source of oscillations, and also takes into account the flooding and drainage of coastal areas. On the basis of the developed software package, further studies of the calculation of the wave force effects on surface objects and objects of coastal infrastructure and bottom surface geometry are possible.

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