Properties of eco-colonies

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Abstract. Eco-colonies are new grammar systems with very simple regular grammars called agents. Every agent generates its own finite language, all agents cooperate on the shared environment. The environment is not changed only by agents, but it can develope itself.
The generating power of eco-colonies was discussed in several papers, eco-colonies were compared especially with various types of colonies, but not all relations were proved. In this paper we summarize the published results and we present some other non-published results about the generating power of eco-colonies.

Keywords: Eco-colony, 0L eco-colony, E0L eco-colony, property, colony, agent, component, grammar system

1 Introduction

Colonies were introduced in [4] as collections of simple grammars (called components) working on common environment. A component is specified by its start symbol (an object or another symbol from the environment, the component can find its start symbol in the environment and process it) and by its finite language. This language determines actions to do with the start symbol, it is usually a list of words, the component substitutes its start symbol by some of these words. The environment is static, only the components can modify it.

There are several variants of colonies with various types of derivation. The original model was sequential (only one component works in one derivation step), the other basic types of derivation are sequential with parallely working components or parallel. Parallel colonies were introduced in [3], parallel behaviour of a colony means working all the components that can work (the component whose start symbol is in the environment and no other component is occupying this symbol for the actual derivation step), one component processes one occurrence of its start symbol.

Eco-colonies were first studied in [7], their E0L form in [5, 6]. Eco-colonies are colonies with developing environment. The concept of developing of the environment is inspired by another type of grammar systems, eco-grammar systems ([2]). The environment is not specified only by its alphabets V and T but as 0L (one alphabet) or E0L (two alphabets) scheme. Every symbol of the environment not processed by agents (components) is overwritten by some of the developing rules of this scheme.

Eco-colonies are useful for modelling some simple processes in nature and it is not so hard to programme the model. Here are some elementary examples of models:

- rabits (agents) on a meadow (environment), the developement of the environment means growth of grass, the pieces of soil without grass are (or are not) replaced by the grass-blades, the grass-blades not eaten by rabbits can grow,
- colony of ants (the ants – agents – work on the shared environment – ant-hill and its neighbourhood),
- drug (agents) acting on a bacterial culture.

2 Definitions

In this section we define colonies, two types of eco-colonies and then two types of derivation in eco-colonies.

Definition 1. *A colony is an 4-tuple* $\mathcal{C} = (V, T, \mathcal{R}, w_0)$*, where*

- V *is a finite non-empty alphabet of the colony,*
- T *is a non-empty terminal alphabet of the colony,* $T \subset V$,
- $\mathcal{R}$ *is a finite (multi)set of components,*
  $\mathcal{R} = \{(S, F) \mid S \in (V - T),\ F \subseteq (V - \{S\})^*,\ F$ *is finite and non-empty*$\}$,
  S *is the start symbol of the component* (S, F) *and* F *is the finite language of this component,*
- w_0 *is the axiom.*

We know several types of derivations for colonies, here are the basic types:

b-**mode** is sequential type of derivation, only one component works in one derivation step, the active component chooses one occurrence of its start symbol and replaces this occurrence by some of the words of its finite language F,

t-**mode** is sequentially-parallel – only one component is active in one derivation step and this component rewrites all occurrences of its start symbol by words of its language,

wp-**mode** is parallel mode, where every component which can work must work; a component can work if its start symbol is in the environment and any other component with the same start symbol does not occupy this occurrence of the symbol, a component rewrites only one occurrence of its start symbol,

sp-**mode** is parallel mode similar to *wp*, but if there is an occurrence of a symbol in the environment, every component with this start symbol must work – if all occurrences of this symbol are occupied by another components with the same start symbol, the derivation is blocked.

The exact definitions of basic types of derivation in colonies are in [1, 3, 4, 7, 5], definitions of an eco-grammar system and its type of derivation are in [2].

Definition 2. *Let $\mathcal{C}$ be a colony, $\mathcal{C} = (V, T, \mathcal{R})$. The language generated by the derivation step $x, x \in \{b, t, wp, sp\}$ in $\mathcal{C}$ is*

$$L(\mathcal{C}, x) = \{w \in T^* \mid w_0 \overset{x}{\Longrightarrow}{}^* w\}$$

Definition 3. *An E0L eco-colony of degree n, $n \geq 1$, is an $(n+2)$-tuple $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$, where*

- *$E = (V, T, P)$ is E0L scheme, where*
  - *V is a finite non-empty alphabet,*
  - *T is a non-empty terminal alphabet, $T \subseteq V$,*
  - *P is a finite set of E0L rewriting rules over V,*
- *$A_i = (S_i, F_i)$, $1 \leq i \leq n$, is the i-th agent, where*
  - *$S_i \in V$ is the start symbol of the agent,*
  - *$F_i \subseteq (V - \{S_i\})^*$ is a finite set of action rules of the agent (the language of the agent),*
- *w_0 is the axiom.*

An *0L eco-colony* is defined similarly, only the environment is 0L scheme $E = (V, P)$, P is a finite set of 0L rewriting rules over V.

As we can see, agents are defined such as components in colonies, an environment is determined by the alphabets in colonies, and by E0L or 0L scheme in eco-colonies.

We define two derivation modes for eco-colonies – the first one, *wp*, is inspired by the *wp* mode for colonies, we only add possibility of developing for the environment. In every derivation step each agent (S, F) looks for its start symbol S. If it finds some occurrence of this symbol not occupied by any other agent, the agent becomes active, occupies this symbol and rewrites it by some of words of its language F.

Definition 4. *We define a weakly competitive parallel derivation step in an eco-colony $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$ by $\alpha \overset{wp}{\Longrightarrow} \beta$, where*

- *$\alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \ldots \gamma_{r-1} S_{i_r} \gamma_r$, $r > 0$,*
- *$\beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \ldots \gamma'_{r-1} f_{i_r} \gamma'_r$, $A_{i_k} = (S_{i_k}, F_{i_k})$, $f_{i_k} \in F_{i_k}$, $1 \leq k \leq r$ (the agent A_{i_k} is* active *in this derivation step),*
- *$\{i_1, i_2, \ldots, i_r\} \subseteq \{1, 2, \ldots, n\}$, $i_k \neq i_m$ for every $k \neq m$, $1 \leq k, m \leq r$,*
- *for every symbol $S \in V$ if $|\gamma_0 \gamma_1 \ldots \gamma_r|_S > 0$ then every agent with the start symbol S must be active (if agents can work they must work),*
- *$\gamma_k \overset{E}{\Longrightarrow} \gamma'_k$, $\gamma_k \in V^*$, $0 \leq k \leq r$, is the derivation step of the scheme E.*

The second type of derivation step, *ap*, means that all agents must work in every derivation step and if some agent is not able to work (there is not any free occurrence of its start symbol), the derivation is blocked. This type of derivation is inspired by the basic type of derivation in eco-grammar systems.

Definition 5. *We define a derivation step ap (all are working parallely) in an eco-colony* $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$ *by* $\alpha \overset{ap}{\Longrightarrow} \beta$*, where*

- $\alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \cdots \gamma_{n-1} S_{i_n} \gamma_n$,
- $\beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \cdots \gamma'_{n-1} f_{i_n} \gamma'_n$, $\quad A_{i_k} = (S_{i_k}, F_{i_k})$, $f_{i_k} \in F_{i_k}$, $1 \leq k \leq n$,
- $\{i_1, i_2, \ldots, i_n\} = \{1, 2, \ldots, n\}$ *(every agent works in every derivation step),*
- $\gamma_k \overset{E}{\Longrightarrow} \gamma'_k$, $\gamma_k \in V^*$, $0 \leq k \leq n$, *is the derivation step of the scheme* E.

Definition 6. *Let* Σ *be an 0L eco-colony,* $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$. *The language generated by the derivation step* $x, x \in \{wp, ap\}$ *in* Σ *is*

$$L(\Sigma, x) = \{w \in V^* \mid w_0 \overset{x}{\Longrightarrow}{}^* w\}$$

Let Σ *be an E0L eco-colony,* $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$. *The language generated by the derivation step* $x, x \in \{wp, ap\}$ *in* Σ *is*

$$L(\Sigma, x) = \{w \in T^* \mid w_0 \overset{x}{\Longrightarrow}{}^* w\}$$

Example 1. We create an E0L eco-colony $\Sigma = (E, A_1, A_2, AbB)$, where $E = (\{A, B, a, b\}, \{a, b\}, \{a \to a, b \to bb\})$, $A_1 = (A, \{aB, \varepsilon\})$, $A_2 = (B, \{aA, \varepsilon\})$

Let us construct derivations with ap and wp types of derivations:

$AbB \overset{ap}{\Longrightarrow} aBb^2aA \overset{ap}{\Longrightarrow} a^2Ab^4a^2B \overset{ap}{\Longrightarrow} a^3Bb^8a^3A \overset{ap}{\Longrightarrow} a^4Ab^{16}a^4B \overset{ap}{\Longrightarrow} \ldots$

$AbB \overset{wp}{\Longrightarrow} aBb^2aA \overset{wp}{\Longrightarrow} a^2Ab^4a^2B \overset{wp}{\Longrightarrow} a^2b^8a^3A \overset{wp}{\Longrightarrow} a^2b^{16}a^4B \overset{wp}{\Longrightarrow} \ldots$

The wp derivation allowes "resting" of non-active agents. If we use the ap type of derivation, a terminal word is generated only if the both agents use the ε-rule in the same derivation step, otherwise the derivation is blocked without creating the final word.

We generate these languages:

$$L(\Sigma, ap) = \left\{a^n b^{2^n} a^n \mid n \geq 0\right\}$$
$$L(\Sigma, wp) = \left\{a^i b^{2^n} a^j \mid n \geq 0,\ 0 \leq i, j \leq n\right\}$$

3 Generating power of eco-colonies

We compare the generating power of eco-colonies and colonies, eco-grammar systems and a special type of colonies with the terminal alphabet equal to the alphabet of the system. We use this notation:

$0EC_x$	$x \in \{wp, ap\}$ class of 0L eco-colonies with x type of derivation
EEC_x	$x \in \{wp, ap\}$ class of E0L eco-colonies with x type of derivation
COL_x	$x \in \{b, t, wp, sp\}$ class of colonies with the x type of derivation
COL_x^T	$x \in \{b, t, wp, sp\}$ class of colonies with the x type of derivation, $V = T$
EG	class of eco-grammar systems

3.1 Known results

We proved in [6] these results:

$$COL_{wp} \subset EEC_{wp} \tag{1}$$
$$0EC_{wp} \subset EEC_{wp} \tag{2}$$

In [5] are proved these results:

$$EEC_{wp} - EG \neq \emptyset \tag{3}$$
$$COL_b \subset EEC_{wp} \tag{4}$$
$$EEC_{wp} - COL_t \neq \emptyset \tag{5}$$

In [7] we proved the results:

$$COL_x^T \subset COL_x,\ x \in \{b, t, wp, sp\} \tag{6}$$
$$0EC_{ap} \neq 0EC_{wp} \tag{7}$$
$$COL_x^T \subset 0EC_{wp},\ x \in \{b, wp\} \tag{8}$$
$$0EC_{ap} - COL_x^T \neq \emptyset,\ x \in \{b, wp\} \tag{9}$$
$$0EC_y \subset EG,\ y \in \{ap, wp\} \tag{10}$$

3.2 New results

In the equation (2) we can find the relation $0EC_{wp} \subset EEC_{wp}$. We prove the equivalent relation for the *ap* derivation.

Theorem 1.

$$0EC_{ap} \subset EEC_{ap} \tag{11}$$

Proof. 0L eco-colonies are special types of E0L eco-colonies where $V = T$, so the relation $0EC_{ap} \subseteq EEC_{ap}$ is trivial.

To prove the proper subset we use the language

$$L_1 = \left\{a^{2^n} \mid n \geq 1\right\}$$

This language is generated by the E0L eco-colony $\Sigma = (E, A_1, A_2, UVa)$, where $E = (\{a, U, V\}, \{a\}, \{a \to aa, U \to U, V \to V\})$, $A_1 = (U, \{V, \varepsilon\})$, $A_2 = (V, \{U, \varepsilon\})$.

$UVa \overset{ap}{\Longrightarrow} VUa^2 \overset{ap}{\Longrightarrow} UVa^4 \overset{ap}{\Longrightarrow} VUa^8 \overset{ap}{\Longrightarrow} UVa^{16} \overset{ap}{\Longrightarrow} \dots \overset{ap}{\Longrightarrow} a^{2^n}$

The agents do not generate any word, they exist only because the number of agents must be greater than 0, and with using the *ap* derivation all agents must work in every derivation step, so the agents A_1 and A_2 work with the symbols U, V until the final word is generated.

Suppose that this language can be generated by some 0L eco-colony with *ap* derivation. We need at least one agent and this agent must be active in every derivation step. We have only one alphabet $V = \{a\}$, so the start symbol of this agent is a. But the agent generates only a finite language, so the symbol a is not alowed in the set of action rules, the agent is defined as $A = (a, \{\varepsilon\})$.

The language is exponential and the agent(-s) does not help with growing of the language, so the rule $a \to aa$ is in the environment. It can generate the language, but the agent "eats" one symbol a in every derivation step and the environment cannot correct it (it is only 0L scheme, any other correcting rule such as $a \to aaa$ would be used freely any times and anywhere in one derivation step), so the language L_1 would not be generated, $L_1 \notin 0EC_{ap}$. □

Theorem 2.

$$xEC_y - COL_z \neq \emptyset \tag{12}$$

where $x \in \{0, E\},\ y \in \{wp, ap\},\ z \in \{b, t, wp, sp\}$.

Proof. In this proof we use the language similar to L_1,

$$L_2 = \left\{cda^{2^{2n}}b^{2^{2n}} \mid n \geq 0\right\} \cup \left\{dca^{2^{2n+1}}b^{2^{2n+1}} \mid n \geq 0\right\}$$

This language can be generated by the eco-colony $\Sigma = (E, A_1, A_2, cda)$, where $E = (\{a, b, c, d\}, \{a \to aa, b \to bb, c \to c, d \to d\})$, $A_1 = (c, \{d\})$, $A_2 = (d, \{c\})$.
$cdab \Rightarrow dca^2b^2 \Rightarrow cda^4b^4 \Rightarrow dca^8b^8 \Rightarrow cda^{16}b^{16} \Rightarrow \dots$

Two agents work in every derivation step and we use only one alphabet, so it may be E0L as well as 0L eco-colony and the derivation step may be wp or ap.

The language L_2 is not context-free, so $L_2 \notin COL_b$, and it grows exponentially so $L_2 \notin COL_{wp}$ and $L_2 \notin COL_{sp}$ (proved in [6] and [5]).

Colonies with t type of derivation can generate some exponentially growing languages, but only one component work in one derivation step. This component rewrites all occurrences of its start symbol, and there is only one start symbol in one component. So we can rewrite a or b in one derivation step, but not both. There is not any way to control generating the same exponential number of a-s and b-s, so $L_2 \notin COL_t$. □

Corollary 1.

$$xEC_y - COL_z^T \neq \emptyset \tag{13}$$

where $x \in \{0, E\},\ y \in \{wp, ap\},\ z \in \{b, t, wp, sp\}$.

Proof. Follows from Theorem 2 and Equation (6). □

Theorem 3.

$$COL_x - 0EC_{wp} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \tag{14}$$

Proof. In [7] we proved that the language

$$L_3 = \left\{a^{15-2n}b^ncb^nd \mid 0 \leq n < 7, n \text{ is even}\right\} \cup \left\{a^{15-2n}b^ncb^nd \mid 0 < n \leq 7, n \text{ is odd}\right\}$$

is not in $0EC_{wp}$. It is a finite language, so $L_3 \in COL_x$ for $x \in \{b, t, wp, sp\}$. □

Corollary 2. *The set of languages* $0EC_{wp}$ *is incomparable to the sets of languages* COL_b, COL_t, COL_{wp} *and* COL_{sp}.

Proof. Follows from Theorems 2 and 3. □

Corollary 3.

$$COL_b^T \subset EEC_{wp}, \quad COL_{wp}^T \subset EEC_{wp} \tag{15}$$

Proof. Follows from Equations (6) and (2). □

Theorem 4.

$$COL_x - 0EC_{ap} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \tag{16}$$

Proof. In [7] we proved that the language

$$L_4 = \{a, aa\}$$

is not generated by any 0L eco-colony. But this language is finite, so $L_4 \in COL_x$ for $x \in \{b, t, wp, sp\}$. □

Theorem 5.

$$COL_b \subset EEC_{ap} \tag{17}$$

Proof. We have a colony with the b mode of derivation $\mathcal{C} = (V, T, \mathcal{R}, w_0)$ and we create an E0L eco-colony with the ap derivation $\Sigma = (E, A_1, A_2, BC \circ w_0)$.

The agents A_1 and A_2 work very simply – they rewrite only one symbol to another one: $A_1 = (B, \{C, \varepsilon\})$, $A_2 = (C, \{B, \varepsilon\})$, such as in the proof of the Theorem 1.

We create rules of the environment from the components of $\mathcal{R}$. Suppose that all components in $\mathcal{R}$ have various start symbol, there is not any couple of components with the same start symbol.

For every component $(a, \{\alpha_1, \alpha_2, \ldots, \alpha_k\})$ we create developing rules for the environment:

$$a \longrightarrow a|\alpha_1|\alpha_2|\ldots|\alpha_k$$

and for every symbol b which is not the start symbol in any component we create one rule $b \longrightarrow b$.

So the environment simulates working of the components in the colony. To simulate the sequential derivation we must allowe to rewrite every symbol to itself. □

Example 2. We demonstrate the construction of the proof on the colony generating this language:

$$L_5 = \{waw^R a^i \mid w \in \{0, 1\}^*, \ i > 0\}$$

We have a colony $\mathcal{C} = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, \mathcal{R}, S)$ generating the language, the set of components $\mathcal{R}$ is
$\mathcal{R} = \{(S, \ \{HA\}), \ (H, \ \{0H'0, 1H'1, a\}), \ (H', \ \{H\}), \ (A, \ \{aA', a\}), \ (A', \ \{A\})\}$

Now we create an E0L eco-colony with ap derivation $\Sigma = (E, A_1, A_2, BCS)$,
$E = (\{B, C, S, H, H', A, A', 0, 1, a\},\ \{0, 1, a\},\ P)$,
$A_1 = (B, \{C, \varepsilon\})$, $A_2 = (C, \{B, \varepsilon\})$,
the set of rules P in the environment is

$$P = \{\ H \to H|0H'0|1H'1|A,\quad H' \to H'|H,\quad 1 \to 1,\quad a \to a,$$
$$A \to A|aA'|a,\quad A' \to A'|A,\quad 0 \to 0,\quad S \to S|HA\ \}$$

One of derivations in $\mathcal{C}$:

$$S \overset{b}{\Rightarrow} HA \overset{b}{\Rightarrow} 1H'1A \overset{b}{\Rightarrow} 1H1A \overset{b}{\Rightarrow} 10H'01A \overset{b}{\Rightarrow} 10H01A \overset{b}{\Rightarrow} 10a01A \overset{b}{\Rightarrow}$$
$$\overset{b}{\Rightarrow} 10a01aA' \overset{b}{\Rightarrow} 10a01aA \overset{b}{\Rightarrow} 10a01aa$$

Two of possible derivations of the same word in Σ:

$$BCS \overset{ap}{\Longrightarrow} CBHA \overset{ap}{\Longrightarrow} BC1H'1A \overset{ap}{\Longrightarrow} CB1H1A \overset{ap}{\Longrightarrow} BC10H'01A \overset{ap}{\Longrightarrow}$$
$$\overset{ap}{\Longrightarrow} CB10H01A \overset{ap}{\Longrightarrow} BC10a01A \overset{ap}{\Longrightarrow} CB10a01aA' \overset{ap}{\Longrightarrow} BC10a01aA \overset{ap}{\Longrightarrow}$$
$$\overset{ap}{\Longrightarrow} 10a01aa$$

$$BCS \overset{ap}{\Longrightarrow} CBHA \overset{ap}{\Longrightarrow} BC1H'1aA' \overset{ap}{\Longrightarrow} CB1H1aA \overset{ap}{\Longrightarrow} BC10H'01aa \overset{ap}{\Longrightarrow}$$
$$\overset{ap}{\Longrightarrow} CB10H01aa \overset{ap}{\Longrightarrow} 10a01aa$$

Corollary 4.

$$COL_b^T \subset EEC_{ap} \tag{18}$$

Proof. Follows from Theorem 5 and Equation (6). □

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