Model of distribution of homogenous resources between suppliers and consumers

Mikhail Matveev[1[0000-0002-6528-6420]], Semen Podvalny[2[0000-0003-1260-4883]], Victor Taratukhin[1[0000-0002-7619-9269]]

[1]Voronezh State University Voronezh, Russia

[2]Voronezh State Technical University Voronezh, Russia

mgmatveev@yandex.ru, spodvalny@yandex.ru

Abstract. The model of distribution of homogenous resources from set of suppliers to set of consumers. The distribution is performed by the center that represents consumer interests, according to formal criteria of matching their commercial and technical requirements. The formalization of criteria is based on fuzzy logical statements. The choice is performed based on solution of modified transportation problem with intermediate points. The result is distribution of the procured resources by suppliers and purchased resources by consumers, which ensures maximum accordance to presented requirements. Analysis of the co-influence of requirements on the distribution results is performed.

Keywords: Resources Distribution, Fuzzy Statements, Indicators Aggregation, Transportation problem.

1 Introduction

The task of rational choice suppliers of necessary resources by consumers from given set is well known [1]. Usually the selection is performed by commercial and technical requirements. Such choice can be described by schema “one consumer – many suppliers”. In case of purchasing homogenous (interchangeable) resources procuring activity could be efficiently organized using schema “many consumers – many suppliers”. In this case the consumers’ requests of purchasing resources come to the centralized purchaser (center), which consolidates them into wholesale shopping lot with corresponding discount. It is required to allocate those shopping lots between suppliers with maximum correspondence between commercial requirements given by the center and within restriction of technical requirements. After that purchased resources is necessary to optimally allocate between consumers with maximum accordance to technical requirements. The purchaser’s role can be performed by commercial or public electronic trading platform. Despite of obvious profitability of the schema “many-to-many”, the realization becomes difficult due to high labor intensity of processes of requirements’ synthesis and analysis on the set of consumers and suppliers as well as impossibility of manual optimization of resource assignments per consumers and suppliers. The automation of decision making based on corresponding mathematical models and methods helps to solve those problems.

ICID-2019 Conference

Many researches are dedicated to modelling the process of purchasing resources with suppliers' selection and corresponding inventories. As of rule, those researches consider two autonomous groups of problems:

- Distribution of potentially allowed resources per suppliers based on commercial requirements with accordance to technical restrictions.
- Distribution of existing resources per consumers based on technical requirements with accordance to commercial restrictions.

As an example, within the first group S.H. Amin et al. [2,3] considered the integrated mathematical model with fuzzy parameters for vendor selection in the form of a closed-network configuration. A. Mendoza and J.A. Ventura [4,5] proposed two combined integer non-linear programming models intended to help managers to make reasonable decisions during the process of selecting fixed set of vendors and in process of planning of resource supplement with accordance to commercial criteria. S.A. Moosavi [6] proposed a multipurpose mathematical vendor selection model, presented as a fuzzy linear programming task, which takes into account importance of commercial criteria weights for different resources. Within the second group K. Pavlov and E. Khobotov [7] considered the problem of selection and modernization of equipment for production systems. As a solution authors proposed discrete production models and methods that make it possible to create several production system projects and select the best in terms of technical characteristics. F.T.S. Chan and B. Jiang [8] proposed a set of multi-criteria models and artificial intelligence techniques for production resource parameters selection which maximizes technical efficiency and flexibility.

Therefore, two approaches to solve the problem of selection of production resources were defined: the way of optimization of commercial activities (as choosing resources from concrete suppliers) within restrictions of technical parameters or optimization of technical systems within economic restrictions (as choosing resources for concrete technical conditions).

In the context of realization of schema "many consumers – many suppliers" it looks feasible to combine those two approaches with saving and using their results, for example through defining reasonable set of requirements for commercial and technical characteristics of required resources.

In this paper the approach for formalizing set of rational requirements for commercial and technical characteristics of resources and mathematical model of organizing the centralized procurement with maximum accordance to those requirements are presented.

2 Models and methods of solving the optimal procurement organization problem.

Commercial and technical requirements of resources are proposed to be formulated as vectors of characteristic parameters of homogenous resource $\tilde{z}^1 = (z_1^1, \dots, z_n^1)$, which components are fuzzy logical statements with membership function $f_i(z_i^1) \in [0; 1]$ [9,10]. Membership functions are defined by experts as well as based on information, obtained with results of above-mentioned groups of problems. Summary of parameters specifies set of local (for each parameter) requirements to resources. Resources, offered by supplier, are also described by corresponding parameters vectors $\tilde{z}^2 = (z_1^2, \dots, z_n^2)$. As opposed to requirements, the concrete resource parameters are defined by crisp quantitive or qualitive values. Membership functions of the fuzzy statements define degree of functional matching of the resource type to given local requirements $s_i = f_i(z_i)$.

For defining of the generalized degree of matching of j-type of resource to technical or commercial requirements it's necessary to aggregate all local matchings of those type. There are different well-known methods of such aggregation [11, 12, 13], suitable to solve this problem. In the current paper the aggregation operator for the multiplicative form of composite fuzzy statement for commercial and technical requirements is proposed in the form:

$$\mu_{ij} = \min\{f_1^i(z_1^j); \dots; f_n^i(z_n^j)\} \tag{1}$$

– aggregated matching of j-resource for commercial requirements of i-supplier;

$$\eta_{jk} = \min\{f_1^k(z_1^j); \dots; f_m^k(z_m^j)\} \tag{2}$$

– aggregated matching of j-resource for technical requirements of k-consumer;

The task of purchasing equipment which should meet both commercial and technical requirements is proposed to be formulated as a transportation problem with intermediate points [12]. The fitting criteria are defined as normalized matchings to commercial and technical requirements, which are in general case contradicting. The normalization ensures keeping the values of criterion in the "matching" category.

Formalized balanced mathematical model of the problem is presented as [10]:

$$\frac{\lambda \cdot \sum_i \sum_j \mu_{ij} x_{ij}}{\sum_i \sum_j x_{ij}} + \frac{(1-\lambda) \cdot \sum_j \sum_k \eta_{jk} y_{jk}}{\sum_j \sum_k y_{jk}} \to \max_{x,y}, \tag{3}$$

$$\sum_j y_{jk} = v_k; \forall k, \ \sum_j x_{ij} = a_i; \forall i, \tag{4}$$

$$\sum_i x_{ij} = r_j, \ \sum_i x_{ij} = \sum_k y_{jk}; \forall j, \tag{5}$$

$$x_{ij}; y_{jk} \in \{0; N\}, \tag{6}$$

where $I = \{i\}, i = 1, \dots, n$ – set of suppliers, $J = \{j\}, j = 1, \dots, m$ – set of alternative resources, $K = \{k\}, k = 1, \dots, d$ – set of consumers, $\mu_{ij} \in [0,1]$ – matching value of proposal from i-consumer to commercial requirements from j-resource; $\eta_{jk} \in [0,1]$ – matching value of j-resource to technical requirements of k-consumer; x_{ij}– amount of j-equipment purchased from the i-supplier; y_{jk}– amount of j-equipment purchased for the k-consumer); v_k– amount of equipment required by the k-consumer; a_i– amount of equipment offered by the i- supplier; r_j– total equipment offered by j-supplier; $\lambda \in [0;1]$ – weight factor that determines domination of the requirement.

Sums $\sum_i \sum_j x_{ij} = const$ and $\sum_j \sum_k y_{jk} = const$ stay constants while changing corresponding variables, which identifies the affiliation of the problem with linear programming. If amount of resources is defined as counting value, the model (3-6) should be complemented with restriction $x_{ij}; y_{jk} \in \{0; N\}$ and the task becomes integer valued.

Expressions under the first and second sum in criterion (3) represent traces of the matrixes A $= \|\mu_{ij}\| \cdot \|x_{ji}\|$ and B $= \|\eta_{jk}\| \cdot \|y_{kj}\|$: $tr(A)$ and $tr(B)$ correspondingly. Those elements are interpreted as weighted values of matching distribution of resources for commercial and technical requirements. Matrixes $\|\mu_{ij}\|$ and $\|\eta_{jk}\|$ are formed based on expressions (1,2), which includes expert evaluations. Matrixes $\|x_{ij}\|$ and $\|y_{jk}\|$ are chosen to ensure maximum of a sum $tr(\|\mu_{ij}\| \cdot \|x_{ji}\|) + tr(\|\eta_{jk}\| \cdot \|y_{kj}\|)$, or minimum of aggregated matching.

Centralized procurement can include one of the conditions:

- Ensure equilibrium of commercial and technical requirements;
- Ensure certain degree of dominance of one requirement above another.

The condition of equilibrium of two types of requirements is due to structurally innate into problem (3-6) dominance of the commercial requirements. Such dominance is related to the point that to perform balance conditions we introduced simulated consumer with null matchings η_{jk} and all distributed to it resources give null products $\eta_{jk} y_{jk}$, as opposite to corresponding products $\mu_{ij} x_{ij}$ which are different from zero. Moreover, the more the excess of supply over the demand of consumers (which is typical for the market), the more the dominance of commercial requirements. In this case the deals will be catted with lower correspondence to technical requirements. The equilibrium is reached through introducing in problem (3-6) additional restriction:

$$tr(A) = tr(B). \tag{7}$$

Given degree of dominance of one requirement above another is reached through corresponding choice of parameter λ of linear combination:

$$\lambda \cdot tr(A) = (1 - \lambda) \cdot tr(B); \quad 0 \le \lambda \le 1. \tag{8}$$

Obviously, in this case one should exclude restriction (7) as its presence automatically neutralize the dominance:

$$\alpha \cdot tr(A) + (1 - \alpha) \cdot tr(A) = tr(A). \tag{9}$$

3 Numerical approbation and discussion

Numerical approbation of the proposed model should answer following questions:

- Is it possible to interpret modelling results of resources distribution appropriately?
- Is it reachable to distribute resources with respect to balance between commercial and technical requirements?
- How sensitive is the solution to the disturbance of the balance of requirements?

Numerical approbation method assumes obtaining solution on the set of real numbers.

The task of organization of purchasing is formulated as follows. In the Tables 1 and 2 the initial data are presented: compliance with the technical requirements and compliance with the commercial requirements. As seen from initial dataset, the most acceptable resources for the consumer (C and D) are proposed by suppliers under the worst commercial conditions, compare to A and B.

Table 1. Consumers requests with compliance matrix of the technical requirements.

Resource	Consumers		
	C1	C2	C3
A	0,45	0,7	0,55
B	0,35	0,45	0,65
C	0,95	0,85	1
D	1	1	0,85
Declared amount	10	5	8

Table 2. Suppliers proposals with matrix of the commercial compliances.

Supplier	Resource				Assumed amount
	A	B	C	D	
S1	1	0	0	0	5
S2	0	1	0	0	4
S3	0	0	0,45	0	2
S4	0	0	0	0,3	5
S5	0	1	0	0	15
S6	0	0	0,5	0	4
S7	0	0	0	0,35	5

The optimal distribution of the purchases per suppliers and purchases per consumers stated as (2-5) is shown in the Tables 3 and 4. Quantitative value of resources amount is presented in the numerator while degree of matching the requirements is stated in the denominator. In the Table 4 the simulated consumer for the balancing is introduced (CS).

Table 4 allows to determine from which supplier and how many resources should be purchased (underlined in table 3). So, proposed resource B is redundant (as Table 4 shows that requirement is 2 times less).

Table 3. Optimal distribution of purchases per suppliers (λ=0,5).

	S1	S2	S3	S4	S5	S6	S7
A	5/1	0	0	0	0	0	0
B	0	4/1 2/1	0	0	15/1	0	0
C	0	0	2/0,45	0	0	4/0,5	0
D	0	0	0	5/0,3	0	0	5/0,35

Table 4. Optimal distribution of resources per consumers (λ=0,5).

	A	*B*	*C*	*D*
C1	0	0	0	10/1
C2	5/0,7	0	0	0
C3	0	2/0,65	6/1	0
CS	0	17/0	0	0

Analysis of the Tables 3 and 4 provides reasonable solution interpretation – trade-off between commercial and technical requirements forced to refuse from mainly commercially efficient resource B.

The problem of achieving balance of requirements can be checked with the same test data but using constraint (7). The result solution shows that resource distribution per consumers has not changed, while purchases distribution per suppliers has modified as presented in Table 5.

Table 5. Optimal distribution of purchases per suppliers having requirements equilibrium from table 2

	S1	S2	S3	S4	S5	S6	S7
A	1/1	0	0	0	0	4/0	0
B	0	4/1 2/1	0	0	15/1	0	0
C	0	0	0	5/0	0	0	1/0
D	4/0	0	2/0	0	0	0	4/0,35

Distribution obtained in the Table 5 is unrealizable as contains supplies with zero membership. That means either infeasibility of the resource or its absence on suppliers' side. From Criteria (3) point of view the requirement of using only suitable (or available) resources can be enhanced by switching zero values in the Table 2 to large negative numbers. Practical result is obtained e.g. when changing all zero values on –1000 as shown in Table 6.

It should be observed that there is an overlap between distribution of purchases per suppliers when having the requirements equilibrium (balance) and having $\lambda=0,5$ (Table 3). Such coincidence allows to assume low sensibility of the solution to change

Table 6. Optimal distribution of purchases per suppliers having requirements equilibrium from Table II and -1000 instead of 0

	S1	S2	S3	S4	S5	S6	S7
A	5/1	0	0	0	0	0	0
B	0	4/1 2/1	0	0	15/1	0	0
C	0	0	2/0,45	0	0	4/0,5	0
D	0	0	0	5/0,3	0	0,005/0	4,995/0,35

equilibrium of the requirements. To check this assumption above mentioned problem was sold with $\lambda \in \{0;1\}$.

When $\lambda=0$, i.e. while ignoring commercial requirements, but fulfilling all constraints, the obtained solutions are similar with those presented in Tables 3 and 4 having $\lambda=0,5$. When $\lambda=1$ the technical requirements are ignored. In this case the distribution of purchases per suppliers doesn't change but deviates towards degradation of resource distribution per consumers according to Table 7.

Table 7. Optimal distribution of resources per consumers (λ=1).

	A	*B*	*C*	*D*
C1	0	0	5/0,95	10/1
C2	0	0	0	5/1
C3	0	7/0,65	1/1	0
CS	5/0	17/0	0	0

Obtained result proves the assumption about low sensibility of solutions to change equilibrium of the requirements. At the same time presented quantitative results cannot provide unambiguous answer on actual practical questions related to taking into account dominance of commercial or technical requirements. Research in this direction will be continued based on theoretical principals' analysis.

4 Conclusion

The approach for centralized procurement organization for resources, homogenous in the context of matching contradicting commercial and customers' requirements was proposed. The ability to describe requirements as parameters vector with fuzzy logic statement and apply fuzzy logic for calculation of corresponding fuzzy components of requirements and crisp values of characteristics alternative resources were shown. The distribution model in the form of modified transport problem which maximizes the degree of matching was developed.

Suggested approach allows to set a basis for creation of automated decision support system when performing centralize purchasing processes. It will allow to drastically reduce effort of those processes, improve quality of the decisions and lower the subjective component while selecting alternative variants.

Test example case study has shown distribution resources models' reliability and defined direction for the future research as impact evaluation from some requirements type domination. Obtaining such evaluation is relevant for practical applications working in financial or production environments.

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