About φ-Transformation Graphs as a Tool for Investigations

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Abstract. The mathematical method φ-transformation in which large structures are regarded as a set of small and simple substructures, which may have some common parts that can be identified and amalgamate when constructing or reconstructing an entire structure from a finite number of substructures has presentation here.

Keywords: Graph representations,geometrc and yopological aspects graph theory,expander graph

1. Introduction

Let's dissolve the problem of modeling a complex system in general form and propose a theoretical and graphical approach as a way of thinking with artificial images-structures. In systems modeling theory, there are mathematical methods in which large structures are regarded as a set of small and simple substructures, which may have some common parts that can be identified when constructing or reconstructing an entire structure from a finite number of substructures. The main object of φ-method is creating graph (graph model) obtained as a pair of finite sets: sets of vertices and sets of edges to determine the relationships between structure of vertices as objects. The basic idea of the method φ-transformation can be interpreted as a way to inherit a particular property of substructures throughout the structure, depending on the properties of the connection (identification of given parts of substructures). An example of this is the transformation of basic system programming problems into graph theory problems, with mathematical support for their solution algorithms.

The graph model of a mathematical model of a complex system is presented in the form of an undirected graph G without multiple edges and loops and is studied by studying the structured properties of a graph embedded in a closed surface S of an undirected genus $\gamma(S)$; the graph edges placed on the S will be located at least on the projective plane or the Mobius band glued to the oriented surface and will have no common points except the vertices of the graph G with genus γ(G) and may not be located only on the handles. A graph G is said to be minimal over S ($\gamma(S)$ -no irreducible) if for each proper subgraph H of graph G there is an inequality $\gamma(H) \leq \gamma(S) < \gamma(G)$. A minimal graph over S is called a graph G that decreases $\gamma(G)$ after the edge is removed or the edge is reduced to a point. For sphere S such are K_5 and $K_{3,3}$. The following results can be used for systematic analysis of graph models.

2. Main Definitions and Results

For a graph $\Im$ (obtained as a φ -image $G + St_n(g_0)$ with n vertices of the star $St_n(g_0)$ amalgamate with vertices of the set X having the number of reachability $t_G(X)$ and characteristics $\theta_G(X), \partial\theta_G(X)$, [3-4] the following inequality holds:

$$\gamma(\Im) \leq \gamma(G) + t_G(X) - \theta_G(X) - \partial\theta_G(X) - 1.$$

Was introduced a characteristic at $\theta_G(X)$ is a measure of the cyclic connectivity of 2-cells of set $S_G(X)$ as opposed to $\partial\theta_G(X)$ which characterizes the cyclicity of the set $S_G(X)$.They can be used in the analysis of graph models of linguistic circuits which know that vertex and vertex links have some common property-context and some pairs of vertexes may conflict or contradict each other. To resolve these conflicts, we suggest placing graph models on the surface of another kind without crossing the edges at the inner points. In order to investigate the behavior of a mathematical model of a complex system placed on the orienting surface S, its graph model G without multiple edges and loops is considered. Then it is possible to use the transform method created for graphs to solve modeling problems by splitting into "simpler" submodels with further identification of elements made with predefined properties. So the expansion of model G can be determined by the following transformation:

$$\varphi : (G, St_n(g_0), \sum_{i=1}^{n} (\mathrm{g_i} \underset{\bullet}{+} a_i)) \to (\Im, \{a_i^*\}_{i=1}^{n})$$

where $\{a_i\}_{i=1}^{n}$ is the set X of points of graph G with the number of reachability $t_G(X)$, which is one set for identification and amalgamation, and the other $\{g_i\}_{i=1}^{n}$ is the set of end vertices of the star $St_n(g_0)$ with center g_0. Generalization of the characteristic relating to the cyclic structure of the set X points of the graph G embedded in the surface S. Introduction of a new characteristic that measures the chain structure of the set X of points of graph G on S. This result will be useful in the systematic analysis of both graph models and their topological aspect. which will have common properties at the edges and vertices of the graph model. The solution to our problem is based on the method of graph transformations [1-2], whose founder is M.P. Khomenko, and the concepts he introduced. For the take of completeness, we present the most important part of them.

Definition 1.1. A φ-transformation of space X into X relative homeomorphism $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$ which is the sum $\varphi_0 + \sum_{i=1}^{q} \varphi_i$ of $q+1$ homeomorphisms:

1. $\varphi_0 = \varphi|_{\mathrm{X} \setminus \mathrm{A}} : \mathrm{X} \setminus \mathrm{A} \cong X \setminus A$, φ_0 is a homeomorphism;

2. $\varphi_j : \mathrm{A}_{\mathrm{j}} \to A_j$;

3. $\sum\limits_{j=1}^{q} \varphi_j = \varphi|_{\mathrm{A}}, \sum\limits_{j=1}^{k-1} \varphi_j + \sum\limits_{j=k+1}^{q} \varphi_j \neq \varphi|\,\mathrm{A},\ k_j = 1(1)q$;

4. $\varphi_j = \sum\limits_{i=1}^{d_j} \varphi_{ji};\ \varphi_{ji} = \varphi|_{\mathrm{A}_{ji}} : \mathrm{A}_{ji} \to \bigcup\limits_{i=1}^{d_j} A_{ji};\ d_j \geq 2;\ j = 1(1)q$;

5. $-1 \leq \dim(\mathrm{A}_{ji} \cap \mathrm{A}_{ji''}) \leq \dim(\mathrm{A}_j), i \neq i'', i, i'' = 1(1)q$:

6. $\mathrm{A}_{ji} \neq \mathrm{A}_{ij}, j \neq i, i, j = 1(1)q.$

An important class φ-transformations are φ-transformations satisfying the condition: $\mathrm{A}_{\mathrm{j\,i}} \cap \mathrm{A}_{\mathrm{j\,i'}} = \varnothing$ at $(i \neq i') \cup (j \neq j')$. Then the subspace A is decomposed into the sum q of the subspace systems $\mathrm{A}_{\mathrm{j\,i}}$ homeomorphic to each other within each system. Thus, on the subspace A, the relation R - equivalence is given, i.e. $\mathrm{R} = \sum\limits_{j=1}^{q} \mathrm{R}_{\mathrm{j}}$, moreover $\mathrm{R}_{\mathrm{j}}\left[\mathrm{A}_{\mathrm{j\,i}}\right] = \sum\limits_{i=1}^{d_j} \mathrm{A}_{\mathrm{j\,i}}$. Let $\mathrm{X} = \sum\limits_{r=1}^{m} \mathrm{X}_r$, $X = \sum\limits_{l=1}^{m_0} X_l$, $p_0(\mathrm{X}_r) = p_0(X_l) = 1$ for $l = 1(1)m_0$, $r = 1(1)m$.

Define φ-transformation $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$ in accordance with definition 1.1. We introduce the following characteristics φ-transformation:

$$k_r^{j\,j'} = \left|\left\{\mathrm{A}_{j\,i} \mid \mathrm{A}_{j\,i} \subseteq \mathrm{A}_{j'\,i'} \subseteq \mathrm{X}_r \quad i = 1(1)d_j, i' = 1(1)d_{j'}\right\}\right| \; k_{r\,j} = \sum_{\substack{j'=1 \\ j' \neq j}}^{q} k_r^{j\,j'},$$

$$\mathrm{A}(\varphi_j) = \left\{k^{j\,j'} / \left(k^{j\,j'} \neq 0\right) \wedge \left(\forall j'', j'' \in \{1, 2, \ldots q\}\right)\left[\left(j'' \neq j, j'\right) \Rightarrow \left(\left(k^{j\,j''} = 0\right) \vee \left(k^{j''\,j'} = 0\right)\right)\right]\right\} k^{j\,j'} = \sum_{r=1}^{m} k_r^{j\,j'},$$

$$j \neq j',\ j' = 1(1)q.$$

Possible causes are shown in figure 1.

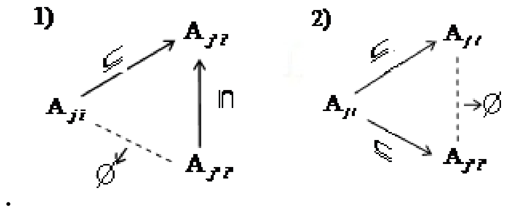


Fig. 1. Possible causes

The set $\mathrm{A}(\phi_j)$ is uniquely defined. We denote k_r^j by a number $k_r^{j\,j}$.

3. Main Three Graphs

Definition 3.1. The φ- base $\mathrm{B}_j = \mathrm{B}(\varphi_j)$ of reflection $\varphi_j : \mathrm{A}_j \to A_j$ with given φ-transformation $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$ is the sum of those components of the subspace X that intersect with the subspace A_j, that is $\mathrm{B}_j = \sum_{r \in J_j} \mathrm{X}_r$, $J_j = \{r / k_r^j > 0\}$.

Definition 2.2. The complex φ-base $B_l = B(X_l)$ over X_l is called the prototype of this component at a given φ -transformation, i.e. $B_l = \varphi^{-1}(X_l)$.

Statement 2.1. If fixed φ-transformation $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$, $J_l = \{j / \mathrm{A}_j \subseteq B_l\}$, $\nu_l = |J_l|$, $l = 1(1)m$, then

1) $B_l = \bigcup_{j \in J_l} B_j$, $l = 1(1)m_0$,
2) $B_l \cap B_{l'} = \varnothing$, $l \neq l'$, $l, l' = 1(1)m_0$,
3) $\sum_{l=1}^{m_0} \nu_l = q$.

Proof of this statement follows from the fact that B_l - the set of components of spaces X "glued" into a component X_l on the subsystem A_j .

Definition 2.3. The graph of the complex φ -base B_l φ - transformation $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$ is called a graph Z_l, $Z_l = (Z_l^0, Z_l^1)$, where $Z_l^0 = \{x_r / \mathrm{X}_r \subseteq B_l\}$ the vertices x_r are joined by edges so that $k_r^j \neq 0$ a tree with a $k_r^j - 1$-loop in x_r for all j, $j = 1(1)q$ is formed on all vertices.

Definition 2.4. The graph φ -base φ -transformation is called a graph $Z(\mathrm{X}, X) = \sum_{l=1}^{m_0} Z_l$.

Statement 2.2. The graph $Z(\mathrm{X}, X)$ is defined uniquely if and only if, when $p_0(B_j) \leq 2$ for $j = 1(1)q$, i.e. we have no more than two connected components that intersect with the system A_j . If $(\mathrm{A}_j = \mathrm{A}_{j1} \cup \mathrm{A}_{j2}) \wedge (d_j = 2)$ for all $j = 1(1)q$, then the graph $Z(\mathrm{X}, X)$ is uniquely defined.

Theorem 2.1. For each graph $Z(\mathrm{X}, X) = Z$ φ -bases φ -transformations $\varphi : (\mathrm{X}, \mathrm{A}) \to (X, A)$ we have:

1) $p_0(Z) = p_0(X)$;
2) $p_1(Z) = \sum_{j=1}^{q} d_j + p_0(X) - p_0(\mathrm{X}) - q$;

In order to ensure that these properties are valid, it is sufficient to calculate

$$\alpha_1(Z(\mathrm{X},X)),\ \alpha_1(Z)=\sum_{j=1}^{q}\left(p_0\left(\mathrm{B}_j\right)-1\right)+\sum_{j=1}^{q}\sum_{r=1}^{m}(k_r^j-1)+\sum_{j=1}^{q}\left|\left\{k_r^j/k_r^j=0\right\}\right|,$$

where $m=p_0(\mathrm{X})$ and use the formula $p_1(Z)=\alpha_1(Z)-\alpha_0(Z)+p_0(Z)$.

Theorem 2.2. The graphs of the φ-bases $Z(\mathrm{X},X)$ are simple (i.e. without multiple edges and loops) if and only if, when $k_r^j\le 1$ and $\left|\left\{\varphi_j/\left(k_r^j\ne 0\right)\wedge\left(k_{r'}^j\ne 0\right)\right\}\right|\le 1$, where $r\ne r',\ r,r'=1(1)m,\ j=1(1)q$. In other words, the graphs $\{Z\}$ are simple if and only if, when:

1) we have only one subspace A_{ji} on each component X_r;

2) there no more than one system $\sum_{i=1}^{d_j}\mathrm{A}_{ji}$ for each pair of such components.

Definition 2.5. The graph φ-transformation $\varphi|B_l:\left(B_l,B_l\cap\mathrm{A}\right)\to\left(X_l,X_l\cap A\right)$ of a complex φ-base B_l at a given φ-transformation of space X is called a graph Λ_l, where

$$\Lambda_l^0=\{x_r/X_r\subseteq B_l\}\cup\{y_j/A_j\subseteq B_l\},$$

$$\Lambda_l^1=\{(k_r^j-k_{r'}^j)(x_r,y_j)/(X_r\subseteq B_l)\wedge(A_j\subseteq B_l)\}\cup$$

$$\{(y_j,y_j)/(k_r^j-k_r^{j'}\in A(\varphi_j))\wedge(A_j\subseteq B_l)\}$$

Definition 2.6. Graph φ-transformation of space X is the graph

$$\Lambda(\mathrm{X},X)=\sum_{l=1}^{m}\Lambda_l$$

Statement 2.3.

1. The arbitrary ϕ-transform graph $\Lambda(\mathrm{X},X)$ is uniquely defined and is simple if and only if, when: $k_r^j-k_{r'}^j\le 1,\ j=1(1)q, r=1(1)m;$

2. There is a connection between $p_1(Z)$ and $p_1(\Lambda)$.

Consider the following example in figures 2,3, where:

$$\mathrm{A}_1=\bigcup_{j=1}^{3}\mathrm{A}_{1j},\ \mathrm{A}_2=\bigcup_{j=1}^{3}\mathrm{A}_{2j},\ \mathrm{A}_3=\bigcup_{j=1}^{2}\mathrm{A}_{3j}.$$

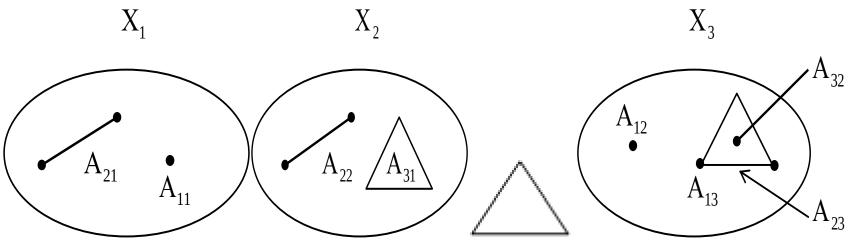
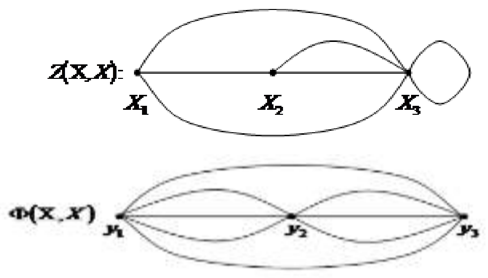
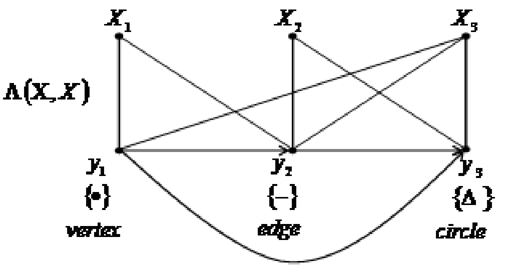


Fig. 2. Consider the following example





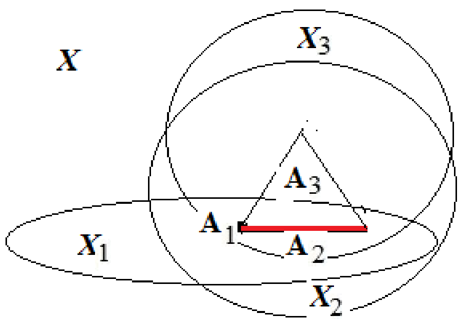


Fig. 3. Consider the following example

4. φ-Transformations for Graphs on Some Surfaces

4.1. Projective Plane

The problem of studying the structure of all minimal non-planar projective graphs is solved by sorting through all the different variants of removing one of the vertices of one of the 35 minors of the projective plane and selecting non isomorphic graphs of nonorientable genus 1. Since [5] does not show the diagrams of these graphs, the construction of all minimal non-planar projective graphs, and in the study of the properties of these subgraphs of the minors of the projective plane relative to the number of reachability of the set of points and the genus of graph.

The solution of this problem is to construct all minimal non-outer projective-planar graphs by sorting out all the different variants of removing one of the vertices of a graph - minor of a projective plane and selecting non isomorphic graphs of nonorientable genus 1. Constructing similarly to how minimally projective non-planar graphs K_5 or $K_{3,3}$ are formed from minimal non-outer planar graphs K_4 or $K_{2,3}$ by gluing a simple star $St(v)$ to the minimum power subset of points of graphs K_4 or $K_{2,3}$ with number reachability equals 2. Main results: theorem 3.1 and diagrams of 118 non-outer projective-planar graphs are given and the numbers of reachability of sets of vertices of minors of a projective plane and sets with points of attachment of a star to subgraphs of these minors are calculated. The full list of thess non-outer projective-planar graphs will be published as soon as possible.

Theorem 3.1. For an arbitrary graph - obstruction G of the projective plane N_1 and each of its vertices v with the set $M(v)$ of all vertices of the incident occur the following statements:

1. For the subgraph $G \setminus v$ of the nonorientable genus, the following relations will take place:

a) If $\gamma(G \setminus v) = 1$, then we have the following relations a1) and one of a2) or a3):

a1) $t_{G\setminus v}(M(v), N_1) = 2$, wherein the set $M(v)$ belongs to the boundaries $\partial s_1, \partial s_2$ of two cells s_1, s_2 of the projective plane having at least one common vertex;

a2) each edge of the subgraph $G \setminus v$ is significant in relation genus $\gamma(G \setminus v)$ with respect to the removing the edge or compressing it in point;

a3) each edge of a subgraph $G \setminus v$ is significant with respect to the removal or compression operations of an edge;

b) If $\gamma(G \setminus v) = 0$ then, one of the following two relationships will occur:

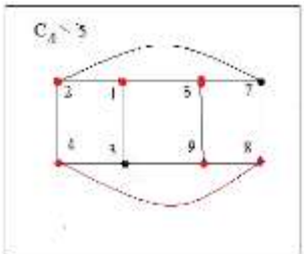
b1) $t_{G\setminus v}(M(v), N_1) = 3$ and the set $M(v)$ is located on the boundaries of three cells s_1, s_2, s_3 of the projective plane satisfying the relation $\partial s_3 \cap \partial s_1 \cap \partial s_2 \neq \varnothing$, each edge of the subgraph $G \setminus v$ being significant relative $t_{G\setminus v}(M(v), N_1)$ to the operations of removing the edge or compressing it to a point, and each point w of the set $M(v)$ satisfies equality $t_{G\setminus v}(M(v) \setminus \{w\}, N_1) = t_{G\setminus v}(M(v), N_1) - 1$;

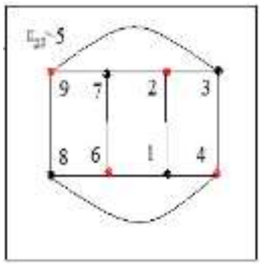
b2) $t_{G\backslash v}(M(v),\Sigma_0)=2$, where $t_{G\backslash v}(M(v),\Sigma_0)$ is the number of reachability of the set $M(v)$ relative to the euclidean plane Σ_0 , is realized by minimal embedding $f:(G\setminus v)\to\Sigma_0$ at the boundaries $\partial s_1,\partial s_2$ of the cells s_1,s_2, where $\{s_1,s_2\}\subset\Sigma_0\setminus f(G\setminus v)$, which satisfies equality $\partial s_1\cap\partial s_2=\varnothing$, that is, separated by a ring from the cells, then relative to the projective plane, the set $M(v)$ will have a number of reachability $t_{G\backslash v}(M(v),N_1)=2$, with each point w of the set $M(v)$ satisfies equality $t_{G\backslash v}(M(v)\setminus\{w\},N_1)=t_{G\backslash v}(M(v),N_1)$ and the set $f(M(v)\setminus\{w\})$ by some embedding $f':G\setminus v\to N_1$ is placed at the boundaries $\partial s_1',\partial s_2'$ of two cells s_1',s_2' having at least one common point where $\{s_1',s_2'\}\subset\Sigma_0\setminus f'(G\setminus v)$, and equality $\partial s_1'\cap\partial s_2'\neq\varnothing$ is satisfied.

2. Each minor G of the nonorientable genus 2 (except G_3,E_1,G_4) is covered by a maximum of 4 (eg, graphs A_2, G_1) subgraphs or parts homeomorphic to one of the following graphs: $K_{2,3}$, K_4, $K_5\setminus e$, $K_{3,3}\setminus e$, K_5, $K_{3,3}$ and relatively Klein surface N_2 the number of reachability 2 for the set of vertices (for $G\in\{G_3,E_1,G_4\}$ we have), and for each removed edge e the graph $G\setminus e$ will have at N_1 the number of reachability equals 2 for the set of vertices;

3. The presence of the coating specified in the statement 2 is not sufficient to make the graph an obstruction of nonorientable genus 2.

4. If $\gamma(G\setminus v)=0$ and on the Euclidean plane Σ_0 made up a set $M(v)$ of points of a graph G formed from the obstruction graph of a projective plane N_1 by removal of a vertex v and adjacent edges is given by an arbitrary minimal embedding $f:G\setminus v\to\Sigma_0$ on the boundaries of two cells that have no common points and have end points that does not belong to their borders. Removing an arbitrary point from the set M leads to the failure of relation 4.





Fig, 4. Illustrates the relation b) of statement 1 of theorem 3.1, where sets {1,2,4,5,8,9} for graph $C_4\setminus 5$, {2,4,6,9} for graph $E_{22}\setminus 5$.

Proof. We prove statements 1 of Theorem 3.1. Suppose that for each vertex v of the graph - obstruction G for a projective plane N_1 with the set $M(v)$ of all vertices incident v, there is a subgraph $G\setminus v$ of a nonorientable genus $\gamma(G\setminus v)$. Then we will either $\gamma(G \setminus v) = 1$ have and $G \setminus v$ contain a subgraph or part homeomorphic K_5, or $K_{3,3}$, or $G \setminus v$ that subgraph does not contain these partial subgraph, where K_5 has two non-isomorphic embeddings in N_1 and $K_{3,3}$ has one non isomorphic embedding in N_1. Prove the relation a1) relation a) statement 1, namely $\gamma(G \setminus v) = 1$, if, then inequality $t_{G\setminus v}(M(v), N_1) = 2$ holds. Using the opposite method, suppose that $t_{G\setminus v}(M(v), N_1) > 2$, that is, the set $M(v)$ is placed by some minimal embedding f of a graph $G \setminus v$ in N_1 the boundaries of at least three cells of the projective plane, namely s_1, s_2, s_3. Let the graph G be the φ-image of the graph $G \setminus v$ and $St_G(v)$, if the pairs of vertices (v_{1i}, v_{2i}) are identified, where $v_{1i} \in M(v)$, $v_{2i} \in St_G{}^0(v) \setminus \{v\}$, $i = 1(1)\deg_G(v)$, To continue embedding $f : G \setminus v \to N_1$ on the graph G, it is necessary and sufficient to attach all the edges of the star and its center to one cell formed of two cells s_1, s_2, where $\{s_1, s_2\} \subseteq N_1 \setminus f(G \setminus v)$ whose boundaries have at least one , the common vertex w, where $w \in G^0 \setminus \{v\} \cap \partial s_1 \cap \partial s_2$, and contain the set $M(v)$. To form a single surface cell from these cells s_1, s_2, we attach on N_1 a Mobius strip L on which we place $f'(N(w))$ by new embedding , $f' : G \setminus v \to N_2$, where $f'(N(w) \subset L$, $f'|_{G^1 \setminus St^1(v) \setminus N(w)} = f|_{G^1 \setminus St^1(v) \setminus N(w)}$, $N(w)$ is the smallest subset of the set of adjacent edges belonging to the boundary of one or to the boundaries of several cells, which on N_1 at least one side separate the cell s_1 from cells s_2, $N(w) = N_1(w, s_1, s_2)$ Note that the insertion of an edge adjacent w to the Mobius strip will be to separate some of the inner points of the edge, which it splits into two parts, and to place its copies on diametrically opposite parts of the circle, and the edges will have endpoints of these copies and the boundary of that edge. As a result, we get a sell s_0 where $\partial s_0 = \partial s_1 \cup \partial s_2$, $\{s_0\} \subseteq N_2 \setminus f'(G^1 \setminus St^1(v))$ in which we put vertex v the center of the star and the subset $St^1(v)$ of its rays of edges that terminate as a bundle of straight segments finished on ∂s_0. Then we will have at least one edge (v,u), where $u \in \partial s_3 \setminus (\partial s_1 \cup \partial s_2)$ there is no investment $f'(v,u)$ in N_2, that is $\gamma(G \setminus (v,u)) = 2$. This contradicts the condition that the graph is an obstruction graph of type 2, the assumption is incorrect. Then the assumption is wrong, we will have equality $t_{G\setminus v}(M(v), N_1) = 2$.

We prove the relation a2) of the statement 1. We give the graph $G \setminus e$ as an φ-image of the graph $(G \setminus v) \setminus e$ and in the identification of pairs of vertices (v_{1i}, v_{2i}), where $v_{1i} \in M(v)$, $v_{2i} \in St_G{}^0(v) \setminus \{v\}$, $i = 1(1)\deg_G(v)$, which satisfies the equality $\gamma(G \setminus e) = 1$, since the graph is an obstruction of nonorientable genus 2. Since $(G \setminus v) \setminus e = (G \setminus e) \setminus v$ and by Theorem 1[5] $\gamma(G \setminus e) \le \gamma((G \setminus v) \setminus e) + t_{(G\setminus v)\setminus e}(M(v), N_1) - 1$, then we will have inequality $\gamma((G \setminus e) \setminus v) + t_{(G\setminus e)\setminus v}(M(v), N_1) \ge 2$, so deleting an arbitrary edge leads either to a decrease of 1 genus of subgraph $(G \setminus e) \setminus v$ and then $\gamma((G \setminus e) \setminus v) = 0$, or to a decrease in the number $t_{(G \setminus e) \setminus v}(M(v), N_1)$ of reachability by 1 and then $t_{(G \setminus e) \setminus v}(M(v), N_1) = 1$. The materiality of the edges of the subgraph relative to the genus upon removal is proved. We will prove other statements similarly and presented proofs as soon as possible.

4.2. Klein surface

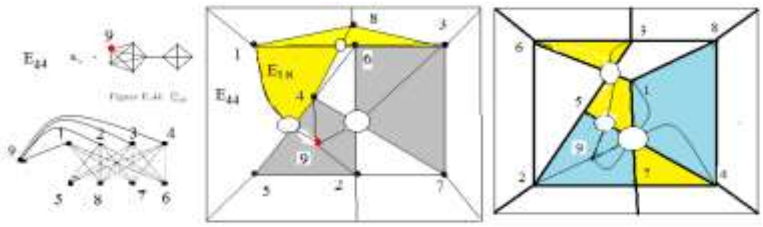
Another problem is constructing all non-outer Klein-planar graphs. In [8] a solution to a similar problem of constructing non-Klein surface graphs by the method of relativistic components was presented.

Theorem 3.2. Each graph obstruction H for N_2-surface of the nonorientable genus 2 satisfies the following statements:

1. An arbitrary edge $u, u = (a, b)$ is placed on the Mobius strip by some minimal embedding of the graph H in N_3 and there is a minimum on inclusion projective-planar subgraph K of the graph or a part of it satisfying the condition: $(t_K(\{a,b\}, N_3) = 1) \wedge (t_{K \setminus u}(\{a,b\}, N_2) = 2)$;

2. There is a finite smallest inclusion set of different subgraphs K_i covering the set of edges of a 2-connected graph H, where K is a local projective-planar subgraph or partial subgraph $H \setminus e$ of a graph, homeomorphic $K_5 \setminus e$ or $K_{3,3} \setminus e$;

3. Every 8-vertex graph - obstruction of non-oriented genus 3 is covered by a minimum of 5 or a maximum of 6 subgraphs or parts of a homeomorphic planar graph with sets of points with reachability 2, or projective-planar, or non-projective-planar graphs (possibly without an edge) from the list of 118 non outer projective planar graphs or set of 103 graphs - obstructions of the projective plane [4].



Fig, 4. φ- transformation of the non-outer projective planar graph E_{18} and $St_4(9)$ give non-outer Klein planar graph E_{44}

5. Conclusions

The use of the φ-transformation of graphs method for the above problems for the projective plane and Klein surface can be generalized to an arbitrary nonrientable surface.

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