Bi-Soft Open Sphere Topology Model of Configuration Space for Reactive Joint Motion Planning of Unmanned Vehicles

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Abstract. This work presents a model of configuration space for the reactive joint motion planning of unmanned vehicles' ensemble. This model is based on a dynamic bi-soft topology, which allows describing the safety conditions of the joint motion of a multitude of vehicles. The approximation space of the topology is constructed in the system of nested open spheres based on an angular coordinate system. The spheres are discretized onto sector-like cells. The proposed bi-soft topology is based on a two-level breakdown of the configuration space. On the first level, the configuration space is broken down onto "free to move" and obstacle subspaces taking into account uncertainties of observations. On the second level, the obtained soft subspaces are broken down onto safety zones with blurred boundaries. The offered topological model can be used in a hybrid motion planner combining potential field and random sampling methods. The use of the proposed bi-soft topology allows reducing the computational complexity of the motion planning task using partitioning of the configuration space and path search heuristics based on the calculation of the cell volume.

Keywords: soft set, soft topology, open sphere, unmanned vehicles, reactive path planning, collision avoidance, safety zones

1 Introduction

The great progress of the latest years in the development of unmanned vehicles (UV) has led to their use in large teams to address a sufficient number of important real-time tasks. UVs belonging to the large teams can be characterized by different sizes, functionalities, roles, motion parameters, and ever environments. For example, forest fire-fighting [1] has recently become an important domain to use the large teams of UVs where unmanned aerial vehicles of various types can provide fire detection, localization, and monitoring missions, while a variety of unmanned ground vehicles (e.g. bulldozers, excavators, tanks, etc.) can be simultaneously involved in fire extinguishing missions. Another example is a smart fishery task, where unmanned aerial vehicles can be involved in searching for fishing flocks, unmanned underwater vehicles can identify fish in flocks, direct and accompany them, as well as unmanned boats can carry fishing gear.

The authors are concerned with the teams of UVs of various types providing different roles and functions while executing their missions jointly and simultaneously to achieve the common objective. Such a team is often called a heterogeneous ensemble due to differences in vehicles' features and their roles in the team [2]. Unlike the widely used concepts of swarms, which bring together a significant number of homogeneous UVs performing the same role, the concept of the ensemble has a crisp structure represented by its spatial and functional configurations taking into account different roles of involved UVs. Regarding the given role, each UV executes a certain scenario of the mission together with other members of the ensemble complementing each other.

However, the missions usually involve a certain number of UVs in the joint simultaneous movement within a confined space, which is open to any other moving objects (manned or unmanned), which may disturb not only the movement of the certain UVs but also the execution of the ensemble mission at all. For example, in the smart fishery mission, the invasion of third-party moving objects (MOs) into the area of mission can require changes in the trajectories of some UVs, which is not an easy task because of the limitations of safe motion, communication distance, fishing gear cover, etc. Furthermore, the change of the trajectories of some members of the fishery ensemble due to safety reasons entails the changes of the trajectories of other members of the ensemble due to requirements on keeping the spatial configuration.

The presented considerations pose the problem of dynamic control of the joint motion of the ensemble members. Thus, using the "sense and avoid" techniques [3], UVs must move to the given target avoiding obstacles and collisions but keeping the predetermined spatial configuration. The more complicated the structure and joint mission of the ensemble, the more difficult the motion control of UVs.

The problem addressed in this paper relates to the coordinated joint motion of heterogeneous UV ensembles. Further, we will consider any obstacles as static objects and any vehicles as moving objects (MO) irrespective of the environment.

2 Literature analysis

The problem of dynamic control of the joint motion of UVs is the subject of sufficient interest of many researchers investigated and reflected in numerous publications. In general, features of the joint ensemble motion differ from the other known formation structures such as swarms, flocks, leader-follower structures, and virtual structures [4] where the entire formation is always considered as a single virtual body [5].

Let us consider joint UV motion within a space C. Executing a given scenario, each UV A_i moves along a pre-planned path P_i represented as a sequence of waypoints (*WP*) or as a sequence of pairs "timepoint-waypoint" (*TP*-*WP*), which define desired spatial locations at the given time moments described by *TP*s. Thus, the path P_i can be almost always represented as a time-arranged sequence

$P_i = \left[(TP_{i1}, WP_{i1}), \ldots (TP_{ij}, WP_{ij}), \ldots (TP_{in}, WP_{in}) \right]$, $TP_{i1} < TP_{ij} < TP_{in}$, which can be conveniently used to control the A_i motion.

In the literature, the most of the suitable approaches to build the path are inspired from global path planning methods, the systematic review of which is presented in [6]: the roadmap approach based on the visibility graph algorithm or Voronoi diagram algorithm, cell decomposition approach, potential field approach, and random sampling approach including probabilistic roadmap method (PRM) and Rapidly Exploring Random Trees (RRT). The last two methods are used most often.

Recent motion planning developments focused on environments that change over time or are not quite known are more tangible to the problem of the joint UV motion. When UV moves, it is exposed to a significant number of both dynamic (e.g., wind) and situational (MOs that disrupting its trajectory) perturbations. When such a perturbation is observed, UV must react and maneuver to avoid the collision. At the same time, it should be worried about keeping both the given spatial configuration (i.e., hold its relative position during the mission execution) and the safe distance from other MOs including the members of the ensemble and various obstacles. Therefore, the initial path P_i can be changed by the displacement of certain WPs forcing other ensemble members to adjust their motion trajectories.

Thus, we need to update the motion paths to adapt them to dynamic changes in the environments. It can be reduced to the dynamic path planning task [7], which can be solved by re-planning some fragments of the path or adjusting its parameters (in our case – locations of WPs). It allows us to obtain a new path ahead of time.

Taking into account the restricted computing capabilities of UVs' on-board computers as well as a lack of time to avoid collisions and obstacles, we should use quick enough methods to re-plan the path P_i. If UV needs to be capable of planning the path of its motion in time to react to dynamic environments as well as situational perturbations, we should provide reactive (real-time) planning immediately during the motion [7]. Thus, the real-time motion planning algorithm could not be iterative.

The potential field method considers obstacles as repulsive fields while targets as attractive fields, so the UV motion is guided to the attractive points avoiding the repulsive points. However, due to the use of iterative optimization techniques this method is so computationally intensive that it is not suitable for reactive collision avoidance in general [7].

Random sampling has emerged as a powerful tool for path planning in high-dimensional configuration spaces, its algorithms are both efficient and simple to implement. However, multi-query planning algorithms (PRM, DRM) have important issues such as a straightforward uniform sampling distribution and the implicit representation of space that need a pre-computation [8]. In contrast to multi-query planning, there is no pre-computation in the single-query algorithms like RRT, which can construct small roadmaps on the fly. However, this algorithm is prone to oversampling while the sampling distribution should be defined explicitly. Unfortunately, random sampling algorithms are not complete in the sense that they cannot consider such a situation when no existing paths are found [6]. Due to the use of lookup tables saving motion primitives, such algorithms are infeasible for resource-limited UVs [9].

To overcome the above-mentioned disadvantages, it is advisable to develop a hybrid method combining RRT and potential field complementing each other. To level the sampling issues, we should develop a model of the configuration space that can reduce the computational complexity by abandoning iterative calculations. A key aspect to combine the RRT and potential field methods is to build an approximate topological model endowed with metric properties to avoid intensive calculations. Well-known geometric approaches based on relative spatial estimates (so-called "point of collision") such as “cones”, “safety domains”, “closest points to approach”, etc. [9-12] can be used to do this.

This work aims to develop the approximate model of the configuration space in the context of the control of the joint motion of unmanned vehicles within confined areas. This model is needed for subsequent implementation of a hybrid reactive path-planning method combined with both RRT and potential field methods. To overcome the computational complexity issue, we must develop a topological model using soft discretization. The developed model allows analyzing big data streams coming from remote sensors and representing them in a user-friendly style.

3 Formal problem statement

Let configuration be a set of k parameters that determine the location of the UV in the space of the joint motion (and, possibly, its motion parameters) uniquely. Each UV can be described by a moving point in a k-dimensional space C called the configuration space. A configuration q is a single point within C. A certain configuration q is free if the UV located at q does not collide with any obstacles or MOs.

A “free to move” subspace $\mathcal{F}$ is a subset of all free configurations in C, while an obstacle subspace $\mathcal{B}$ is a complement of $\mathcal{F}$ to C: $\mathcal{B} = C \setminus \mathcal{F}$ [6].

Planning is always performed within the configuration space C.

The motion planning problem concerning the UV A_i can be defined as finding a path P_i from a start configuration q_s to a target configuration q_t such that P_i lies entirely within the free subspace $\mathcal{F}$. A path P_i is defined by a continuous sequence of configurations [7]. A path planning algorithm is complete if it finds a path whenever one exists and reports none exists otherwise [6].

Reactive motion planning at a time t can be defined as changing the path P_i from a current configuration $q(t)$ to the target configuration q_t whenever a change of the configuration space C takes any fragments of the path P_i beyond the free subspace $\mathcal{F}$ (i.e., a fragment of P_i collides with the obstacle subspace $\mathcal{B}$).

An example is shown in Fig. 1, where UV A_i moving to the target configuration q_t along the path P_i with velocity v meets an obstacle U at current configuration $q(t)$. The collision conditions can be described by a collision cone constructed by dropping tangents r_1, r_2 from $q(t)$ to B representing the safety zone around U.

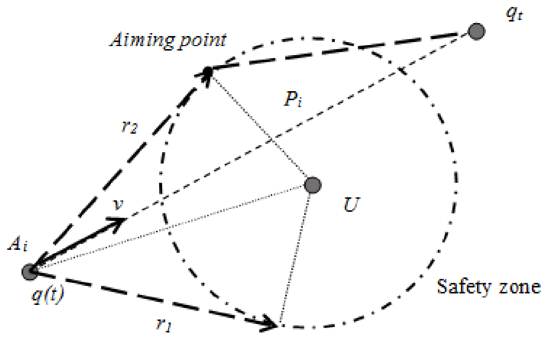

Fig. 1. Reactive motion planning

If the velocity vector v lies within the collision cone, A_i violates B. Decomposition of v in terms of the tangents r_1, r_2 gives us $v = ar_1 + br_2$. If $a > 0$ and $b > 0$, then the obstacle U is critical to A_i. Thus, P_i must be replanned by adding a new waypoint, which should be the nearest aiming point determined on B by tangents r_1, r_2.

4 Building a basic spatial model

Consider a three-dimensional linear uniform space C. Let Y be a set of certain elements, and T be a set of time points t strictly ordered by $<_T$ having the initial count t_0. Let us introduce a norm $\|y\|_c = \min_{t\in[0,T)}(y(t))$ within C, where $y \in Y$, $t \in T$, and a corresponding metric $\xi_C(y_1, y_2) = \|y_1 - y_2\|$ such as:

1. $\xi_C(y_1, y_2) = \|y_1 - y_2\| = 0 \Leftrightarrow y_1 = y_2$;
2. $\xi_C(y_1, y_2) = \|y_1 - y_2\| = \|y_2 - y_1\| = \xi_C(y_2, y_1)$;
3. $\xi_C(y_1, y_2) = \xi_C(y_1 + a, y_2 + a)$;
4. $\xi_C(\lambda y_1, \lambda y_2) = \lambda \cdot \xi_C(y_1, y_2)$.

Define the basis e_1, e_2, e_3 within C so that the metric ξ_C remains uniform. Therefore, the decomposition of a vector $v = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$ gives the coordinates $v(\alpha_1, \alpha_2, \alpha_3)$ of a certain point of the space C describing the position of MO. Currently, we have built a basic continuous spatial model based on the Cartesian coordinate system that needs to be sampled to provide sufficient performance.

Using the metric ξ_C, we impose a metric grid of coordinate lines of size $\delta = \Delta\alpha_1 = \Delta\alpha_2 = \Delta\alpha_3$ onto space C with an origin $\alpha_1 = \alpha_2 = \alpha_3 = 0$, so that the coordinate lines form a set D of isometric cubic cells d of size $\delta \times \delta \times \delta$. Thus, we obtain a space discretized by the grid $D=\{d_{xyz}\}$, in which the cells d_{xyz} are the smallest homogeneous spatial objects, whose coordinates x,y,z correspond to e_1,e_2,e_3.

Although the basic model we obtained is consistent with the information received from the sensors observing the navigation situation, including the rectangular cell coordinate system, however, to represent the safe motion conditions adequately, this model must be adequately transformed.

5 Building an open sphere topology

Let us choose the position of the UV as an observer and endow it with the properties of origin. Let us construct a sphere $V \subseteq C$ with an infinite radius around this position and define the angular coordinate system, as it is shown in Fig. 2.

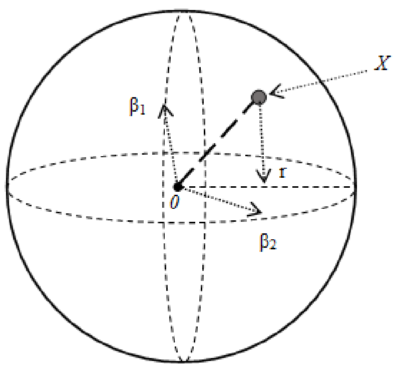

Fig. 2. Angular coordinates on a sphere

Accordingly, the coordinates of UV can be defined as $Crd(A)=(\beta_1,\beta_2,r)$, where β_1 is latitude, β_2 is longitude, and r is a distance from the origin to the UV position.

Let's create a metric ξ_B within the angular coordinate system with the properties similar to ξ_C using an isometric bijection $\chi:\xi_C \to \xi_B$. Obviously, $\chi: v(\alpha_1,\alpha_2,\alpha_3) \to Crd(\beta_1,\beta_2,r)$, so we can transform the coordinates of the observation space C to the angular coordinates in the constructed sphere V.

At the next stage, we construct a discrete sphere model using metric ξ_B and mapping φ, and place an angular grid of coordinate lines with equal angles and equal discrete of the radius $\phi=\Delta\beta_1,\Delta\beta_2,\Delta r$ on the sphere V with the origin point $Crd(0,0,0)$, so that the coordinate lines form a set W of cells w (Fig. 3).

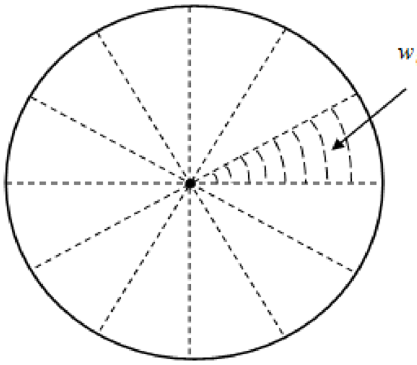

Fig. 3. Discretization of a sphere with infinite radius

Thus, we have got a sampled sphere $W=\{w_{ijk}\}$, whose cells w_{ijk} are the smallest sectors of the sphere V with the angular coordinates i, j, k. The cells w_{ijk} are homogeneous objects concerning their interior.

Obviously, the sampling of a sphere V by a set of cells W constitutes a topological space $\mathcal{T}_V^W=(V, Def(W))$ if the cells of each pair (w_l, w_m) are internally homogeneous and disjoint, that is $\forall l, m \; w_l \cap w_m = \varnothing$, and their union completely covers the sphere V, so that $V=\cup_{w_l \in W} w_l$. The resulting topology $Def(W)$ is nonlinear since the further from the center of the sphere, the larger the volume of each subsequent cell.

6 Building the first level of soft open sphere topology

Let $Y=\{y_i\}_{i=0}^{k}$ be a set of $k+1$ possible states of the cell $w_{ijk} \in W$. For example, the state y_0 corresponds to an "empty" cell containing no objects, the state y_1 corresponds to a cell containing a certain obstacle, the state y_2 corresponds to a cell that is a target for the UV motion, the state y_3 corresponds to a cell containing UV from its ensemble, and the state y_4 corresponds to a cell containing MO considered as "intruder" because it does not belong to the ensemble.

Thus, a subset of states $\{y_1, y_2, y_3, y_4\} \in Y$ attributes the appropriate cells to the category of "occupied", as well as $y_0 \in Y$ attributes the cell to the category of "free to move". Correspondingly, the set of cells having the state y_0 constitutes the sampling of "free to move" subspace $\mathcal{F}$ of configuration space C, while the set of cells having the state $\{y_1, y_2, y_3, y_4\}$ constitutes the sampling of the obstacle subspace $\mathcal{B}$ of configuration space C.

Furthermore, the set of cells having the state y_2 corresponds to an attractive manifold in terms of potential fields while the set of cells having the state $\{y_1, y_3, y_4\}$ corresponds to a repulsive manifold respectively. Since the state of a cell depends on time, each above-mentioned is dynamic, consequently, the partitioning of configuration space C onto $\mathcal{F}$ and $\mathcal{B}$ subspaces is also dynamic.

Assume that the set W is a universe and the set Y is a set of parameters. A pair (Υ, Y) constitutes a soft set of cells if Υ is a certain mapping Y into the set of all subsets of the set W, $\Upsilon : y_i \to 2^W$ [14]. In other words, the soft set is a parameterized family of subsets of the set of cells W. A certain set (Υ, y_i), $y_i \in Y$ from such family can be regarded as the set of y_i-approximated elements of the soft set [15], or, in other words, as an y_i-element of the soft set denoted by Υ_i.

Thus, the universe W can be partitioned by the soft set (Υ, Y) that is the union of all k of its y_i-elements, where $\Upsilon = \cup \{\Upsilon_i\}_{i=1}^{k}$, which constitute a multitude of pairs $\Upsilon_i = \{(\Upsilon, y_i) : y_i \in Y, (\Upsilon, y_i) \in 2^W\}$. The soft set is often associated with the set of equivalence classes induced by a certain indiscernibility relation that depends on time [16]. We can define such relation of y_i-indiscernibility dynamically given on the set of cells W by $(\forall y_i \in Y)\ \Re_W^{y_i}(t) = \{(w_m, w_n) \in W \times W \mid y_i(w_m, t) = y_i(w_n, t)\}$.

Therefore, each y_i-element of the soft set Υ_i provides the partitioning of the set of cells W into equivalence classes given by the y_i-indivisibility relation $\Re_W^{y_i}(t)$ at the moment t. In other words, the parameterized family of subsets of the set W, which constitutes the y_i-element of the set Υ_i, can be considered as a factor-set $W / \Re_W^{y_i}(t)$ consisting of all equivalence classes of W induced by relation $\Re_W^{y_i}(t)$. Thus, the pair $apr_W = (W, \Re_W^{y_i}(t))$ defines the dynamic approximation space [17].

Let $\varnothing$ be an empty set, W be the universe, and elements $W / \Re_W^{y_i}(t)$ are elementary sets so that the finite union of one or more of elementary sets is a compound set. If the family of all compound sets is $Def(apr_W)$, then the dynamic approximation space uniquely defines the dynamic topological space $\mathcal{T}_W^{\Re_W^{y_i}}(t) = (W, Def(apr_W))$.

It is known that $Def(apr_W)$ is a topology on W if its subsets satisfy the following conditions for any $A, B \in W$ [18]:

1. $\varnothing \in Def(apr_W)$, $A \in Def(apr_W)$;
2. $A, B \in Def(apr_W) \Rightarrow A \cap B \in Def(apr_W)$;
3. $A, B \in Def(apr_W) \Rightarrow A \cup B \in Def(apr_W)$.

Consequently, $Def(apr_W)$ is the family of open sets and $\mathcal{T}_W^{\Re_W^{y_i}}(t) = (W, Def(apr_W))$ is the dynamic topological space. The mapping $\Upsilon(t)$ in the given interpretation is also dynamic and uniquely assigns each cell of the universe W to a certain y_i-element of the soft set $\left(\Upsilon(t),Y\right)$ at the time moment t. If the results of the observation are uncertain, we can interpret them by "degrees of confidence" that the cells are "free to move" or contain obstacles at the moment of consideration. For this purpose, the function $\Upsilon(t)$ can be represented as a fuzzy set, so that $\tilde{\Upsilon}(t): y_i \to [0,1]$, where the degree of confidence has a range of values in the interval $[0,1]$. Accordingly, we need to soften the relation $\Re_W^{y_i}(t)$ using a tolerance relation $\tilde{\Re}_W^{y_i}(t)$ instead of the equivalence one. Thus, we have got a dynamic fuzzy soft set of cells $\left(\tilde{\Upsilon}(t),Y\right)$ instead of the soft set [19]. Specifying a certain threshold $\tau \in [0,1]$, we will be able to cut off from the consideration all those cells $w \in W$, for which the degree of confidence at the moment t is lower than a given threshold τ, i.e. $\Upsilon_\tau(y_i,t) = \left\{w \in W : \left(\tilde{\Upsilon}, y_i\right)(w) \geq \tau\right\}$ [20]. Each y_i-element $\left(\Upsilon_\tau(t), y_i\right)$ of the corresponding fuzzy soft set consists of only those cells $w \in W$, for which the degree of confidence that their state is $y_i \in Y$, is greater than the threshold τ at the moment t. As a result, we obtain the approximation space $apr_W = \left(W, \tilde{\Re}_W^{y_i}(t)\right)$ and the dynamic fuzzy soft topology $\mathcal{T}_S(t)$ that describe configuration space C at the moment t.

7 Safety zones assessment

Safety domains usually defined around obstacles and moving objects significantly affect the solution of the reactive motion planning problem.

Let $\rho = \{\rho_0, \dots \rho_m\}$ be a set of distance limits such that $\xi_B(A,B) = \left\|Crd(A) - Crd(B)\right\| \to \rho_i$ for each pair (A,B) of moving objects.

Let $þ = \{ю_0, ю_1, ю_2\}$ be a set of time limits and ξ_T be a metric given on T as $\left\|t_i - t_j\right\|_T \to þ$ with the following properties $\forall t_i, t_j, t_k \in T$:

1. $\xi_T\left(t_i,t_j\right) = 0 \Leftrightarrow t_i = t_j$;
2. $\xi_T\left(t_i,t_j\right) = \xi_T\left(t_j,t_i\right)$;
3. $\xi_T\left(t_i,t_k\right)$„ $\xi_T\left(t_i,t_j\right) + \xi_T\left(t_j,t_k\right)$.

In the case of the stationary obstacle, the assessment of the safety motion based on the collision cone is illustrated in Fig. 1 (Section 3). In the case of two moving objects, the assessment becomes more complex and usually is based on evaluation of the closest approach point (Fig. 4) concerning the time T_{CPA} and distance D_{CPA} [9]. It is possible to evaluate the potential collision based on time T_{CPA} and distance D_{CPA} to the closest approach point using both metrics ξ_B and ξ_T, for example, if $\xi_B(D_{CPA}, A_1) < \rho_m$ and $\xi_T(T_{CPA}, D_1 / V_1) \leq ю_1$, there is a threat of collision.

If $\xi_B\left(D_{CPA}, A_i\right) < \rho_{mj}$, we assume that A_i interacts with A_j. It interacts with A_j dangerously with danger degree k, if $\xi_B\left(D_{CPA}, A_i\right) < \rho_{kj}$, $k > 0$, and $\xi_T\left(T_{CPA}, D_1 / V_1\right) \leq \text{ю}_2$. Furthermore, A_i interacts with A_j critically, if $\xi_B\left(D_{CPA}, A_i\right) < \rho_{0j}$ and $\xi_T\left(T_{CPA}, D_1 / V_1\right) \leq \text{ю}_1$. Finally, the motion of the A_i is forbidden if $\xi_T\left(T_{CPA}, D_1 / V_1\right) \leq \text{ю}_0$, so A_i must turn off immediately.

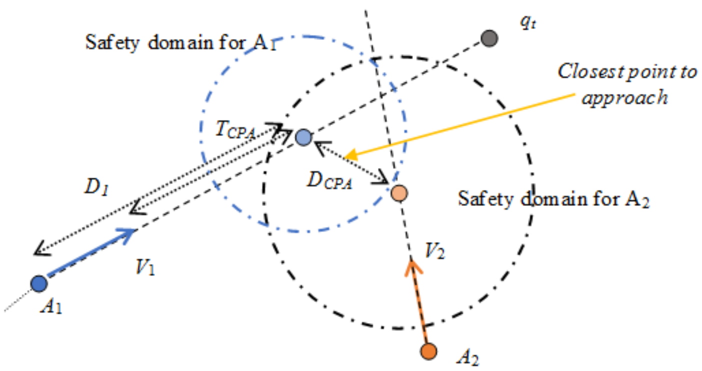

Fig. 4. Safety Assessment based on the Closest Point to Approach

It should be noted that such interactions are not always symmetric. In general, it depends on the vehicle safety domains, which usually are calculated based on its course and velocity, as well as several environmental factors such as weather [13]. Thus, in Fig. 4 vehicle A_1 interacts with the vehicle A_2 dangerously while the vehicle A_2 interacts with the vehicle A_1 not dangerously due to the difference in the size of their determined safety domains.

To take into account spatial zones with different danger levels related to the safety domain of the vehicle, we can define a set of safety assessments $\lambda = \{\lambda_0, \ldots \lambda_k, \ldots \lambda_n\}$ and assessment function $\gamma(A, B) \rightarrow \lambda_i$ that returns safety assessment λ_i for a vehicle A to the moving object B using both metrics ξ_B and ξ_T, as well as respective courses and velocities of A and B.

Based on the set of safety assessments λ, we can restrict a discretized sphere W by representing it as a system of $m+1$ nested spheres $W_n, \ldots W_0$ having a common center at the origin and corresponding radiuses from λ_m to λ_0. Using such a system of nested spheres, we will be able to take into account certain spatial zones, which are determined by the safety conditions of the UV joint motion. For example, based on the UV motion parameters with respect to the conditions of collision avoidance, there can be defined forbidden h_A, dangerous h_B, restricted h_C, and free to move (unlimited) h_D zones bounded by corresponding spheres with boundary lines B_3, B_2, B_1 that meet the assessment levels $\lambda_3, \lambda_2, \lambda_1, \lambda_0$ respectively (Fig. 5) [21].

8 Building a second level of soft open sphere topology

At the first level, we have parameterized the y_i-subset family of the set of cells W, which constitutes the soft set of cells $(\Upsilon(t),Y)$ (or possibly fuzzy soft one $(\tilde{\Upsilon}(t),Y)$) related to the partitioning the cells into equivalence classes by their category.

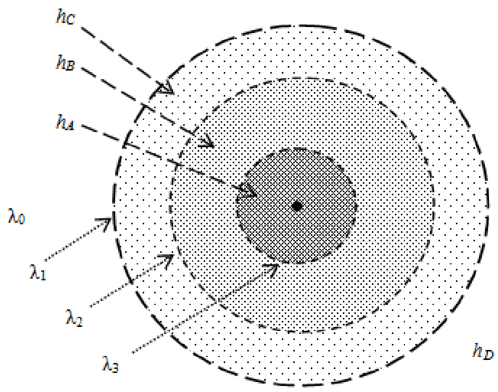

Fig. 5. Definition of safety zones

At the second level, it is necessary to break down each y_i-subset of the set W into safety zones according to the constructed system of spheres having radiuses $\lambda_0,\ldots\lambda_n$ to take into account the distribution of cells between the corresponding safety zones of the nested spheres $W_n,\ldots W_0$. Since the safety assessments for interacting vehicles can differ due to the differences in the size of their safety domains, there is no possibility to break down the entire set of cells W into safety zones. Thus, we need to partition each y_i-subset of W onto each $\lambda_j \in \lambda$ separately.

Let $(\Upsilon(t),Y)$ be a soft set on W such that $\Upsilon(t) = \cup\{\Upsilon_i(t)\}_{i=1}^{k}$, which consists of the union of all k of its y_i-elements $\Upsilon_i(t) = \{(\Upsilon(t),y_i) : y_i \in Y, (\Upsilon(t),y_i) \in 2^W\}$.

Let $\Re_{\Upsilon_i(t)}^{\lambda_j}$ be an λ_j-indiscernibility relation given on the y_i-element $\Upsilon_i(t)$ of the soft set $(\Upsilon(t),Y)$, such that $(\forall\lambda_j)_{i=0}^{m}\ \Re_{\Upsilon_i(t)}^{\lambda_j} = \{(w_m,w_n) \in W_j \times W_j \mid w_m \in \Upsilon_i(t), w_n \in \Upsilon_i(t)\}$. Thus, we can construct an λ_j-approximation of each y_i-element of the set $(\Upsilon(t),Y)$. Therefore, each λ_j-element of the soft set $(\Upsilon_i(t),Y)$ provides the partitioning of the y_i-element of the set $(\Upsilon(t),Y)$ into correspondent equivalence classes given by the λ_j-indivisibility relation $\Re_{\Upsilon_i(t)}^{\lambda_j}$ at the moment t. In other words, we parameterize first the family of subsets of the set W, which constitutes the y_i-elements of the set $\Upsilon_i(t)$, then we parameterize the family of subsets of each set $(\Upsilon_i(t),Y)$, which constitutes the λ_j-elements of the set $\Upsilon_i(t)$. Just like the pair $apr_W = \left(W, \Re_W^{y_i}(t)\right)$ defines the dynamic approximation space at the first level, the pair $apr_{\Upsilon_i(t)} = \left(\Upsilon_i(t), \Re_{\Upsilon_i(t)}^{\lambda_j}\right)$ defines the dynamic approximation space at the second level.

Thus, we obtain a two-leveled soft set of cells $\left(\left(\Upsilon(t), Y\right), \lambda\right)$ also named as a bi-soft set. The correspondent dynamic bi-soft topology $\widehat{\mathcal{T}}(t) = \left(apr_W, Def\left(apr_{\Upsilon_i(t)}\right)\right)$ is the partition of y_i-elements of the soft set of cells $\left(\Upsilon(t), Y\right)$ into approximated subsets of cells $\Upsilon_{ji}(t)$, which are λ_j-elements of the y_i-elements of the set $\left(\left(\Upsilon(t), Y\right), \lambda\right)$. Clearly, $\widehat{\mathcal{T}}(t) = \left(\left(W, \Re_W^{y_i}(t)\right), Def\left(\Upsilon_i(t), \Re_{\Upsilon_i(t)}^{\lambda_j}\right)\right)$.

Each element Υ_{ji} forms a soft topology $\mathcal{T}_{ji}(t)$. Thus, each λ_j-element is represented by the soft set of cells $\left(\Upsilon(t), Y\right)$ contained in the corresponding sphere V_j. The sphere is broken down in turn into y_i-elements of the set $\left(\left(\Upsilon(t), \lambda\right), Y\right)$, each of which contains the cells belonging to a certain category $y_i \in Y$. The dynamic bi-soft topology that defines configuration space C can be represented as $\mathcal{T}_S = \cup_{j=0}^{m}\left(\cup_{i=0}^{k} \mathcal{T}_{jiS}\right)$. In the case of the fuzzy soft set of cells $\left(\tilde{\Upsilon}(t), Y\right)$ be a carrier set for the second level of topology, we have got the dynamic fuzzy bi-soft set of cells $\left(\left(\tilde{\Upsilon}(t), Y\right), \lambda\right)$ and the corresponding topology.

9 Implementation of the model

The proposed model has been implemented in the reactive path planning module within UV onboard control system prototype based on embedded microcontroller STM32F429 (180 MHz Cortex M4, 2Mb Flash/256Kb RAM internal, QSPI Flash N25Q512). Information about the navigational situation comes from onboard sensors while information about team members and their targets is received from the radio communication channel.

Using the proposed model, a reactive planner transforms coordinates of all observed objects into the angular coordinate system within the configuration space. Then it creates an open sphere for each observed vehicle and discretizes it separately. Using the safety domains constructed for each observed vehicle, the planner creates the open sphere bi-soft topology. Since the environment can change, each observed event updates the partitioning of the configuration space to distinguish “free to move” subspace, in which the planner search for the next fragments of the path if a potential collision has been detected or the spatial configuration of the UV ensemble has violated.

The planner superposes open sphere topologies for each vehicle and eliminates all insignificant subsets of the cell. The set of target cells having the state equal to y_2 is considered as attractive manifold while the set of “occupied” cells having the state equal to y_1, y_3, or y_4 is considered as repulsive manifold (Fig. 6).

Then the search for a path fragment to the next waypoint is provided using the potential fields-based algorithm, which builds a path choosing the cells of the constructed bi-soft topology that have the highest degree of safety motion.

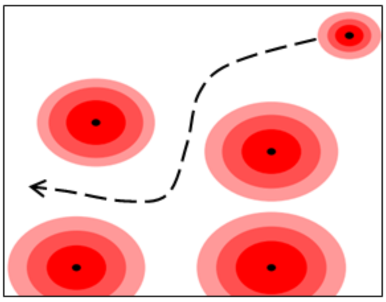

Fig. 6. Search for a path using potential fields

Finally, the path fragments will be transformed back to the cartesian coordinate system. The search algorithm is driven by the heuristic based on the fact that the greater the volume of the cells the farther the cell from the center of the sphere. It allows reducing the computational complexity essentially.

The efficiency of the proposed model has been examined compared to using the ordinary cartesian spatial model during the computer simulation based on the UV onboard control system and GIS-based model of the real terrain having the square 4 km2. During the simulation, the number of vehicles has been varied from 10 to 100. The total time of the path search has been evaluated, the results are shown in Fig. 7.

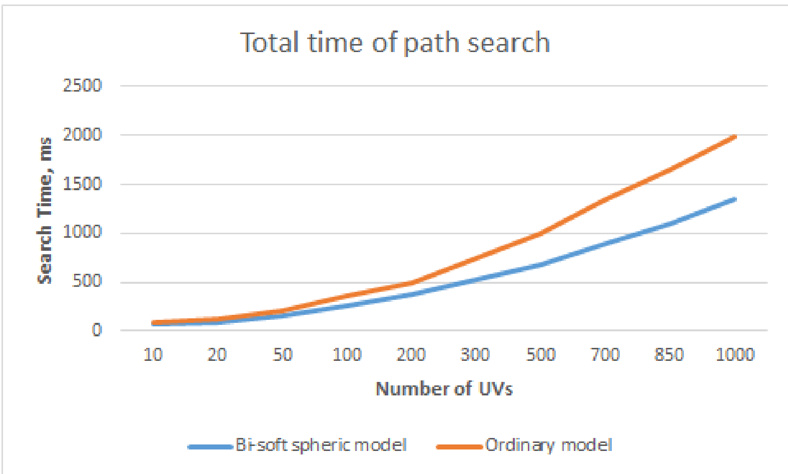

Fig. 7. The results of the simulation

The results of the experiment show that the proposed bi-soft topological model provides acceptable performance in terms of the path search time, its efficiency is about 45 percent higher than the efficiency of the ordinary model.

10 Conclusions

The problem of modeling of configuration space for the reactive joint motion planning of unmanned vehicles' ensemble is addressed in the paper.

The proposed model is based on a novel dynamic bi-soft topology, which allows describing the safety conditions of the joint motion of a multitude of vehicles. The two-leveled approximation space of this topology is constructed within the system of nested open spheres based on an angular coordinate system. The spheres are sampled into sector-like cells. The first level of the soft topology breaks down the configuration space onto "free to move" and obstacle subspaces represented by soft sets of cells taking into account uncertainties of observations. The second level of the soft topology breaks down the determined soft subspaces into safety zones also represented as soft sets. As a result, the bi-soft (fuzzy) set of cells is defined and corresponding dynamic (fuzzy) bi-soft topology is constructed.

The proposed topological model can be used in a hybrid motion planner combining potential field and RRT methods. The use of the proposed bi-soft topology allows reducing the computational complexity of the motion planning task using sophisticated partitioning of the configuration space as well as path search heuristics based on the calculation of the cell volume. The computer simulation has shown that the proposed model provides enough performance, its efficiency is about 45 percent higher than the efficiency of the ordinary cartesian model.

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